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CURRENT ALGEBRA, PCAC, ITS ANOMALY, AND THE TWO-PHOTON PROCESS (SOFT-PION PRODUCTION BY TWO PHOTONS)*,**

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ABSTRACT

Various applications of the current algebra, PCAC (partially conserved axial-vector current hypothesis) and its anomaly to the hadron production by two photons $\gamma + \gamma \rightarrow$ hadrons are reviewed. They include the processes (1) $\gamma + \gamma \rightarrow \pi^0, \eta, \eta'$ (or X^0) and ϵ, (2) $\gamma + \gamma \rightarrow 3\pi^0, \pi^+ + \pi^- + \pi^0$, and (3) $\gamma + \gamma \rightarrow n\pi^+ + n\pi^-$ ($n = 1, 2, 3 \cdots$), where all produced pions are soft. Asymptotic behavior of the processes (4) $\gamma^* + \gamma^* \rightarrow \pi^+ + \pi^-$ and (5) $\gamma^* + \gamma^* \rightarrow \pi^0$ for highly virtual photons is also discussed.

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In this talk I shall review recent work on the application of the current algebra, PCAC and its anomaly to the hadron production by two photons. I shall talk about the following subjects:

(1) $\gamma + \gamma \to \pi^o, \eta, \eta'$ (or X^o) and ϵ

(2) $\gamma + \gamma \to 3\pi^o, \pi^+ + \pi^- + \pi^o$

(3) $\gamma + \gamma \to n\pi^+ + n\pi^-$ $(n = 1, 2, 3, \cdots)$

(4) $\gamma^* + \gamma^* \to \pi^+ + \pi^-$

(5) $\gamma^* + \gamma^* \to \pi^o$

where all the produced pions are soft (in other words, their energies are small compared to, say, 1 GeV), and γ and γ^* denote an almost real photon and a highly virtual photon, respectively.

I shall try to be consistent in such a way that all the predictions for these processes are based on the following three assumptions and on nothing else:

(a) SU(2) × SU(2) current algebra[1]

$$\delta(x_0)[V_0^a(x), V_\mu^b(0)] = i\epsilon_{abc} V_\mu^c(0)\,\delta(x), \text{ etc.},$$

(b) "strong" PCAC,[2] and (c) the possible existence of its anomaly[3]

$$\partial_\mu A_\pi^\mu(x) = f_\pi m_\pi^2 \phi_\pi(x) \quad \left(+ \frac{e^2 S}{16\pi^2} \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta}(x) F^{\gamma\delta}(x) \text{ for } \pi^o\right),$$

where f_π ($\simeq$ 93 MeV) is the pion decay constant and S is the anomalous constant.

By the "strong PCAC" I mean that the divergence of the axial-vector current is dominated by a pion pole and that the extrapolation of amplitudes needed from the off-shell point at $p_\pi^2 = 0$ to the on-shell point at $p_\pi^2 = m_\pi^2$ is smooth so that the physical amplitude can be approximated well by the soft-pion amplitude calculated at the unphysical point where $p_\pi^2 = 0$.[4]

In order to save our time, I shall skip rigorous derivation of the results without any exception but, instead, give a rough idea about how to derive them. Therefore, for those who are interested in the more details, I strongly recommend that they refer

to the original papers or my review article[5] which will appear in the Reviews of Modern Physics soon.

Now let me start with the first subject.

(1) $\gamma + \gamma \to \pi^0$, η, η' (or X^0) and ϵ.

The production of π^0 by colliding beams, $e^{\pm} + e^- \to e^{\pm} + e^- + \pi^0$, was first proposed by F. E. Low[6] in 1960 as an experiment for measuring the life time of π^0. In the equivalent-photon approximation the total cross section for this process is proportional to the decay width $\Gamma_{\pi^0 \to \gamma\gamma}$ as follows:

$$\sigma_{ee \to ee\pi^0} \cong \frac{64\alpha^2 \Gamma_{\pi^0 \to \gamma\gamma}}{m_\pi^3} \left(\ln \frac{E}{m_e}\right)^2 \ln \frac{2E}{m_\pi} + \cdots \simeq 1 \times 10^{-33} \mathrm{cm}^2 \text{ for } E \simeq 3 \text{ GeV}$$

The latest experimental value[7] for the width is $\Gamma_{\pi^0 \to \gamma\gamma} = 7.8 \pm 0.9$ eV. In order to obtain a better accuracy by two-photon experiments, I urge to observe scattered leptons within a small angle so that both theoretical and experimental uncertainties may be eliminated. It seems fairly easy to obtain better values for the decay width of $\eta \to \gamma\gamma$, $\eta' \to \gamma\gamma$, and $\epsilon \to \gamma\gamma$ in this way. Why is the measurement of these decay widths important? I shall give one reason here. I expect the next speaker, Dr. Brodsky, will present us another reason.

The vertex function of $\pi^0\gamma\gamma$ is defined by

$$M_{\mu\nu}(q,k) = \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta F(q^2, k^2, P^2) = i \int dx e^{-iqx} \langle P | T^*(J_\mu(x) J_\nu(0)) | 0 \rangle .$$

The decay width is given by $\Gamma_{\pi^0 \to \gamma\gamma} = \dfrac{|e^2 F(0, 0, m_\pi^2)|^2}{64\pi} m_\pi^3$. By using the LSZ reduction formula and the modified PCAC including the anomalous term, one can obtain the Ward-Takahashi identity at $P^2 = 0$:

$$M_{\mu\nu}(q,k)\Big|_{P^2=0} = -\frac{S}{2\pi^2 f_\pi} \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta + \frac{P^\lambda}{f_\pi} i \int dy dx e^{iPy - iqx} \langle 0 | T(A_\lambda^\pi(y) J_\mu(x) J_\nu(0)) | 0 \rangle \Big|_{P^2=0} .$$

The equal-time commutator terms vanish because the electromagnetic current commutes with the neutral current A_λ^π. This identity can be transformed into the

relation

$$F(q^2,k^2,0) = -\frac{S}{2\pi^2 f_\pi} + \frac{1}{f_\pi}\left[-\frac{1}{2}(q^2+k^2)G_3(q^2,k^2,0) + k^2 G_4(q^2,k^2,0)\right.$$
$$\left. -q^2 G_5(q^2,k^2,0) + \frac{1}{2}(q^2+k^2)\, G_6(q^2,k^2,0)\right]$$

where G's are the form factors of the AVV vertex. Now one can see that

$$F(0,0,0) = 0 \qquad \text{if } S = 0,$$

which is the old Sutherland-Veltman theorem.[8] According to Bell-Jackiw and Adler,[3] the anomaly exists and the low-energy theorem

$$F(0,0,m_\pi^2) \cong F(0,0,0) = -\frac{S}{2\pi^2 f_\pi}$$

holds. The anomalous constant can be calculated by the triangle diagram [see Fig. 1(a)] with a fermion quark loop to be

$$S = \begin{cases} \frac{1}{2} \text{ for the Han-Nambu model and the three-triplet quark model.} \\ \frac{1}{6} \text{ for the original Gell-Mann-Zweig triplet quark model.} \end{cases}$$

The present experimental data shows

$$S_{exp} \cong 0.5,$$

which strongly favors the former two models. The anomaly also affects the low energy theorem on the $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ vertices. However, the low energy theorem is less practical in these cases than in the case of $\pi^0 \to \gamma\gamma$ because the extrapolation needed is much more demanding.

The coupling constant of $\epsilon(700)$ and two photons is theoretically as interesting as the $\pi^0 \to \gamma\gamma$ decay constant. In order to measure this, it is desirable to do either the experiment of the type

$$e^\pm + e^- \to e^\pm + e^- + \epsilon$$

in which both scattered lepton momenta are measured to find a missing mass or the one of the type

$$e^\pm + e^- \to e^\pm + e^- + \pi^+ + \pi^-$$

in which both pion momenta are measured to find a broad ϵ (or σ) enhancement in the invariant mass distribution of the pion pair. It should be noticed here that the $\epsilon\gamma\gamma$ coupling constant can be directly measured in the first type of experiments while what can be measured in the second type is the product of the $\epsilon\gamma\gamma$ and $\epsilon\pi^+\pi^-$ coupling constants but not the $\epsilon\gamma\gamma$ coupling constant alone. Theoretical predictions for the product of these coupling constants and their consequences will be discussed by the next speaker.

For the $\epsilon\gamma\gamma$ coupling constant we can play the same game as we just did for the $\pi^0\gamma\gamma$ decay constant. What we need is to replace PCAC by PCDC (partially conserved dilation current).[9] Kleinert, Staunton and Weisz[10] showed that if the $\epsilon(700)$ meson dominates the trace of energy momentum tensor θ^λ_λ, then the $\epsilon\gamma\gamma$ coupling constant vanishes in the soft-meson limit. However, Crewther[11] and, independently, Chanowitz and Ellis[12] have recently pointed out that the PCDC anomaly[13] affects the low energy theroem and that the $\epsilon\gamma\gamma$ coupling constant does not vanish [see Fig. 1(b)]. Furthermore they have predicted the coupling constant defined by

$$\mathscr{L}_{\epsilon\gamma\gamma} = -\frac{e^2 g_{\epsilon\gamma\gamma}}{2}\,\phi_\epsilon F_{\mu\nu}F^{\mu\nu}$$

to be

$$g_{\epsilon\gamma\gamma} \cong \frac{R}{12\pi^2 f_\epsilon}$$

where R is the asymptotic ratio of $\sigma(e^+e^- \to \text{hadrons})$ to $\sigma(e^+e^- \to \mu^+\mu^-)$ and f_ϵ is defined by

$$\langle 0 | \theta^\mu_\mu(0) | \epsilon \rangle = m_\epsilon^2 f_\epsilon .$$

From this result Chanowitz and Ellis[12] have estimated the $\epsilon \to \gamma\gamma$ decay width to be

$$\Gamma_{\epsilon \to \gamma\gamma} \cong 0.2\, R^2 \text{ keV} \qquad \text{for } m_\epsilon \cong 700 \text{ MeV} \qquad \text{and } f_\epsilon \cong 150 \text{ MeV}.$$

This can be checked by two-photon experiments.

Next, I shall discuss the production of an odd number of soft pions by two real photons.

(2) $\gamma + \gamma \to 3\pi^0$, $\pi^+ + \pi^- + \pi^0$.

Aviv, Hari Dass, and Sawyer[14] first found that the amplitudes for $\gamma + \gamma \to 3\pi^0$ vanishes in the soft-pion limit ($p_{\pi^0} \to 0$) and that the amplitude for $\gamma + \gamma \to \pi^+ + \pi^- + \pi^0$ in the same limit is related to the PCAC anomaly and, therefore, can be written solely in terms of the $\pi^0 \to \gamma\gamma$ amplitude. Their first result: the vanishing $\gamma + \gamma \to 3\pi^0$ amplitude was confirmed by Abers and Fels[15] and by many others. However, their second result was controversial for a whole. For example, Yao[16] obtained a different result for the $\gamma + \gamma \to \pi^+\pi^-\pi^0$ amplitude. This problem has finally been solved by Terent'ev,[17] Wong,[18] Adler, Lee, Treiman, and Zee,[19] Bacry and Muyts,[20] and Hari Dass,[21] who pointed out that the amplitudes for $\gamma + \gamma \to \pi^+ + \pi^- + \pi^0$ written previously are not gauge-invariant [see Fig. 2]. The conclusion of these various authors is the following: (1) Both of the amplitudes for $\gamma + \gamma \to 3\pi^0$ and $\pi^+ + \pi^- + \pi^0$ vanish when the π^0 momentum vanishes, while the charged-pion momenta are on the mass shell, (2) the amplitude for $\gamma + \gamma \to \pi^+\pi^-\pi^0$ can be expressed in terms of those for $\pi^0 \to \gamma + \gamma$ and $\gamma \to \pi^+ + \pi^- + \pi^0$, and (3) the latter two amplitudes are simply related by

$$F_{\pi^0 \to \gamma\gamma} = f_\pi^2 F_{\gamma \to \pi^+\pi^-\pi^0}$$

due to the gauge invariance of the whole amplitude, where $F_{\pi^0 \to \gamma\gamma} = F(0, 0, 0)$ and $F_{\gamma \to \pi^+\pi^-\pi^0}$ is defined by the soft-pion production amplitude

$$\mathcal{M}(\gamma(k) \to \pi^0 + \pi^+(p) + \pi^-(q)) = i e k^\alpha \epsilon^\beta p^\gamma q^\delta \, \epsilon_{\alpha\beta\gamma\delta} F_{\gamma \to \pi^+\pi^-\pi^0} .$$

The proof of (3) in the presence of the PCAC anomaly has recently been given by Terent'ev[17] and Adler, Lee, Treiman and Zee[19] although the relation between $\pi^0 \to \gamma\gamma$ and $\gamma \to \pi^+ + \pi^- + \pi^0$ was approximately derived by Kawarabayashi and Suzuki[22] several years ago.

With the corrected version of the amplitudes for $\gamma + \gamma \to 3\pi^0$ and $\pi^+ + \pi^- + \pi^0$, actual calculations of the cross sections for $e + e \to e + e + \pi^+ + \pi^- + \pi^0$ and $e + e \to e + e + 3\pi^0$ have been done by Pratap, Smith and Uy,[23] Köberle[24] including hard-pion terms, and by Zee.[25] Unfortunately, these predicted cross sections are

small ($\sim 10^{-36}\text{cm}^2$ for E = 1 GeV) in the soft pion region where these soft-pion results should be tested. The result of Köberle[24] shows, however, that the hard-pion cross section for these processes calculated from vector meson dominance may be large enough ($\sim 10^{-34}\text{cm}^2$ for $e + e \rightarrow e + e + \pi^+ + \pi^- + \pi^0$ at E = 1 GeV and $\sim 10^{-35}\text{cm}^2$ for $e + e \rightarrow e + e + 3\pi^0$ at E = 2 GeV) to be measured in the near future.

I shall now move to the production of even number of charged soft pions by two photons.

(3) $\gamma + \gamma \rightarrow n\pi^+ + n\pi^-$ (n = 1, 2, 3, ···)

Which is more practical experimentally than the production of odd number of soft pions. We need not worry about the PCAC anomaly in this case. Instead, there are a few ambiguities[26] in taking the soft-pion limit and in extrapolating the off-shell amplitude to the physical one. However, I will take gauge invariance and the Thomson limit for forward Compton scattering as guiding principles to obtain a consistent result.

Let us start with the definition of the amplitude for $\gamma + \gamma \rightarrow n\pi^+ + n\pi^-$

$$M_{\mu\nu} = i \int dx e^{-ik_1 x} \langle n\pi^+(p_i),\ n\pi^-(q_i) | T(J_\mu(x) J_\nu(0) | 0 \rangle .$$

The successive application of the PCAC hypothesis, the soft-pion technique, and the algebra of currents makes it possible to reduce a pion pair in the limit $p, q \rightarrow 0$. Repeating the same procedure n times, I shall end up with

$$\langle n\pi^+(p_i),\ n\pi^-(q_i) | T(J_\mu(x) J_\nu(0)) | 0 \rangle$$

$$\xrightarrow[\overrightarrow{p_i, q_i} \rightarrow 0]{} \frac{-2(-3)^{n-1}}{f_\pi^{2n}} \left[\langle 0 | T(V_\mu^3(x)\, V_\nu^3(0)) | 0 \rangle - \langle 0 | T(A_\mu^3(x) A_\nu^3(0)) | 0 \rangle \right]$$

where V_μ^3 and A_μ^3 are the strangeness conserving vector and axial-vector currents (their third components in the isospin space). Using the spectral representations of the propagators, one can obtain the following expression for the matrix element

$$M^{\mu\nu} \rightarrow \frac{2(-3)^{n-1}}{f_\pi^{2n}} \left[\int dm^2 \frac{\rho_V(m^2) - \rho_A(m^2)}{k_1 \cdot k_2 + m^2 - i\epsilon} \left(g^{\mu\nu} + \frac{k_2^\mu k_1^\nu}{m^2} \right) - f_\pi^2 \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2 - i\epsilon} \right.$$

$$\left. - g^{\mu o} g^{\nu o} \int dm^2 \frac{\rho_V(m^2) - \rho_A(m^2)}{m^2} + g^{\mu o} g^{\nu o} f_\pi^2 \right]$$

We can clearly see that not only Lorentz covariance but also gauge invariance is maintained if and only if Weinberg's first sum rule[27]

$$\int dm^2 \frac{\rho_V(m^2) - \rho_A(m^2)}{m^2} = f_\pi^2$$

is valid. Therefore I shall assume the validity of the sum rule hereafter. Thus I have obtained the soft-pion theorem[28]

$$M^{\mu\nu} \xrightarrow[\text{soft pion}]{} 2 \left(\frac{-3}{f_\pi^2} \right)^{n-1} \left(g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2} \right) F(k_1 \cdot k_2)$$

and

$$F(Q^2) = \frac{1}{f_\pi^2} \int dm^2 \frac{\rho_V(m^2) - \rho_A(m^2)}{Q^2 + m^2 - i\epsilon} \qquad \text{with } F(0) = 1,$$

which gives a correct Thomson limit for $n = 1$ when k_1 and k_2 vanish. This relation has also been obtained by Terent'ev[29] for $n = 1$ and has been confirmed by Goble and Rosner[30] for $n = 2$ and in the limit of k_1 and $k_2 \rightarrow 0$.

Now we can apply this soft-pion theorem to various physical processes. For example,

$$\sigma(\gamma\gamma \rightarrow 2\pi^+ 2\pi^-) \cong 2.1 \times 10^{-33}\ \text{cm}^2 \qquad \text{at } s = (6\, m_\pi)^2$$

and

$$\sigma(\gamma\gamma \rightarrow 3\pi^+ 3\pi^-) \cong 0.54 \times 10^{-35}\ \text{cm}^2 \qquad \text{at } s = (8 m_\pi)^2.$$

We should notice that the cross section for two-pion-pair production is two orders of magnitude larger than that for $\pi^+\pi^-\pi^o$ production $\sim 10^{-35}\text{cm}^2$ at $s = (4m_\pi)^2$. More detailed and precise numerical results for $n = 2$ can be found in the paper by Goble and Rosner.[30]

Within the equivalent-photon approximation we can estimate the cross section for $e + e \rightarrow e + e + n\pi^+ + n\pi^-$. Since the low-s region dominates the total cross sections, the soft-pion results should give a reasonable estimate for these processes. For example, we have

$$\sigma^{\text{soft pion}}_{(ee \rightarrow ee\, 2\pi^+ 2\pi^-)} = 3.5 \times 10^{-36} \text{cm}^2 \qquad \text{at } E = 1 \text{ GeV}.$$

The next subject is how to extend the soft-pion results obtained for real photons to those for virtual photons.

(4) $\gamma^* + \gamma^* \rightarrow \pi^+ + \pi^-$ and (5) $\gamma^* + \gamma^* \rightarrow \pi^0$.

As we have just seen, the PCAC hypothesis and the algebra of currents determine the amplitude for $\gamma^*(k_1) + \gamma^*(k_2) \rightarrow \pi^+(q_1) + \pi^-(q_2)$ at the unphysical point

$$k_1 + k_2 = 0 \quad \text{and} \quad q_1 = q_2 = 0.$$

From this point we must extrapolate to the nearest physical point, namely, the production threshold

$$q_1 = q_2 = (m_\pi, 0, 0, 0) \text{ and } \quad k_1 + k_2 = (2m_\pi, 0, 0, 0).$$

In addition to k_1^2 and k_2^2, there are two invariant variables available, namely, $s = (k_1 + k_2)^2 = (q_1 + q_2)^2$ and $k_1 \cdot q_1$. These two quantities vary from zero to $4m_\pi^2$ and m_π^2, respectively, in the minimum extrapolation. Thus T. M. Yan[31] has found that the soft-pion amplitude derived for real photons can be extrapolated into the threshold amplitude for any highly virtual photons without losing the validity of soft-pion approximation. Therefore, if we keep the kinematical condition of Calogero and Zemach[32] [see Fig. 3(a)]

$$\vec{p}_1' + \vec{p}_2' = 0$$

as strictly as possible, we can measure the theoretically interesting quantity

$$F(Q^2) = \frac{1}{f_\pi^2} \int dm^2 \, \frac{\rho_V(m^2) - \rho_A(m^2)}{Q^2 + m^2 - i\epsilon} \quad \text{with } Q^2 = -k_1^2 = -k_2^2$$

for arbitrary values of Q^2. The differential cross section is proportional to $[F(Q^2)]$.

$$d\sigma(ee \to ee\pi^+\pi^-) \propto [F(Q^2)]^2 .$$

On the other hand, several years ago Das, Guralnik, Mathur, Low, and Young[33] derived an expression for the electromagnetic mass difference of the pions based on the same assumptions as we have just made. It is given by

$$m^2_{\pi^+} - m^2_{\pi^0} = \frac{3\alpha}{4\pi} \int_0^\infty dQ^2 F(Q^2)$$

Combining these two things together, Yan[31] has arrived at a sum rule which looks as follows [see Fig. 3(b)]:

$$m^2_{\pi^+} - m^2_{\pi^0} = \int_0^\infty dQ^2 \lim_{s \to 4m^2_\pi} \left[\frac{d\sigma(ee \to ee\pi^+\pi^-)}{(\text{known function})}\right]^{1/2} .$$

Therefore, the electromagnetic mass difference can be determined by the process $e^\pm + e^- \to e^\pm + e^- + \pi^+ + \pi^-$. This experiment is extremely interesting for the following reasons: (1) a measurement of the function $F(Q^2)$ is of great theoretical interest because it may answer such questions as the convergence of Weinberg's second sum rule, the behavior of the spectral function of the axial-vector current, and so on. (2) The origin of the pion mass difference is not known. Most people may believe that it is electromagnetic. I think, however, that it is still an open and interesting question whether the pion mass difference is really and entirely electromagnetic or not.

Unfortunately, the effective cross section for large $Q^2 (> 0.5\ \text{GeV}^2)$ is too small ($\sim 10^{-38}\ \text{cm}^2$) to be measured in the near future. Detailed calculations of the cross section for this experiment can be found in the paper by Isaev and Khleskov.[34]

Next, I shall talk about the asymptotic behavior of the $\pi^0\gamma\gamma$ vertex function for large mass of virtual photons. It can be studied by the two-photon process

$$e^\pm + e^- \to e^\pm + e^- + \pi^0 .$$

Let us go back to the definition of the vertex function

$$M_{\mu\nu}(q,k) = \epsilon_{\mu\nu\alpha\beta} q^{\alpha} k^{\beta} F(q^2, k^2, P^2) = i \int dx e^{-iqx} \langle P | T^*(J_\mu(x) J_\nu(0)) | 0 \rangle$$

$$= i \int dx e^{-iQx} \langle P | T^*\left(J_\mu\left(\frac{x}{2}\right) J_\nu\left(-\frac{x}{2}\right)\right) | 0 \rangle \qquad \text{and } Q = \frac{q-k}{2} \ .$$

Several years ago, Cornwall[35] showed that the $\pi^0\gamma\gamma$ vertex function approaches the limit

$$F(q^2, k^2, m_\pi^2) \to \frac{2}{3} \frac{f_\pi}{q^2} \text{ as } q^2 \to \infty \quad \text{and } q^2/k^2 \to 1$$

if there is no q-number Schwinger term, if the BJL theorem

$$M_{\mu\nu}(q,k) \xrightarrow[\substack{Q_0 \to \infty \\ P, \vec{Q} \text{ fixed}}]{} \frac{1}{Q_0} \int dx e^{-iQx} \delta(x_0) \langle P | [J_\mu\left(\frac{x}{2}\right), J_\nu\left(-\frac{x}{2}\right)] | 0 \rangle$$

is valid, and if the quark model for the space-space component of the equal-time current commutators

$$\delta(x_0) [J_i(x), J_j(0)] = 2i \epsilon_{0ijk} A^k_{Q^2}(0) \delta(x)$$

holds. Furthermore, Gross and Trieman[36] have predicted the scaling of $F(q^2, k^2, m_\pi^2)$

$$q^2 F(q^2, k^2, m_\pi^2) \to f(\omega)$$

in their scaling limit, $q^2 \to \infty$ with $\omega = k^2/q^2$ fixed, by assuming the gluon quark model for the light-cone current commutator[37]

$$[J_\mu(x), J_\nu(y)] \underset{(x-y)^2 \cong 0}{\cong} \partial^\alpha D(x-y) \left\{ s_{\mu\nu\alpha\beta} [V^\beta_{Q^2}(x,y) - V^\beta_{Q^2}(y,x)] + i\epsilon_{\mu\nu\alpha\beta} [A^\beta_{Q^2}(x,y) + A^\beta_{Q^2}(y,x)] \right\} ,$$

$$s_{\mu\nu\alpha\beta} = g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\beta} ,$$

and

$$D(x) = \frac{1}{2\pi} \epsilon(x_0) \delta(x^2).$$

Since these results are not only model-dependent but limited to the special kinematical region where both q^2 and k^2 are large with the ratio k^2/q^2 fixed, it is desirable to find a less model-dependent and more widely applicable prediction for the asymptotic behavior of the vertex function. I have found that the vertex function, if it decreases at all, should decrease not slower than $1/\sqrt{-q^2}$ for any fixed values of k^2. I have shown this by using the Schwartz inequality and unitarity only (see Fig. 4). More detailed discussion on the consequences of this inequality can also be found in the recent paper by West.[39]

How does the PCAC anomaly affect the asymptotic behavior of the $\pi^0\gamma\gamma$ vertex function? To see this, let me remind you of the following expression:[40]

$$F(q^2,k^2,0) = -\frac{S}{2\pi^2 f_\pi} + \frac{1}{f_\pi}\left[-\frac{1}{2}(q^2+k^2)\,G_3(q^2,k^2,0) + k^2 G_4(q^2,k^2,0) - q^2 G_5(q^2,k^2,0) + \frac{1}{2}(q^2+k^2)G_6(q^2,k^2,0)\right].$$

We can clearly see that, if the $\pi^0\gamma\gamma$ vertex function decreases at all, a combination of the form factors of the AVV vertex should decrease as fast as $(S/2\pi^2)(q^2+k^2)^{-1}$:

$$-\frac{1}{2}G_3(q^2,k^2,0) + \frac{k^2}{q^2+k^2}G_4(q^2,k^2,0) - \frac{q^2}{q^2+k^2}G_5(q^2,k^2,0) + \frac{1}{2}G_6(q^2,k^2) \to \frac{S}{2\pi^2}\,\frac{1}{q^2+k^2},$$

which is a very strong constraint on the form factors.

Although the effective cross section is small for large $-q^2$ and $-k^2$ [$\sim 10^{-37}\,\mathrm{cm}^2$ for $E = 3$ GeV and $-q^2$, $-k^2 > 1\ \mathrm{GeV}^2$], it seems easy to find whether the vertex function decreases or whether it scales.

In conclusion, I think that the asymptotic behavior of the inclusive process

$$\gamma^* + \gamma^* \to \text{any hadrons}$$

in which both photons are highly virtual is theoretically most interesting.[41-45] It is related to and completely determined by the algebra of bilocal currents instead of the algebra of currents. I expect that the speakers for this afternoon, Dr. Walsh and Dr. Stern will talk about this in great detail.

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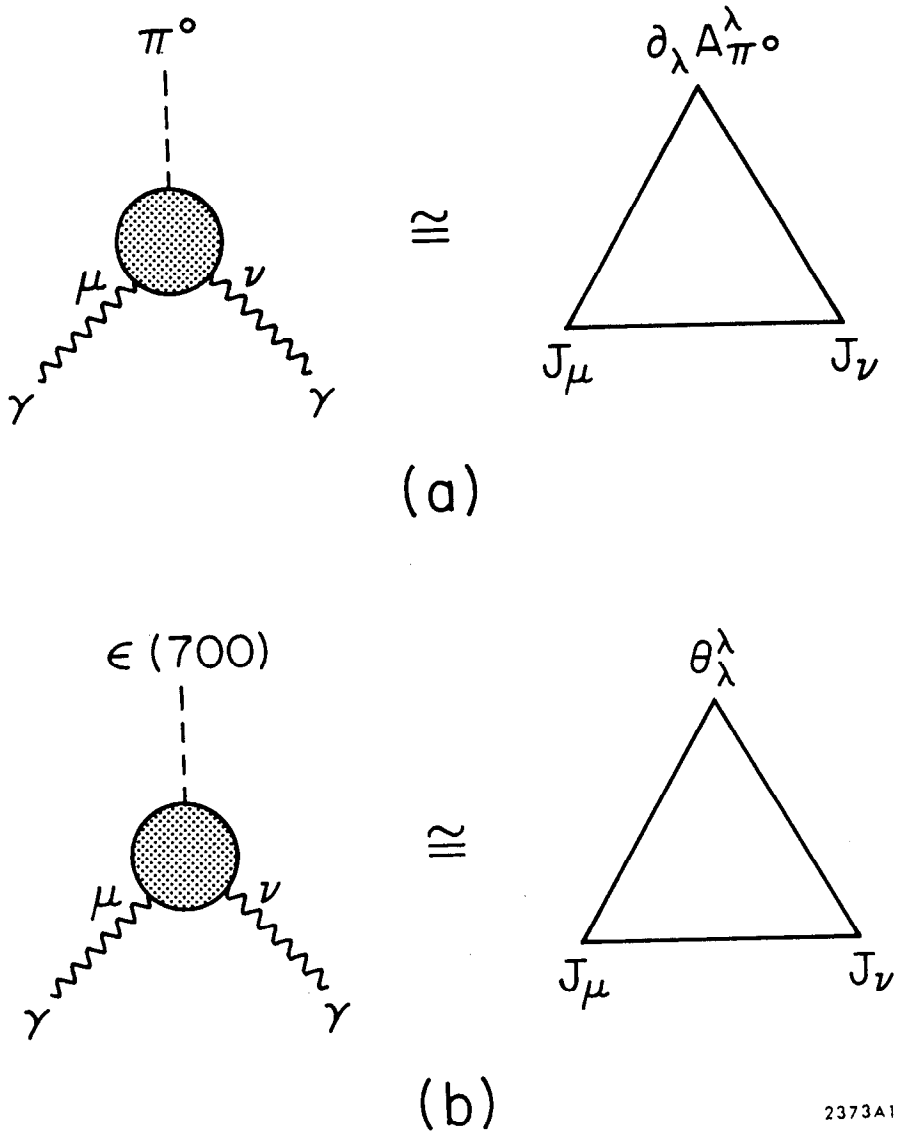

π°
μ
ν
γ
γ
$\cong$
$\partial_\lambda A^\lambda_{\pi^\circ}$
J_μ
J_ν
(a)
$\epsilon(700)$
μ
ν
γ
γ
$\cong$
θ^λ_λ
J_μ
J_ν
(b)
2373A1

FIG. 1

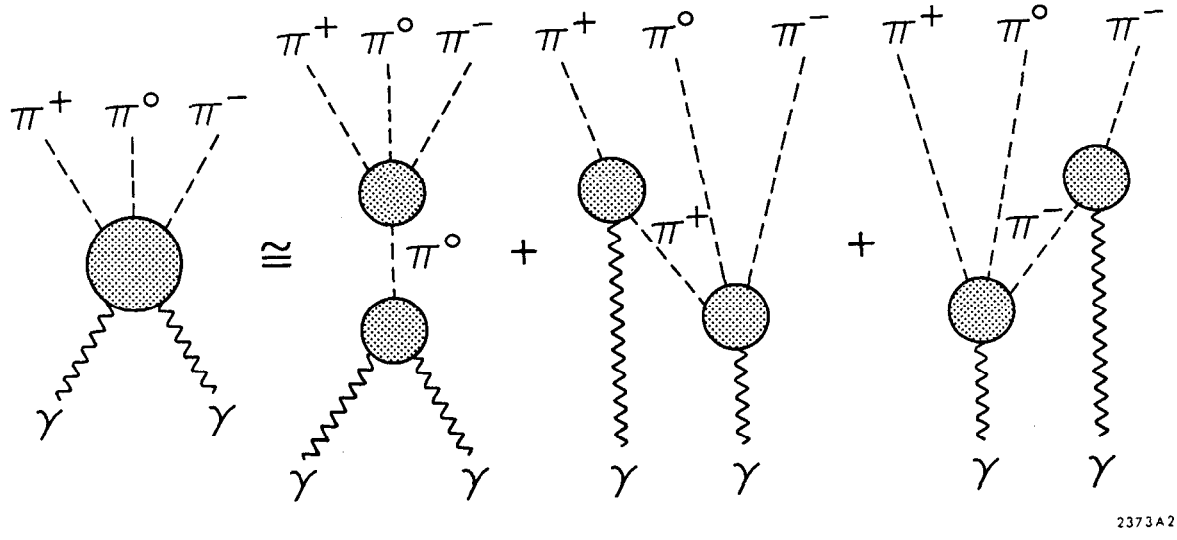
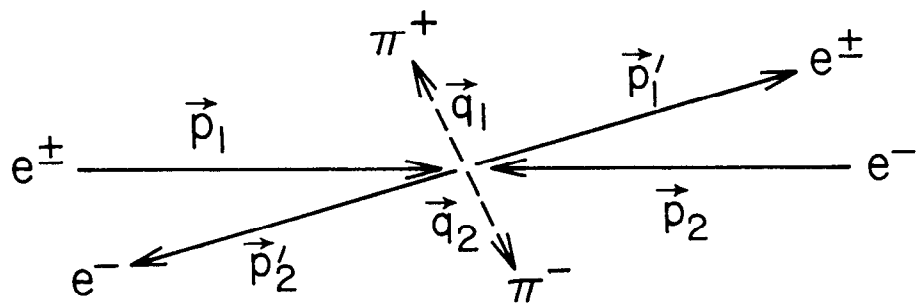
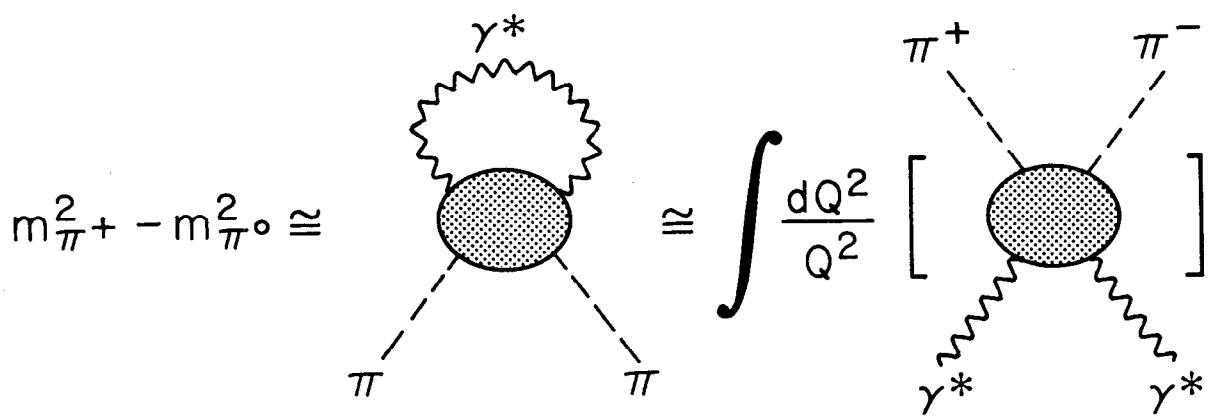


FIG. 2

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2' = \vec{q}_1 + \vec{q}_2 = 0$$



(a)



(b)

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FIG. 3

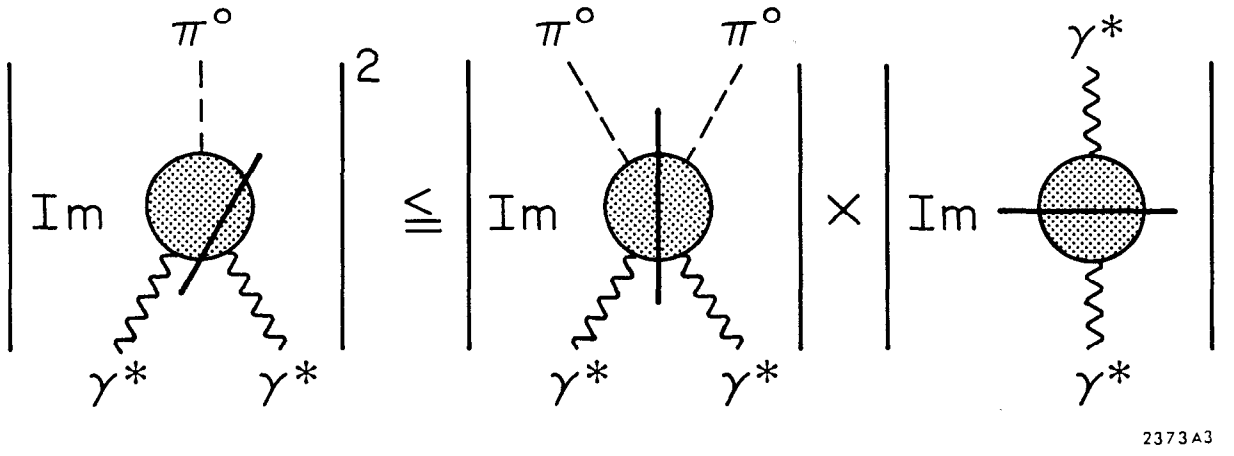

FIG. 4