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THE HADRONIC PHYSICS OF PHOTON-PHOTON COLLISIONS*

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I. Introduction

Although the first observations of the process ee → ee + hadrons have only just now been reported[1,2], the anticipation that definitive measurements of photon-photon cross sections $\sigma(\gamma\gamma \rightarrow \text{hadrons})$ will be possible has stimulated a great deal of theoretical interest.[3-5] Photon-photon physics is enormously interesting and fundamental: it allows the study of the structure and electromagnetic interactions of hadrons via a double electromagnetic probe. In principle, we can even independently "tune" the mass and polarization of both independent particles.

For the dominant part of the ee → ee + hadrons cross section, where both leptons scatter in the near forward direction, the equivalent photons are nearly on-shell and the hadronic aspects of their interactions are expected to dominate. In this case we have the analogue of meson-meson colliding beams. In addition, however, there can be new interactions and effects specific to local two photon interactions. Thus measurements of the total cross section $\sigma_{\gamma\gamma \rightarrow \text{had}}(s)$ are certainly of comparable importance to the total photoabsorption cross section and other total hadronic cross sections. Measurements of $\gamma\gamma$ cross sections as a function of s can determine the leading C = + hadronic states or resonances, and test fundamental ideas of duality and Regge theory. Measurements of the specific channel $\gamma\gamma \rightarrow \pi\pi$ allow a unique probe of the pion Compton amplitude and can determine the $\pi\pi$ scattering lengths and phase shifts. The behavior of this amplitude at large s and t is an important test of scaling laws and fixed pole behavior predicted by quark-parton models.

If a scattered lepton in ee $\rightarrow$ ee + hadron is detected at large angles then we can probe the $\gamma\gamma \rightarrow$ hadron amplitudes with one or both photons off the mass shell. Such measurements can provide critical tests of the scaling and short distance behavior predicted in the quark-parton and light-cone model. A review of these tests, including deep inelastic scattering on a photon target, are reviewed in Walsh's contribution to this conference.[4]

In this talk I will review the theoretical predictions for $\gamma\gamma \rightarrow$ hadrons for the case of (nearly) on-shell photons, and briefly discuss the off-shell behavior for some specific hadronic channels. A review of the applications of current algebra and soft pion theories to various exclusive $\gamma\gamma$ cross sections is presented in Terazawa's contribution to this conference.[5]

II. The Total Photon-Photon Annihilation Cross Section

Perhaps the most basic hadronic physics measurement accessible via the two photon process is the total $\gamma\gamma$ annihilation cross section into hadrons. We will consider here a double-tagged measurement of ee $\rightarrow$ ee + hadron where both leptons are measured in the forward direction, $\theta_{1,2}^{max^2} << 1$, and at least one hadron is detected. Then, to a very high accuracy we can relate σ(ee $\rightarrow$ ee + had) to $\sigma(\gamma\gamma \rightarrow$ had) with nearly real photons using the formula[3,6,7]

$$\frac{d\sigma(\text{ee} \rightarrow \text{ee} + \text{had})}{ds} = \frac{\sigma_{\gamma\gamma \rightarrow \text{had}}(s)}{2s} f\left(\frac{s}{s_0}\right) N_1 N_2$$

where

$$f(x) = \tfrac{1}{2}(2+x)^2 \log\frac{1}{x} - (3+x)(1-x)$$

is a phase-space factor obtained from integrating over the momentum $\omega_1 - \omega_2$ of the $\gamma\gamma$-center of mass, $s = 4\omega_1\omega_2$ is the square of the center of mass energy in the quasi-real $\gamma\gamma$-collision, $s_0 = 4E^2$, and N_1 and N_2 are the equivalent photon spectra obtained by integrating over the scattered lepton angles[7]; e.g. for $m_e^2/E^2 \ll \theta_{min}^2 < \theta_{max}^2$

$$N(\omega, \theta_{min}, \theta_{max}) \cong \frac{\alpha}{\pi}\left[\frac{E^2 + (E-\omega)^2}{E^2} \log \frac{\theta_{max}}{\theta_{min}}\right] .$$

More accurate expressions are discussed in the literature.[7-10] Further, by measuring the scattering planes of the leptons, we can even separate $\sigma_{\gamma\gamma \to had}$ for parallel and perpendicular linearly-polarized quasi-real transverse photons. The corrections terms due to longitudinal currents and $C = -1$ hadron state contributions to $\sigma_{ee \to ee + had}$ only contribute to $O(\theta_{max}^2)$. (The photon mass and longitudinal current contributions vanish to order k^2/M_H^2 and k^2/E_γ^2.[9]) Thus to a very good approximation, we can literally convert an electron-positron or electron-electron storage ring into a facility for the collision of two real photons. In the case where the leptons are not detected, we can use $N \sim \frac{2\alpha}{\pi}\left[\log \frac{E}{m_e} - \frac{1}{2}\right]$, or, e.g., Eq. (A.7) of Ref. 7, but the accuracy of this approximation (usually better than 30%) depends on the process, and the importance of these other contributions. Although somewhat harder to interpret theoretically, measurements of $\sigma_{e^-e^- \to e^-e^- had}$ without lepton detection as a function of $s_0 = 4E^2$ will also be enormously interesting.

The tools of hadronic physics, specifically, our knowledge of Regge behavior, duality, and hadronic symmetries already give us a clear picture of

what to expect for the total $\gamma\gamma$ cross section. Here I will follow a very useful discussion given by Rosner[10]. In a conventional Regge pole model, $\sigma_{\gamma\gamma \to \text{had}}$ is given by a constant (Pomeranchuk) part plus a part decreasing with s due to the non-leading f and A_2 trajectories

$$\sigma_{\gamma\gamma \to \text{had}}(s) = \sigma_0 + \sigma_1 / s^{\frac{1}{2}} .$$

The factorization of the Pomeranchuk trajectory implies[7,11]

$$\sigma_0 = \frac{\left[\sigma_T^{\infty}(\gamma N)\right]^2}{\left[\sigma_T^{\infty}(NN)\right]} = \frac{(95\,\mu b)^2}{37\,\text{mb}} = 0.24\ \mu b$$

and one can estimate[10]

$$\sigma_1 = 0.27\ \mu b\ \text{GeV}$$

using exchange degeneracy and the couplings of the non-leading trajectories to p, n, and γ in other processes.

By duality, we can also identify the non-Pomeranchuk component of $\sigma_{\gamma\gamma \to \text{had}}$ with the average effects of the direct channel resonances. Comparing the magnitude of σ_0 and σ_1, we can construct the following table[10] for $\sigma_{ee \to ee\ \text{had}}$ and the resonance/background ratio

Range of $\sqrt{s}$, GeV	Resonance/Background	$\sigma^{TOT}_{ee \to ee\ had}$ (E = 3 GeV)	(E = 15 MeV)
0.3 to 1	2 to 1	11.0 nb	30.0 nb
1 to 2	1 to 1	1.7 nb	6.9 nb
2 to 6	1 to 3	0.6 nb	5.1 nb
6 to 30	1 to 5	—	1.8 nb

[Here $\sigma^{TOT}_{ee \to ee\,had}$ is constructed by integrating over $4m_\pi^2 < s < 4E^2$ and using $N = \frac{2\alpha}{\pi} \log \frac{E}{m_e}$.]

From this table it is clear that the region $\sqrt{s} < 1$ GeV should be accessible to measurements with the present storage rings; for comparison, we note that the CEA measurement at 2.5 GeV of the $e^+e^- \to$ hadron cross section is 22 ± 5 nb. Note that because of the various experimental cuts and biases on angle, minimum energy, etc., the two photon cross section could be constrained to be small background of the total hadronic sections measured at CEA.[12]

Figure 1 gives a simplified representation of Rosner's predictions for $\sigma_{\gamma\gamma \to had}$. The possible C = + hadronic resonances predicted by the quark model for $q\bar{q}$ ($L \leq 3$), which modulate the decreasing component are[10]:

Final State	Name	Mass	$^{2s+1}L_J(q\bar{q})$	$^{I}G_J{}^{P}$
$\pi^+\pi^-$	ϵ or σ	750	$^3P_0$	0^+0^+
	f_0	1260	$^3P_2$	0^+2^+
	$[\epsilon^*]$	1600 to 1800	$^3F_2$	0^+2^+
	$[f_0^*]$	1900	$^3F_4$	0^+4^+
$\pi^+\pi^-\pi^0$	η	550	$^1S_0$	0^+0^-
	A_2	1300	$^3P_2$	1^-2^+
	π_A	1650	$^1D_2$	1^-2^-
	$[\delta^*]$	1700 to 1800	$^3F_2$	1^-2^+
	$[A_1^*]$	1800	$^3F_3$	1^-3^+
	$[A_2^*]$	1900	$^3F_4$	1^-4^+

All of these resonances with natural $J^P = 0^+, 2^+, \ldots$ will appear in the K^+K^- final state, as well as the s*, f' and their recurrences. The K^+K^- final state can also be used to study the f_0-A_2 interference, to compare with SU(3) predictions for the decay amplitudes. The resonances seem to be fairly well separated implying that an important resonance study may be possible just from the energy dependence of $\sigma_{\gamma\gamma \to \text{had}}$.

Other estimates of the hadronic channels can be given using a variety of theoretical techniques. For example, Walsh[4] simply uses the measured radiative width $\Gamma(f_0 \to \gamma\gamma) \sim 4$ keV plus integration over the resonance width to predict via duality a result in good agreement with the σ_1 estimate given by Rosner. Similarly, a finite energy sum rule approach using the $\omega\pi\gamma$ coupling leads to similar results. The analysis of Schrempp-Otto, Schrempp, and Walsh[5] based on $\Gamma(f \to \gamma\gamma) = 4$ keV, $\Gamma_f = 150$ MeV gives $\sigma(ee \to eef) \sim 0.3$ nb at E = 3 GeV and $\Gamma(\epsilon \to \gamma\gamma) \sim 15$ keV, $\sigma(ee \to ee\epsilon) \sim 2$ nb at E = 3 GeV for $\Gamma_\epsilon \sim 400$ MeV. Also, we can estimate using vector meson dominance $\sigma_{\gamma\gamma \to \rho p} \cong (e/2\gamma_\rho)^2 \sigma_{\rho p \to \rho\rho} \sim (1/200)^2\, 10\,\text{mb} \sim 0.1\,\mu\text{b}$, to get[7] $\sigma_{ee \to ee\rho\rho \to ee\pi\pi\pi\pi} \sim 0.2$ nb at E = 2 GeV. Further, using Low's formula[6] we have for the narrow resonances[7]:

$$\sigma_{ee \to ee\pi^0} \sim 1\,\text{nb}\ (E \gtrsim 2\ \text{GeV})\ \text{assuming}\ \Gamma_{\pi^0 \to \gamma\gamma} \sim 9 \pm 2\ \text{eV}$$

$$\sigma_{ee \to ee\eta^0} \sim 1\,\text{nb}\ (E \gtrsim 2\ \text{GeV})\ \text{assuming}\ \Gamma_{\eta^0 \to \gamma\gamma} \sim 1\ \text{keV}$$

$$\sigma_{ee \to ee\eta^{0\prime}(960)} \sim 1\,\text{nb}\ (E \gtrsim 3\ \text{GeV})\ \text{assuming}\ \Gamma_{\eta^{0\prime} \to \gamma\gamma} \sim 6\ \text{keV}\ .$$

$$\hookrightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$$

The radiative width for the η_0' is simply an estimate from broken SU(3) and may not be reliable.

From a different point of view, Gatto and Preparata[11] have used the Cabibbo-Raddicati sum rules plus a ρ-dominance argument to obtain the sum rule for the resonance contribution

$$\int_{s_{th}}^{s_0} \frac{ds}{s} \sigma^{\gamma\gamma}_{resonance}(s) \simeq \frac{4\pi^2 \alpha^2}{m_\rho^2}$$

not including correction of order (15-20%) from isoscalar contributions. Again, this gives a contribution to the total cross section for ee $\rightarrow$ ee + hadrons of order 10 nb at E = 3 GeV.

Because of the consistency of the various estimates for $\sigma_{\gamma\gamma}$, I think that a hadronic physicist would be surprised if the measured cross section differed by more than 50% from the duality prediction. Further, as emphasized by Bjorken and Kogut[14], the predictions based on quark-parton or light-cone model for the electromagnetic current, are generally expected to agree with predictions based on vector dominance considerations of the hadronic nature of photons in processes where the photon is real. The exceptions to this rule for $\gamma\gamma$ processes are: (i) when impulse approximation results are valid such as when at least one photon is highly virtual[4,15-17]; or (ii) in processes involving two real photons which are sensitive to J = 0 fixed pole behavior arising from local two photon interactions[18]; and also (iii) real photon exclusive processes at large s and t [19] (fixed CM angle). Thus, predictions for $\sigma_{\gamma\gamma \rightarrow had}$ based on hadronic physics considerations should be sound, but it

would be delightfully interesting to be proved wrong! The interesting physics of $\sigma_{\gamma\gamma \to \text{had}}$ when the photons are virtual are reviewed in Walsh's talk.[4]

III. The Production Reactions $\gamma + \gamma \to \pi\pi$, KK

The exclusive two photon process of perhaps the most exciting and critical interest is $\gamma + \gamma \to \pi\pi$. Among the unique features of this reaction are

(1) It is perhaps the simplest production reaction in all of hadron physics: the hadronic interactions are all in the final state.

(2) It is related by crossing to the Compton amplitude $\gamma + \pi \to \gamma + \pi$. The value of $\gamma\gamma \to \pi\pi$ amplitudes are thus fixed below threshold at $s = 0$ by the Thomson limit.

(3) We can finally study $\pi^+\pi^-$ interactions in the absence of other hadrons (p, Δ) in the final state.

(4) Unitarity implies to order e^2,[7]

$$\text{Im } T^J_{\gamma\gamma \to \pi\pi} \propto T^{*J}_{\gamma\gamma \to \pi\pi} \, T^J_{\pi\pi \to \pi\pi},$$

for each angular momentum state and isotopic spin below the inelastic threshold. Therefore the phase of $T_{\gamma\gamma \to \pi\pi}$ is equal (modulo π) to the phase of $\pi\pi \to \pi\pi$. In fact, inelasticity is expected to be small up to the $K\overline{K}$ threshold.

(5) We can study the C = + resonant states of $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, with $J^P = 0^+, 2^+, \ldots, I = 0^+, 2^+$. Linear photon polarization information from the correlation of electron planes can also be used as a check of the spin-parity of the resonances.

(6) The Compton amplitude is fundamental to the study of electromagnetic interactions and specific photon-photon physics such as J = 0 fixed pole behavior[18], and scaling dependence of off-shell photons.

(7) Measurements of $d\sigma/dt$ ($\gamma + \gamma \rightarrow \pi + \pi$) at large s and t (fixed CM angle) provide a critical test of parton model predictions for large angle, and scaling laws based on the degrees of freedom of quark field theory.[19,20]

The kinematics of the process $ee \rightarrow ee\pi\pi$ and the extraction of the $\gamma\gamma \rightarrow \pi\pi$ subprocess has been discussed in many places in the literature.[3,7-9,13,21,22] The general, exact, analysis of the C = + contribution including the contribution of all helicity amplitudes for both real and virtual photons has been given by Carlson and Tung[17,21] and Brown and Muzinich[22]. [See also Ref. (9).] Specific calculations using Born approximation for $\gamma\gamma \rightarrow \pi^+\pi^-$ with elementary point-like pions, which are useful as a theoretical reference point are given in Refs. (7, 21-26), often with no restrictions on the final leptons assumed. Similar calculations for $\gamma\gamma \rightarrow \mu^+\mu^-$ are useful for experimental normalization.[28-30] The effects of meson form factors in the case where one lepton is detected at large angles are presented in an interesting study by Kessler et al.[25]

The experiment of most direct immediate interest will be the measurement of $\gamma\gamma \rightarrow \pi^+\pi^-$ for nearly-real photons. Detailed discussions appropriate to experiments where both leptons are tagged in the near-forward direction so that k_1^2, $k_2^2 \sim 0$, the longitudinal currents may be ignored, and the C = -1 contributions suppressed, are discussed in Refs. 7 and 25. In these experiments the equivalent photon approximation is accurate to order θ^2_{max}. The determination of polarization information from the correlation of the electron scattering planes

discussed by Brown and Lyth[9] and Cheng and Wu[24]. In this type of experiment, the scattered lepton kinematics are completely determined, the production plane of the pion pair is constrained by momentum conservation, and a modified equivalent photon approximation is required.

Let us now turn to the physics of the $\gamma\gamma \to \pi\pi$ cross section. Of basic interest will be the extraction of $\pi - \pi$ phase shifts using the unitarity condition. As Carlson and Tung[21] have emphasized, this can be done in a model-independent way. Using the equivalent photon approximation,

$$\frac{d\sigma(ee \to ee\pi\pi)}{ds\ d\cos\theta_\pi} = \frac{d\sigma_{\gamma\gamma \to \pi\pi}}{d\cos\theta_\pi} \frac{f\left(\frac{s}{4E^2}\right)}{2s} N_1 N_2$$

where

$$\frac{d\sigma_{\gamma\gamma \to \pi\pi}}{d\cos\theta_\pi} = \frac{\pi\alpha^2}{4s}\left[|T_{++}|^2 + |T_{+-}|^2\right]\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{1}{2}}$$

and T_{++}, T_{+-} are the invariant helicity amplitudes, and N_1 and N_2 are defined in Section I. Again, this result is accurate to order θ^2_{max}. If we write

$$T_{mn}(\theta_\pi) = \sum_{\ell \text{ even}} (2\ell + 1)\, a^{(\ell)}_{mn}\, d^{(\ell)}_{m-n}(\theta_\pi)$$

where $d^\ell_{m-n}(\theta_\pi)$ is the usual rotation matrix, and define $a^{(\ell)I}_{mn} = |a^{(\ell)I}_{mn}|\, e^{i\delta^I_\ell}$ $(I = 0,2)$, then δ^I_ℓ is the $\pi - \pi$ phase shift in the region of elastic unitarity. At low s we can use the first two partial waves:

$$T_{++} = |a^{(0)}_{++}|^2 + 10\,|a^{(0)}_{++}|\,|a^{(2)}_{++}|\cos(\delta_0 - \delta_2)\,d^{(2)}_{0,0}(\theta_\pi)$$

$$+ 25\,|a^{(2)}_{++}|^2 \left[d^{(2)}_{0,0}(\theta_\pi)\right]^2 + \ldots$$

$$T_{+-} = 25\,|a^{(2)}_{+-}|^2 \left[d^{(2)}_{2,0}(\theta_\pi)\right]^2 + \ldots$$

Thus the angular dependence in θ_π can determine the modulus of the amplitudes and $\cos(\delta_0 - \delta_2)$, which appears linearly in the cross section. The effects of a large $\pi - \pi$ scattering length in the s-wave, assuming all the other partial waves are given by Born approximation is shown in Fig. 2. The solid curve with small scattering length satisfies current algebra, unitarity and crossing symmetry and has the form for $I = 0$, $\delta_0(s) = 67.6\pi\,(s-4)/(s+28)^2$, with s in GeV^2. The other curve corresponds to $\delta(s) = \frac{5}{2}\sqrt{5}\,\pi\,(s-4)^{\frac{1}{2}}/(s+26)$. Both have resonances at $\sqrt{s} = 700$ MeV. The cross section near threshold does seem to show considerable sensitivity to the presence of an appreciable scattering length, but Carlson and Tung find much less sensitivity to the asymptotic behavior of δ_0, and, in their model, minimal sensitivity to the behavior of the phase shift in the ϵ region ($\sqrt{s} \sim 700$ MeV). A basic question, which we will return to below, is the adequacy of the Born approximation for the other partial waves.

A great challenge to theoretical physics is whether we can predict the amplitudes for $\gamma\gamma \to \pi\pi$ and $\gamma\gamma \to KK$ before the data comes in. There have been several valiant attempts.[21,26-34] The general strategy is to apply all of the relevant knowledge from hadronic physics on direct and cross channel resonance or Regge contributions, and the constraints of analyticity, unitarity, the low energy theorem, as well as assumptions derived from current algebra

and possibly, superconvergence relations. In Yudurain's phenomenological approach[33], the $\gamma\gamma \to \pi\pi$ amplitude is constructed from ϵ and f_0 Breit-Wigner resonances in the direct channel, with crossed channel contributions from the Born terms (π exchange), and elementary ρ and ω exchange (see also Ref. 26). In the papers of Lyth[31], Carlson and Tung[21], Isaev and Kleskov[35], Schierholz and Sundermeyer[32], and Gensini[34], a partial-wave dispersion relation is used for the s-wave incorporating $\pi - \pi$ data, and the higher waves are assumed to be given by Born approximation, plus in some cases, vector meson Regge exchange. Figure 3, which is taken from Schierholz and Sundermeyer's paper, shows the effects of using the "down" $\pi - \pi$ partial wave solution favored by Protopopesev et al.[36] (curve I) which has a very broad inelastic resonance at $m_\epsilon = 420$ MeV, $\Gamma = 380$ MeV, versus an elastic resonance (curve II), the "up" solution. The possibility of a strong ϵ contribution satisfying the constraints of current algebra is discussed by Goble and Rosner.[26]

At larger s the theoretical ambiguities become even worse, since the inelastic effects of other channels, especially $K\bar{K}$ becomes severe, and a multi-channel analysis is required. Model calculations have been given by Isaev and Kleskov[35], Gensini[34], and Sundermeyer[37]. In Gensini's work a two channel ($\pi\pi$, $K\bar{K}$) problem is solved, and the results are calculated assuming partial wave dispersion relations for the s-waves with poles at $m_\epsilon = 660 - i320$ MeV and $m_{S^*} = 997 - i27$ MeV, and Born approximation for the other partial waves. A rather dramatic behavior at K^+K^- threshold is predicted (see Fig. 2), but little sign of the ϵ is seen — in fact, the cross section is depressed below Born approximation in the region $\sqrt{s} \sim 700$ MeV!

An important question is how reliable the Born approximation is as an estimate of the crossed channel contribution to the s-wave, as well as to the higher partial waves. As one indicator, Lyth[38] points out that the cross-channel contribution to the s-wave from ρ and ω exchange alone exceed that of the Born term for $s \gtrsim 0.8\ \mathrm{GeV}^2$, so there must be an inherent smooth background uncertainty in all of the model estimates. But beyond this there is a more basic question. The Born contribution gives a scale-invariant cross section

$$\frac{d\sigma_{\gamma\gamma \to \pi\pi}}{d\cos\theta_{CM}} \propto \frac{1}{s}$$

for $s \gg 4m_\pi^2$, at fixed $\cos\theta_{CM}$. This clearly must be unreliable at large s (unless the pion were a point-like elementary particle with no form factor). In fact, there is strong reason to believe that the asymptotic behavior of $d\sigma_{\gamma\gamma \to \pi\pi}/d\cos\theta_{CM}$ is s^{-3} at fixed angle. This is the prediction of the parton interchange model of Ref. 18, assuming the meson form factors behave asymptotically at t^{-1}. More recently, Glennys Farrar and I[20] have derived a general scaling law for any electromagnetic or strong exclusive process:

$$\Delta\sigma \to \frac{1}{s^{1+N_M+2N_B}}, \qquad s \gg M^2$$

where the cross section is integrated over a fixed cm angular region with finite $p_i \cdot p_j/s$. Here N_M and N_B are the total number of external mesons and baryons. Thus we predict (modulo factors of log s)

$$\frac{d\sigma}{dt} \rightarrow \begin{cases} s^{-10} & pp \rightarrow pp \\ s^{-8} & \pi p \rightarrow \pi p,\ kp \rightarrow kp,\ kp \rightarrow \pi\Sigma \\ s^{-7} & \gamma p \rightarrow \pi p \\ s^{-6} & ep \rightarrow ep \\ s^{-4} & e^+e^- \rightarrow \pi^+\pi^-,\ K^+K^- \end{cases}$$

at fixed t/s or $\cos\theta_{CM}$, and $F_{1p} \sim t^{-2}$, $F_{2p} \sim t^{-3}$ and $F_\pi, F_k \sim t^{-1}$ for the asymptotic dependence of spacelike or timelike form factors. All of these predictions are consistent with experiment. For $\gamma\gamma$ physics we predict at fixed CM angle

$$\frac{d\sigma}{dt} \rightarrow \begin{cases} s^{-4} & \gamma\gamma \rightarrow \pi\pi,\ K\overline{K} \\ s^{-6} & \gamma\gamma \rightarrow p\overline{p} \\ s^{-3} & e\gamma \rightarrow e\pi^o \end{cases} .$$

The last result is for π^o production in the two photon process where the electron is detected at large angles and the other lepton is at small angles. If we define the $\gamma(q)\gamma(k) \rightarrow \pi^o$ transition form factor (q^2 large, $k^2 \sim 0$) via

$$M^{\mu\nu} = F_{\pi^o}(q^2)\, \epsilon^{\mu\nu\sigma\tau}\, k^\sigma q^\tau ,$$

this implies $F_{\pi^o}(q^2) \sim (q^2)^{-1}$.

The scaling laws of Ref. 20 are derived from renormalizable field theories based on quark-field degrees of freedom, assuming two conditions: (i) asymptotically scale-invariant interactions among the constituents within the hadrons, and (ii) Bethe-Salpeter wavefunctions for the composite hadrons which are finite at the origin. The asymptotic behavior of $\gamma\gamma \rightarrow \pi^+\pi^-$ is clearly a crucial test of this scheme. We also note the prediction for the

exclusive process

$$\frac{d\sigma}{d\cos\theta_{CM}}(ee \to ee\pi\pi) \to \frac{1}{s_{\pi\pi}^{3}} \quad \text{at fixed } \cos\theta_{CM}$$

where the electron angles need not be constrained. In this case the scaling comes from the dimension of the $\gamma\gamma \to \pi\pi$ amplitude. The equivalent photon approach is analogous to Feynman scaling; the electron fragmentation into photons is scale invariant. On the other hand, the inclusive process

$$\frac{d\sigma}{d\cos\theta_{CM}}(e^-e^- \to \pi X) \to \frac{1}{p_\perp^2} \text{ at fixed } \cos\theta_{CM}$$

is predicted to be asymptotically scale invariant (see also Ref. 11).

The scaling law for $\gamma\gamma \to \pi\pi$ also implies that the s-wave cross section decreases asymptotically as $d\sigma/d\cos\theta_{CM} \sim s^{-3}$. This condition should imply interesting constraints and cancellations (i. e. superconvergence relations[39,7]) between the cross channel and direct resonance contributions to the s-waves.

In addition, the quark-parton and light-cone models also can give predictions on the spin and angular dependence of the $\gamma\gamma \to \pi\pi$ asymptotic amplitude. The angular dependence will reflect the angular and fixed pole behavior of the underlying $\gamma + \gamma \to q + \bar{q}$ amplitudes.[18,40] Also, there is <u>no</u> off-shell photon mass dependence for $s \to \infty$, t/s fixed, $q^2 \ll s$. On the other hand, for a virtual photon $d\sigma/dt\,(\gamma + \gamma \to \pi\pi) \to (q^2)^{-3} f(\omega_1)$ for $s \to \infty$, at fixed $\omega_1 = 1 - q^2/s$, t, $k^2 = q_2^2$ small.[14,40]

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Figure Captions

Fig. 1: Schematic representation of the total cross sections for $\gamma\gamma \to$ had based on factorizable Regge poles and two component duality. See J. Rosner, Ref. 10. The dashed line represents $\sigma_{\gamma\gamma \to \text{had}} = 0.24\ \mu\text{b} + 0.27\ \mu\text{b GeV s}^{-\frac{1}{2}}$.

Fig. 2: (a) Two sets of $\pi - \pi$ phase shifts used by Carlson and Tung[4], for the calculation of the s-wave amplitude for $\gamma\gamma \to \pi\pi$.

(b) Integrated cross section for $ee \to ee\pi\pi$ at E = 3 GeV as a function of p, the magnitude of the pion momentum in the dipion CM frame. The dashed line corresponds to large scattering length.

Fig. 3: The $\gamma\gamma \to \pi\pi$ cross section calculated by Schierholz and Sudermeyer[32]. Curves I and II correspond to $\pi - \pi$ phase shifts with inelastic and elastic ϵ resonances, respectively.

Fig. 4: The $\gamma\gamma \to \pi\pi$ cross section calculated by Gensini.[34] The quantity $S\, d\sigma/d\Omega$ is plotted versus CM energy. The effects of the $K\bar{K}$ threshold and S^* resonance are included. Curve (a) is Born approximation.

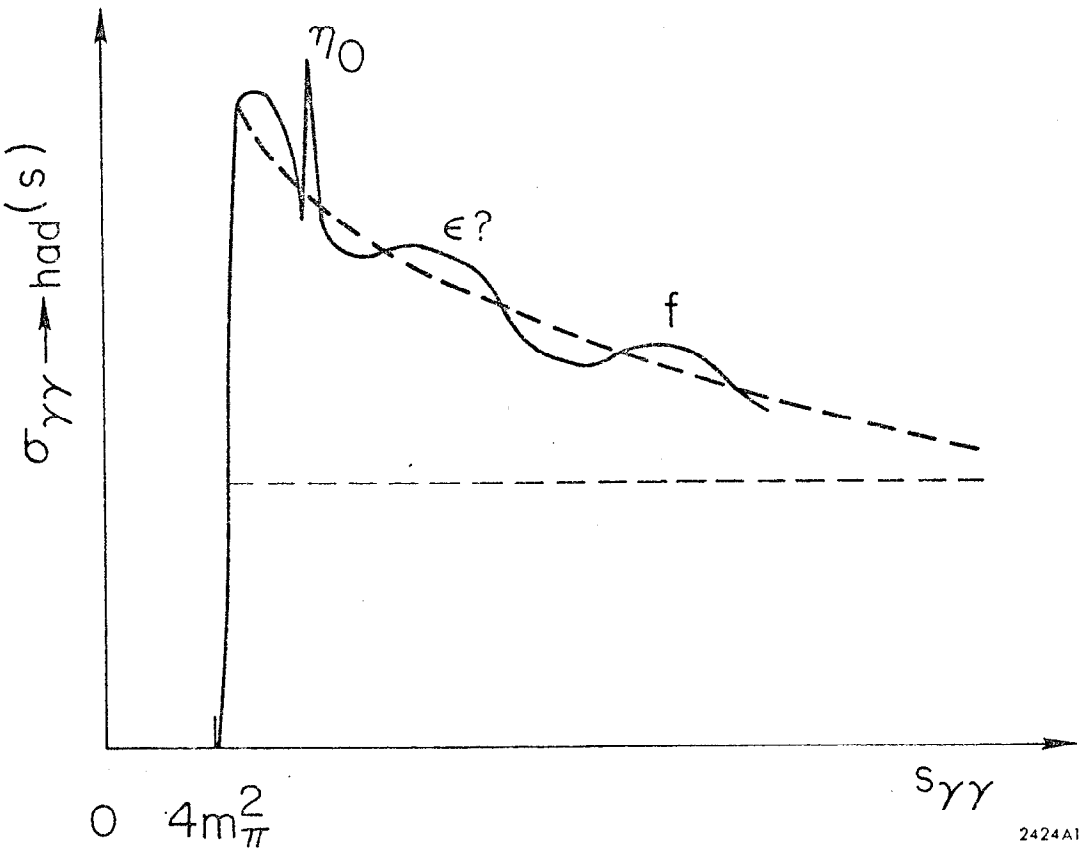
η_0
ϵ?
f
$\sigma_{\gamma\gamma \to had}(s)$
0
$4m_\pi^2$
$s_{\gamma\gamma}$
2424A1

Fig. 1

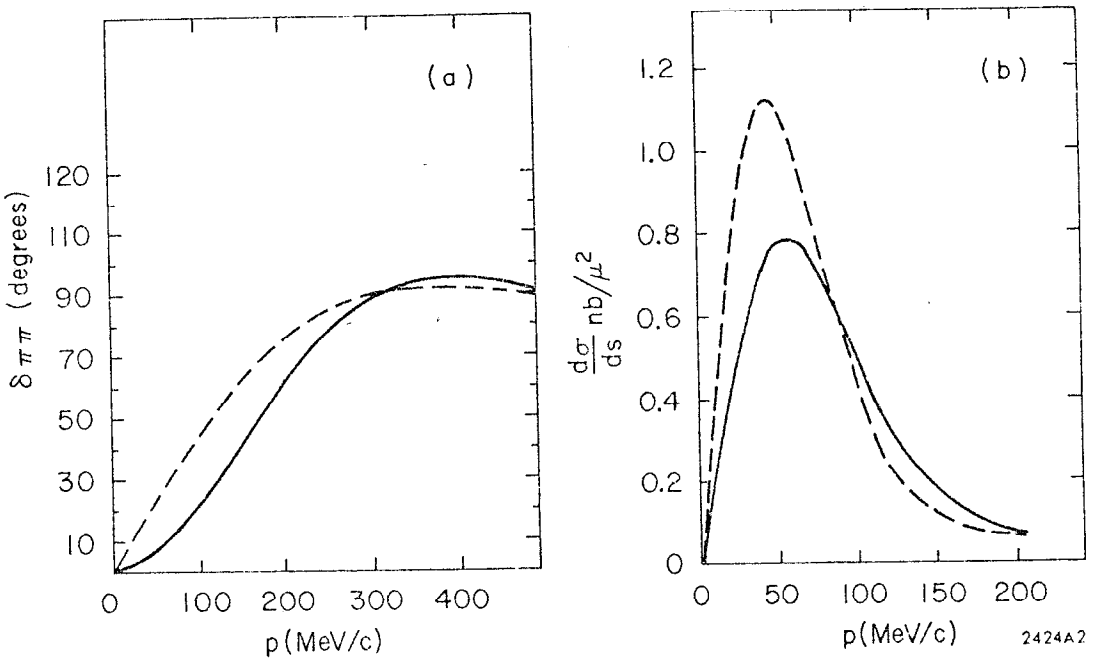
(a)
120
110
90
70
50
30
10
$\delta_{\pi\pi}$ (degrees)
0
100
200
300
400
p(MeV/c)
(b)
1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{d\sigma}{ds}$ nb/μ^2
0
50
100
150
200
p(MeV/c)
2424A2

Fig. 2

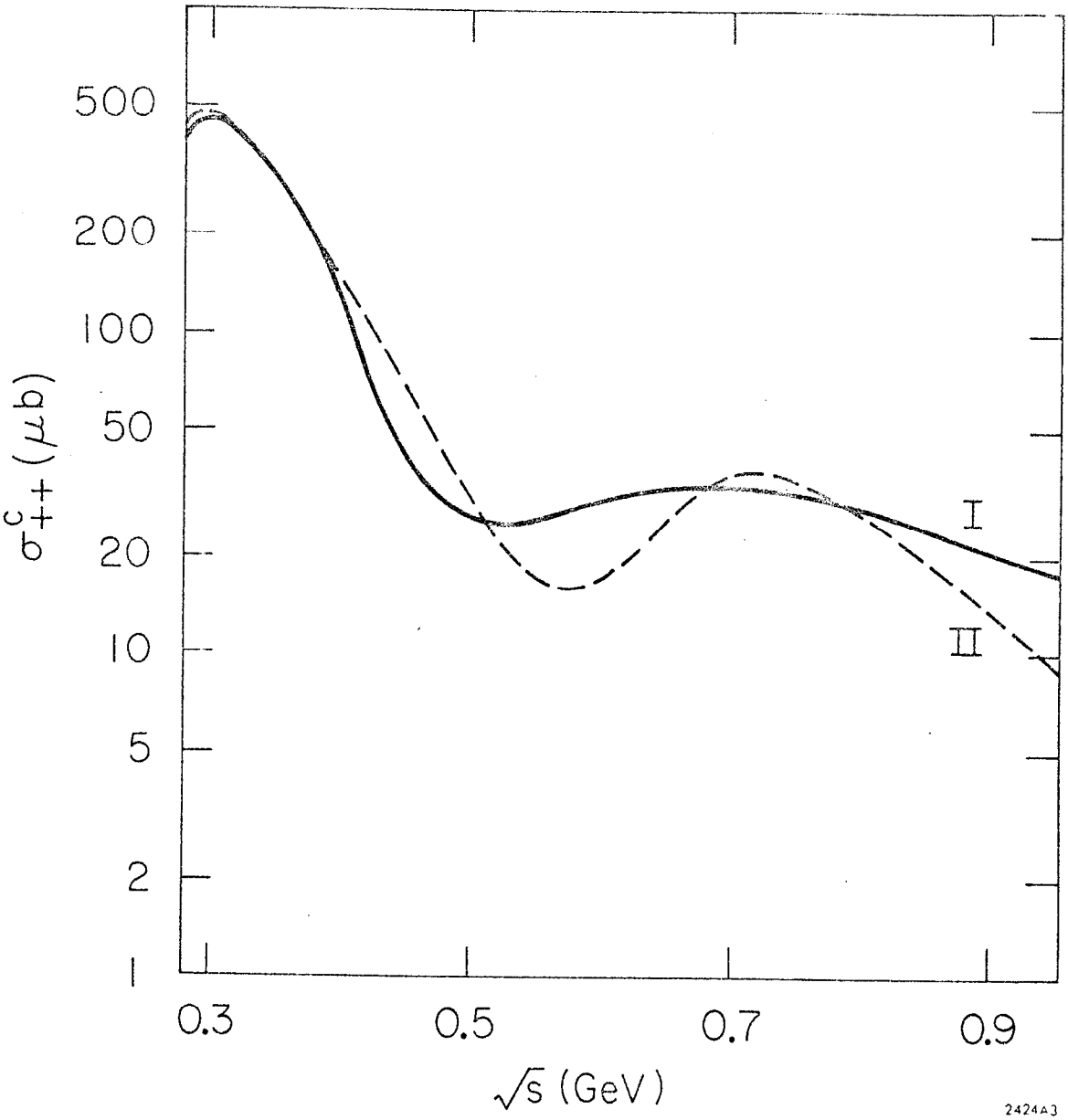
500
200
100
50
20
10
5
2
1
σ^c_{++} (μb)
I
II
0.3
0.5
0.7
0.9
$\sqrt{s}$ (GeV)
2424A3

Fig. 3

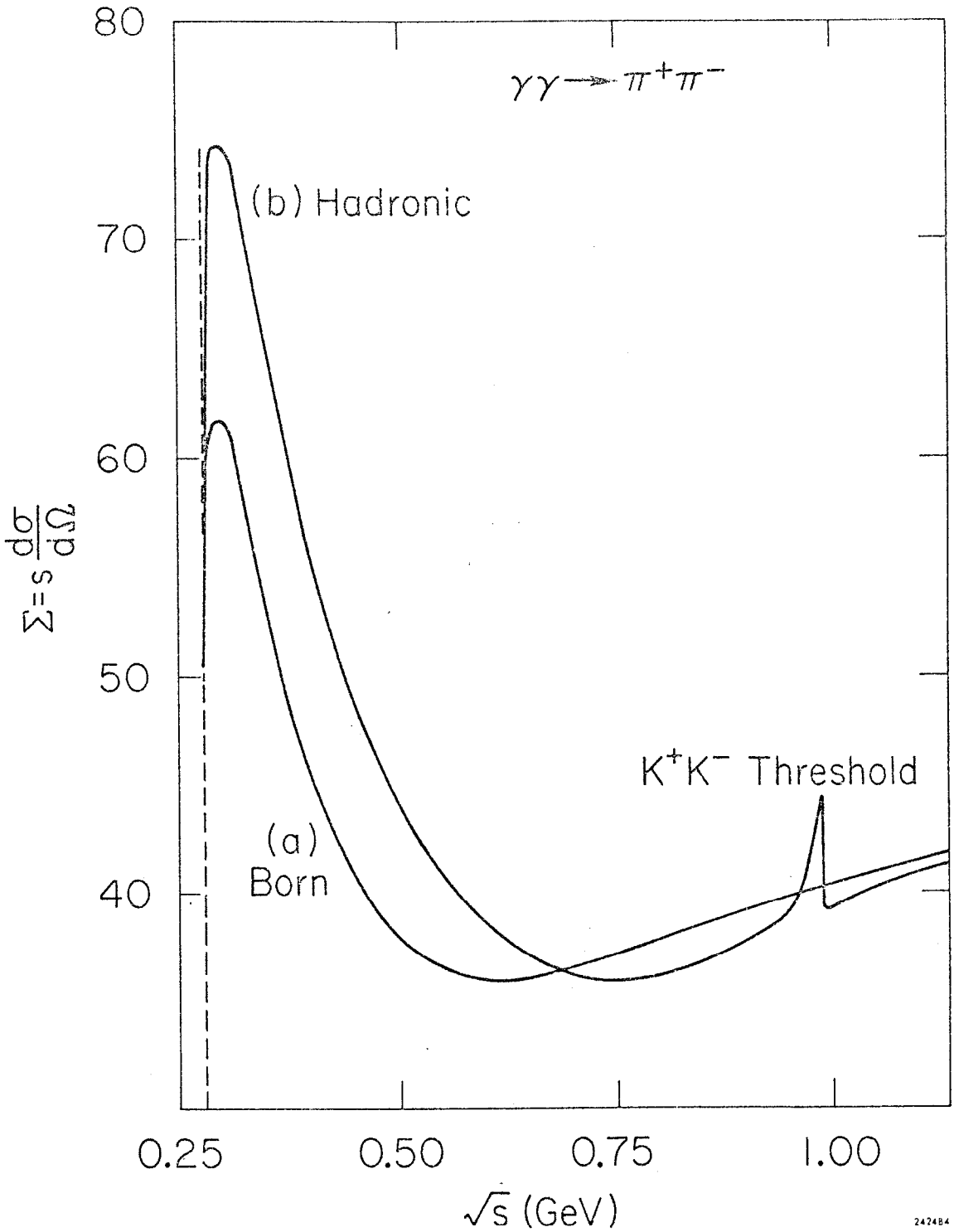
80
70
60
50
40
$\gamma\gamma \to \pi^+\pi^-$
(b) Hadronic
(a)
Born
K^+K^- Threshold
$\Sigma = s \frac{d\sigma}{d\Omega}$
0.25
0.50
0.75
1.00
$\sqrt{s}$ (GeV)
2424B4

Fig. 4