SLAC-PUB-1279
(T)
July 1973

Erratum

The Hadron Constituents and e^+e^- Colliding Beam Experiments

A. I. SANDA

p.11 Equation (23) should read

$$R(Q) = \frac{3}{2} \int_{\frac{M}{Q}}^{1} \omega^2 \left[M\overline{W}_1 + \frac{\omega}{6} \nu \overline{W}_2 \right] d\omega$$

$$\langle n \rangle R(Q^2) = 3 \int_{\frac{M}{Q}}^{1} \omega \left[M\overline{W}_1 + \frac{\omega}{6} \nu \overline{W}_2 \right] d\omega$$

p.12 line 7

"energies of the hadrons" should read

"energies of the partons".

SLAC-PUB-1279
(T)
July 1973

THE HADRON CONSTITUENTS AND e^+e^- COLLIDING BEAM EXPERIMENTS†

A. I. Sanda*
Stanford Linear Accelerator Center
Stanford, California 94305

ABSTRACT

In order to test the idea that hadrons are made out of some fundamental constituents, it is most important to state precisely, in a general way, what we mean by the hadronic constituents. We call the constituents partons and state a set of basic assumptions. They are weaker than any of the assumptions previously used in connection with the model. With these assumptions, we show that if $R \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ increases faster than or equal to log Q/M at large Q, then the multiplicity of hadrons in $e^+e^- \rightarrow$ hadrons must increase at least as fast as $\frac{Q}{M}/\log\frac{Q}{M}$. Q is the center of mass energy of the colliding beam. The increase in R with Q, itself, does not violate the basic concept of the parton model. If above relations between the increase in the cross section and the multiplicity is contradicted by experiments at large Q, however, the parton model requires a major overhaul.

(Submitted to Physics Letters)

† Supported in part by the U. S. Atomic Energy Commission.
* Permanent Address: National Accelerator Laboratory, P.O. Box 500, Balavia, Illinois 60505.

The parton model has generated great interest in connection with the deep inelastic lepton nucleon scattering and e^+e^- annihilation process. There are various different versions of the parton model, some of them, due to their simplified nature, predict various dramatic effects. It is important to obtain a set of most reasonable assumptions of the model and understand their consequences. We will state the assumptions required to prove Bjorken scaling for the structure functions of the deep inelastic ep scattering as well as those of e^+e^- annihilation into one hadron plus anything. Using the scaling property, we obtain a relationship between the multiplicity of hadrons in e^+e^- annihilation and the energy dependence of R. These have severe experimental consequences. The assumptions we use are weaker than any of the assumptions previously used. And they are of such a general character that if their consequences contradict experiments, it will force us to modify the basic ideas of the parton model. We stress however that the increase of R with Q^2 is not directly in contradiction with these assumptions.

The parton model assumptions are: (a) There is some underlying field theory which governs the hadronic physics. The bare particles of the field theory are called partons. (b) The wave functions for a hadron state to be in a certain state of partons and for a parton state to be in a certain state of hadrons are well defined.[1]

(b.1) Define

$$|_{\pm}\langle p_1, \cdots, p_\ell | k_1, \cdots, k_n \rangle'|^2 = \sum_{s=n}^{\infty} \int \prod_{j=n+1}^{s} d^3k_j \, |\langle p_1, \cdots, p_\ell | k_1, \cdots, k_n, k_{n+1}, \cdots, k_s \rangle|^2 \tag{1}$$

This is a probability for a in or out (+, -) hadron state with momenta $\vec{p}_1, \cdots, \vec{p}_\ell$ to be found in a parton state which contains partons with momenta $\vec{k}_1, \cdots, \vec{k}_n$. If the hadronic state is properly normalized, we must have[2]

$$\int |_\pm\langle p_1, \cdots, p_\ell | k_1, \cdots, k_n\rangle' |^2 \, d^3k_i \leq 1 \quad \text{for} \quad 1 \leq i \leq n \tag{2}$$

(b.2) Define

$$|\langle k_1, \cdots, k_n | p_1, \cdots, p_\ell\rangle'_\pm|^2 = \sum_{s=\ell}^{\infty} \int \prod_{j=\ell+1}^{s} d^3p_j \, |\langle k_1, \cdots, k_n | p_1, \cdots, p_\ell, p_{\ell+1}, \cdots, p_s\rangle|^2 \tag{3}$$

This is the probability that a parton state $|k_1, \cdots, k_n\rangle$ is found in a in or out hadron state which contains hadrons with momenta $p_1, \cdots, p_\ell$. Again, if (3) is well defined we must have

$$\int |\langle k_1, \cdots, k_n | p_1, \cdots, p_\ell\rangle'_\pm|^2 d^3p_i \leq 1 \quad \text{for } 1 \leq i \leq n \tag{4}$$

(c) Since the integrands for (2) and (3) are positive definite, only a finite regions out of the entire phase space of d^3k_i and d^3p_i respectively are important. If we expect the parton model to be useful, we must be able to say something about the parton configurations by observing the hadronic configurations. So, we assume that; (i) The character of the strong interaction is such that the contribution of d^3k_i integration in (2) comes from the part of phase space about the direction $p_1, \cdots, p_\ell$. In a particular frame $\vec{p}_i = (\vec{p}_{i\perp}, y_iP)$, $\vec{k}_j = (\vec{k}_{j\perp}, z_jP)$, (2) and this assumption yields

$$|_\pm\langle p_1, \cdots, p_\ell | k_1, \cdots, k_n\rangle'|^2 \leq 0 \left[\mathrm{Min} \left\{ \left(\frac{M}{k_{j\perp} - \frac{z_j}{y_i} p_{i\perp}} \right)^{2+\epsilon} ; \; i = 1, \cdots, \ell \right\} \right] \tag{5}$$

as $k_{j\perp} \to \infty$, $\epsilon > 0$. $k_{j\perp} - \frac{z_j}{y_i} p_{i\perp}$ is the transverse distance between $\vec{p}_i$ and $\vec{k}_j$. That is, the major contribution to the integral (2) comes from the cone around $\vec{p}_1, \cdots, \vec{p}_\ell$. See fig. 1. (ii) The contribution of d^3p_i integration in (4) comes from the part of phase space about the directions $\vec{k}_1, \cdots, \vec{k}_n$. This assumption together with (4) yields

$$|\langle k_1, \cdots, k_n | p_1, \cdots, p_\ell \rangle'_\pm|^2 = 0 \left[\mathrm{Min} \left\{ \left(\frac{M}{p_{j\perp} - \frac{y_j}{z_i} k_{i\perp}} \right)^{2+\epsilon} ; \; i=1, \cdots, n \right\} \right] \tag{6}$$

as $p_{j\perp} \to \infty$.

(d) Consider $\langle k_1, k_2 | p_1, \cdots, p_\ell \rangle_\pm$. For given $|p_1, \cdots, p_\ell\rangle_\pm$, let η be a finite phase space region which contains $p_1, \cdots, p_n$. When k_1 and k_2 are very far away from η, we assume that the amplitude is sufficiently damped so that

$$\int \langle k_1, k_2 | p_1, \cdots, p_\ell \rangle_\pm \; d^3k_1 \tag{7}$$

is convergent. This assumption is supported by the expectation that a parton state $|k_1, k_2\rangle$, when k_1 and k_2 are far away from η, is found to be in a hadron state with all the hadrons in the phase space η must be very small. See Fig. 2.

The assumptions (a), (b.1) and (b.2) must hold in order to start talking about partons. (c) and (d) are not of the same class as the first three but they are implied by any set of parton model assumptions previously used. It is trivial to construct a field theory model in which (c) and (d) are automatically satisfied once (a), (b.1) and (b.2) are assumed.

In Ref. 1, we have used the assumptions (a), (b.1), (b.2) and an experimental observation that in the deep inelastic ep scattering, the cross section for observing a final state hadron with large transversal momentum with respect to the photon direction in the laboratory frame is negligibly small compared to that of observing a hadron with small transversal momentum. With these assumptions we obtained the parton model result for ep deep inelastic scattering

$$\lim_{\substack{\nu,\, Q^2 \to \infty \\ x = Q^2/2M\nu \text{ fixed}}} \nu W(Q^2, \nu) = \sum_i e_i^2 f_i(x) \tag{8}$$

where $f_i(x)\,dx$ is the number of parton "i" with a fraction of momentum between x and x + dx. e_i is the charge of parton "i". It is easily seen that (c) replaces the phenomenological assumption mentioned above. Thus (8) can be obtained from (a), (b.1), (b.2) and (c). We have also observed that the parton model result

$$\lim_{Q^2 \to \infty} R(Q^2) \to \text{const} \tag{9}$$

does not follow from the assumptions (a) ~ (d). But rather a stronger assumption

$$|{}_{-}\langle p_1, \cdots, p_n | k_1, k_2 \rangle|^2 \leq 0 \left[\text{Min} \left\{ \left(\frac{M}{p_{j\perp} - \frac{y_j}{z_i} k_i} \right)^{4+\epsilon} ;\ i = 1,\ 2 \right\} \right] \tag{10}$$

and an assumption about the multiplicity of hadrons was needed to obtain (9).

Consider $e^+e^- \to h$ + anything. h is a hadron. The kinematics is shown in Fig. 3. The structure functions are defined by

$$(2\pi)^6 \int d\alpha\, \delta(q^o + p^o - p^o_\alpha)\, \frac{P}{M} \langle 0 | J^+_\mu(0) | p, \alpha\rangle_{-\,-}\langle p, \alpha | J_\nu(0) | 0\rangle$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \overline{W}_1 + \frac{1}{M^2}\left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \overline{W}_2 \tag{11}$$

$\overline{W}_1$ and $\overline{W}_2$ are functions of Q^2 and ν. The cross section is given by

$$\frac{d\sigma}{d\omega d\cos\theta} = \frac{3\pi\alpha^2}{Q^2}\,\omega\left[\overline{W}_1 + \frac{\sin^2\theta\,\omega}{4}\,\nu\overline{W}_2\right] \tag{12}$$

where θ is the center of mass angle of the detected hadron with respect to the incident beam direction. In the parton model,[1]

$$_{-}\langle\alpha,p|\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, J_\mu(x)|0\rangle = \sum_i \int d^3k\, e_i \sqrt{\frac{m^2}{k^o(q-k)^o}}\;\bar{u}(k)\gamma_\mu v(q-k)\;\;_{-}\langle\alpha,p|i;k,q-k\rangle \tag{13}$$

"i" denotes the type of partons. Consider the problem in a Lorentz frame defined by

$$p = (P + \frac{M^2}{2P},\ 0,\ 0,\ P)$$

$$q = \left(\frac{\nu\tau}{M}P\,\tau + \frac{M\nu}{2P}\ ,\ \eta Q,\ 0,\ \frac{\nu\tau}{M}P - \frac{M\nu}{2P}(1-\tau)\right) \tag{14}$$

where

$$\tau = 1 - \sqrt{1-(1+\eta^2)Q^2/\nu^2}\ \text{ for fixed } \omega,\ \tau \approx (1+\eta^2)\frac{Q^2}{2\nu^2}\ .$$

η is a parameter which defines a boost direction in the rest frame of the detected hadron. Writing out the zero-zero component,

$$(1+\eta^2)^2\ \frac{xM}{2}\ \overline{W}_1 + \left(\frac{1-\eta^2}{2}\right)^2\ \nu\,\overline{W}_2$$

$$= \frac{(2\pi)^3M\nu}{VP}\int d\alpha\ \delta(q^o - p^o - p^o_\alpha)\sum_{ij} e_ie_j \int d^3k d^3k'\ \langle k',\ q-k'\ ;\ j|\alpha,p\rangle_{-}$$

$$_{-}\langle\alpha,\ p|i;\ k,\ q-k\rangle \tag{15}$$

Using (6) we can place upper bounds for large $p_{i\perp}$. Choosing

$$\vec{k} = \left(\vec{k}_\perp, \left(\frac{\nu\tau}{M} - \eta z\right) P\right), \vec{k}' = \left(k'_\perp, \left(\frac{\nu\tau}{M} - \eta z'\right) P\right), \vec{p}_i = (\vec{p}_{i\perp}, y_i p),$$

$$|\langle k, q-k; i|p, p_1, \cdots, p_\ell\rangle_-| < 0\left[\mathrm{Min}\left\{\left(\frac{M}{p_{i\perp} - \frac{y_i k_\perp}{\frac{\nu\tau}{M} - z\eta}}\right)^{1+\epsilon}; \left(\frac{M}{p_{i\perp} - \frac{y_i(\eta Q - k_\perp)}{z\eta}}\right)^{1+\epsilon}\right\}\right]$$

$$|{}_-\langle p, p_1, \cdots, p_\ell|j; k', q-k'\rangle| < 0\left[\mathrm{Min}\left\{\left(\frac{M}{p_{i\perp} - \frac{y_i k'_\perp}{\frac{\nu\tau}{M} - z'\eta}}\right)^{1+\epsilon}, \left(\frac{M}{p_{i\perp} - \frac{y_i(\eta Q - k'_\perp)}{z'\eta}}\right)^{1+\epsilon}\right\}\right]$$

(16)

The numerators on the right side of (16) are the transverse momentum of $\vec{p}_i$ relative to $\vec{k}$, $\vec{q}-\vec{k}$ and $\vec{k}'$, $\vec{q}-\vec{k}'$ respectively. From these bounds, we want to show that the sum over the final state α can be limited to only part of the phase space about $\vec{k}$ and $\vec{q}-\vec{k}$. If $\vec{k}$ and $\vec{k}'$ were approximately parallel, then the product of two amplitudes behaves for example

$$\left(\frac{M}{p_{i\perp} - \frac{y_i k_\perp}{\frac{\nu\tau}{M} - z\eta}}\right)^{2+\epsilon}$$

away from vectors $\vec{k}$ and $\vec{k}'$. With this convergence, the integral $d\alpha$ can be cut off at finite transverse distance away from $\vec{k}$. We will show that the major contribution to (15) comes from the region where k and k' are approximately parallel and along p.

Define cones C_1, C_2, C'_1, C'_2. Consider the final state hadron with momentum $\vec{p}_i$ with $y_i \gg \frac{M}{E}$. For some ξ', $M \ll \xi' \ll Q$, say

$$p_i \in C_i \text{ if } \left|p_{i\perp} - \frac{y_i k_\perp}{\frac{\nu\tau}{M} - z\eta}\right| < \xi', \quad p_i \in C_2 \text{ if } \left|p_{i\perp} - \frac{y_i(Q\eta - k_\perp)}{z\eta}\right| < \xi'$$

and

$$p_i \in C_1' \text{ if } \left| p_{i\perp} - \frac{y_i k_\perp'}{\frac{\nu\tau}{M} - z'\eta} \right| < \xi' \quad p_i \in C_2' \text{ if } \left| p_{i\perp} - \frac{y_i(\eta Q - k_\perp')}{z'\eta} \right| < \xi' . \tag{17}$$

Any hadron with momentum p_i must be contained in at least one of C_1, C_2, C_1' or C_2' otherwise the integrand of (15) is $0\left(\left(\frac{M}{\xi'}\right)^{2+\epsilon}\right)$ and this region gives negligible contribution. Let the detected hadron with momentum $\vec{p}$ be contained in C_1. Consider $p_i \notin C_2$ or C_2'. If $p_i \in C_1'$, then $C_1' \cap C_1 \neq 0$ since if $C_1' \cap C_1 = 0$ then (16) implies that the integrand is $0\left(\left(\frac{M}{\xi}\right)^{2+\epsilon}\right)$ and this region gives negligible contribution. So, either $C_1' \cap C_1 \neq 0$ or C_1' is empty. Similarly, if a particle is contained in C_2, then $C_2' \cap C_2 \neq 0$ or C_2' is empty. Define Ω_1 and Ω_2 such that $\ell \in \Omega_1$, if $|\ell_{i\perp}| < \xi$ and $\ell \in \Omega_2$ if $\left| \ell_\perp - \frac{y_i Q}{\eta z} \right| < \xi$. There is $M \ll \xi \ll Q$ such that all hadrons are included in either Ω_1 and Ω_2. $\vec{k}$ and $\vec{q}-\vec{k}$ are also included in Ω_1 and Ω_2 respectively. Assumption (d) is used to exclude the possibility that C_1' and C_2' are empty. Then

$$(1+\eta^2)^2 \frac{xM}{2} \overline{W}_1 + \left(\frac{1-\eta}{2}\right)^2 \nu \overline{W}_2$$

$$= \frac{(2\pi)^3}{V} \frac{M\nu}{P} \int_\xi d\alpha \int d^3k\, d^3k'\, \delta(q^0 - p^0 - p^0_\alpha) \langle k', q-k'; j|\alpha, p\rangle_- \langle \alpha, p|i; k\, q-k\rangle$$

where

$$\int_\xi d\alpha\, d^3k\, d^3k' = \prod \left[\int_{|p_{i\perp}|<\xi} d^3p_i + \int_{|\widetilde{p}_{i\perp}|<\xi} d^3p_i \right] \int_{|k_\perp|<\xi} d^3k \int_{|k_\perp'|<\xi} d^3k' \tag{18}$$

where $\widetilde{p}_{i\perp} = p_{i\perp} - y_i Q/z$. The process is shown in figure 4. The terms neglected from (18) is $0\left(\left(\frac{M}{\xi}\right)^{\epsilon}\right)$. Consider the state $|\alpha, p\rangle = |p_1, \cdots, p_n, p; (\text{wee})\rangle$. $p_1, \cdots, p_n$ are nonwee hadrons and (wee) denotes collection of wee hadrons with

$y = 0\left(\frac{M}{P}\right)$. The hadrons with $y_i \gg \frac{M}{Q}$ can be clearly separated in to two groups according to (18). The hadrons with $y_i \leq 0\left(\frac{M}{Q}\right)$ can also be separated by hand so that the two groups $p_1, \cdots, p_\nu$ and $p_{\nu+1}, \cdots, p_n$ satisfy

$$\sum_{i=1}^{\nu} p_i = k + 0\left(\frac{M}{Q} P\right) , \quad \sum_{i=\nu+1}^{n} p_i = q - k - 0\left(\frac{M}{Q} P\right) . \tag{19}$$

Then

$$\sum_{i=1}^{\nu} y_i = \frac{\nu\tau}{M} - \eta z + 0\left(\frac{M}{Q}\right), \quad \sum_{i=\nu+1}^{n} y_i = \eta z - 0\left(\frac{M}{Q}\right)$$

$$\sum_{i=1}^{\nu} \ell_i^o \cong E_{wee} + \frac{\nu\tau}{M} P - \frac{M\nu}{2P} + \sum_{i=1}^{n} \frac{M^2 + \xi^2}{2y_i P} + \frac{Q^2}{2zP}$$

The energy conserving δ function in (18) becomes

$$\delta\left(-\frac{M\nu}{P} + E_{wee} + \frac{Q^2}{2zP} + \sum_{i=1}^{n} \frac{m^2 + \xi^2}{2y_i P}\right) \tag{20}$$

Consider a hadron with $y \lesssim 0\left(\frac{M^2}{Q^2}\right)$ and make a transformation to the laboratory frame. It can be seen that such a hadron must have momentum of 0(Q) and must be moving precisely opposite to the direction of the boost into the infinite momentum frame. Therefore, in the limit of large Q, the contribution to the inclusive cross section from such region can be made negligibly small. By similar argument, states which contain wee hadrons with $y = 0\left(\frac{M}{P}\right)$ can be eliminated. States which contain hadrons with $y = 0\left(\frac{M}{Q}\right)$ requires more careful examination. These are very slow hadrons in the center of mass frame of the colliding beams. If the multiplicity of hadron does not grow as fast as Q, then even if some of the y_i's are $0\left(\frac{M}{Q}\right)$, the last term of (20) can be neglected and we have

$$(1+\eta^2)^2 \ \frac{xM}{2}\ \overline{W}_1 + \left(\frac{1-\eta}{2}\right)^2 \ \nu\overline{W}_2$$

$$= \frac{(2\pi)^3}{V} \sum_i \int_{|k_\perp|,\, |k'_\perp| < \xi} d^3k\, d^3k'\ z\ \delta\left(z - \frac{Q^2}{2M\nu}\right) \langle k', q-k'; i| b^+(p)\ b(p) | i; k, k-q\rangle \tag{21}$$

where $b^+(p)$ is a creation operator for the observed hadron.

In case the multiplicity of hadrons is of $0\left(\frac{Q}{M}\right)$, then we can not neglect the term $\sum_{i=1}^{n} \frac{m^2 + \xi^2}{2y_i P}$. From the δ function and the kinematics shown in fig. 4, it is clear that $\frac{Q^2}{2M\nu} < z < \frac{1+\eta^2}{\eta} \frac{Q^2}{2M\nu}$ The upper bound follows from the fact that the matrix element vanishes if $\frac{\nu\tau}{M} < \eta z$. Assimilar bounds are also satisfied for z'.

Then

$$\int_{\Delta\omega} \left[(1 \pm \eta^2)^2 \ \frac{xM}{2}\ \overline{W}_1 + \left(\frac{1-\eta}{2}\right)^2 \nu\overline{W}_2\right] d\omega$$

$$\leq \frac{(2\pi)^3}{V} \sum_{ij} \int_\xi d\alpha\ d^3k\ d^3k' \ \langle k', q-k'; j|\alpha, p\rangle_{--}\langle\alpha, p| i\ ; k,\ q-k\rangle$$
$$\frac{Q^2}{2M\nu} < z < \frac{1+\eta^2}{\eta}\ \frac{Q^2}{2M\nu} \tag{22}$$

where right hand side is obtained by integrating over all ω. $\Delta\omega$ is arbitrary. Summing over all α, we obtain

$$\int_{\Delta\omega} \left[(1+\eta^2)^2 \ \frac{xM}{2}\ \overline{W}_1 + \left(\frac{1-\eta}{2}\right)^2 \ \nu\overline{W}_2\right] d\omega$$

$$\leq \frac{(2\pi)^3}{V} \sum_i \int d^3k\ d^3k' \ \langle k', q-k'; i| b^+(p)\ b(p)| i; k,\ k-q\rangle$$
$$\frac{Q^2}{2M\nu} \leq z \leq \frac{1+\eta^2}{\eta}\ \frac{Q^2}{2M\nu} \tag{23}$$

The matrix element of the right side of (21) and (23)

$$\int d^2k_\perp\ d^2k'_\perp \ \langle k',\ q-k'; i| b^+(p)\ b(p)| i; k,\ k-q\rangle$$

is related to the probability for finding a proton with momentum $\vec{p}$ in a pair of bare parton states with well defined range of z component momenta. As long as $z = 0(1)$, the probability for finding a proton in a state $|i, k, q-k\rangle$ should not change with Q. Since η and $\Delta\omega$ is arbitrary, we have shown that, (i) if the multiplicity of hadrons in the final state does not increase as fast as E, $\nu\overline{W}_2$ as well as $M\overline{W}_1$, is a function of only $\frac{2M\nu}{Q^2}$ and the result is the same as that of the naive parton model. (ii) If the multiplicity of hadrons in the final state increases as fast as Q, then $\nu\overline{W}_2$ and $M\overline{W}_1$, are bounded by a function of only $\frac{2M\nu}{Q^2}$.

The experimental check on scaling of $\nu\overline{W}_2$ and $M\overline{W}_1$ is a direct test of assumptions (a) ~ (d). But the energy momentum conservation sum rule enable us to make a strong statement about the multiplicity of hadrons. The energy conservation sum rule relates the total cross section and the multiplicity to the structure functions[3,4]

$$R(Q^2) = \frac{3}{2} \int_{\frac{M}{Q}}^{1} \omega^2 \left[M\overline{W}_1 + \frac{3}{2} \omega \quad \nu\overline{W}_2 \right] d\omega$$

$$\langle n \rangle\, R(Q^2) = \frac{3}{2} \int_{\frac{M}{Q}}^{1} \omega \left[M\overline{W}_1 + \frac{3}{2} \omega \ \nu\overline{W}_2 \right] d\omega \tag{23}$$

We have shown that for $\omega = 0(1)$, $M\overline{W}_1$ and $\nu\overline{W}_2$ are bounded by functions of only ω. Therefore, any energy dependence must come from the lower end of the integral. In particular, it is clear that if we assume that the total cross section increases at least as fast as $\log \frac{Q^2}{M^2}$ then we must have $\langle n \rangle$ growing as fast as $\frac{Q}{M}/\log\frac{Q}{M}$.

We see that in order to obtain a result that $R(Q^2)$ approaches a constant value as $Q^2 \to \infty$, we must assume a behavior of the structure function in the wee region. This requires a more specific model.

The recent data[5] on R indicates that it is increasing at least as fast as log Q/M. If the trend continues, it is most important to perform a careful study of the multiplicity as well as a check on $M\overline{W}_1$, $\nu\overline{W}_2$ scaling.

The philosophy of our formalism[1] differ greatly with those of Drell, Levy and Yan.[6] It is important to point out, however, one operational difference between our formalism and those of DLY. That is DLY discuss the energy conserving δ function in terms of energies of the hadrons. Restricting ourselves to discussing only hadron states has an advantage that the hadron states have well defined Lorentz transformation properties. The Lorentz transformation property of a parton state is unknown without detailed knowledge of the Hamiltonian. It should also be pointed out that although Bjorken scaling of $M\overline{W}_1$ and $\nu\overline{W}_2$ is predicted by DLY, they did not make any prediction about $R(Q^2)$. That is, they also have not ruled out the possibility that $R(Q^2)$ increases with Q^2 because of the unknown behavior of wees.

ACKNOWLEDGEMENT

It is a pleasure to thank S. D. Drell and H. Terazawa for fruitful and enjoyable discussions.

REFERENCES

1. The consequence of (b) is described in detail in A. I. Sanda, NAL preprint, NAL PUB-73|36-THY (1973).

2. We use the normalization $\langle p_1, \cdots, p_\ell | p_1', \cdots, p_\ell' \rangle = \prod_{i=1}^{\ell} \delta_{p_i p_i'}$ in (1) and (2). Throughout the rest of the paper $\langle p_1, \cdots, p_\ell | p_1', \cdots p_\ell' \rangle = \prod_{i=1}^{\ell} \delta^3(p_i - p_i')$ is used. The statistical factors are ignored.

3. For the case where one species of hadron is produced. The argument can easily be generalized for the case where more than one species of hadrons are present. A discussion of the energy momentum conservation sum rules can be found in C. E. DeTar, D. Z. Freedman and G. Veneziano, Phys. Rev. D4, 906 (1971).

4. The constraint on the multiplicity from the positivity of the spectral function and the energy dependence of the total cross section for various different models has also been studied by J. Pestieau and H. Terazawa, Phys. Rev. Letters 24, 1149 (1970).

5. A. Litke, et al., Phys. Rev. Letters 30, 1189 (1973).

6. S. D. Drell, D. J. Levy and T.-M. Yan, Phys. Rev. D4, 1035 (1970).

FIGURE CAPTIONS

1. The major contribution to (2) comes from the configuration in which $\vec{k}_1, \cdots, \vec{k}_n$ are contained in cones around $\vec{p}_1, \cdots, \vec{p}_\ell$.

2. We expect the overlap between the parton state $|k_1, k_2>$ and the hadron state $|p_1, \cdots, p_\ell>$ where $\vec{p}_1, \cdots, \vec{p}_\ell \epsilon \eta$ is sufficiently small so that (7) converges.

3. $e^+e^- \to h +$ anything.

4. The configuration for $e^+e^- \to$ hadron + anything.

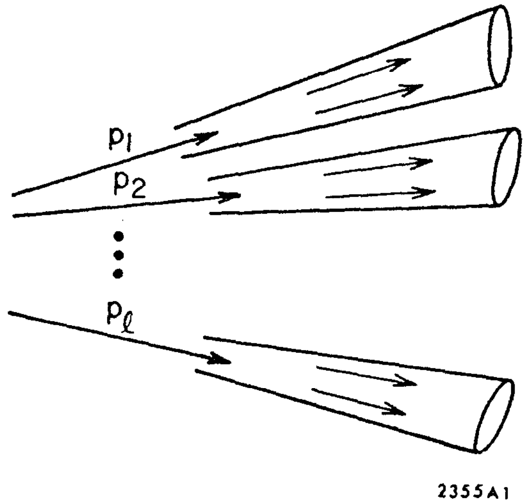

FIG. 1

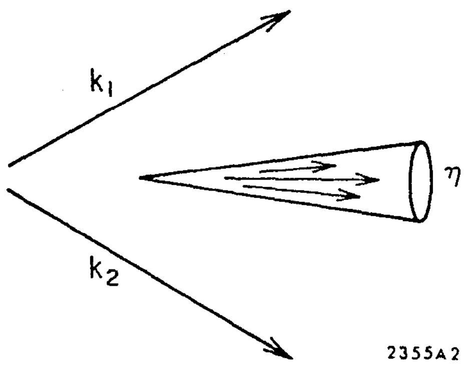

FIG. 2

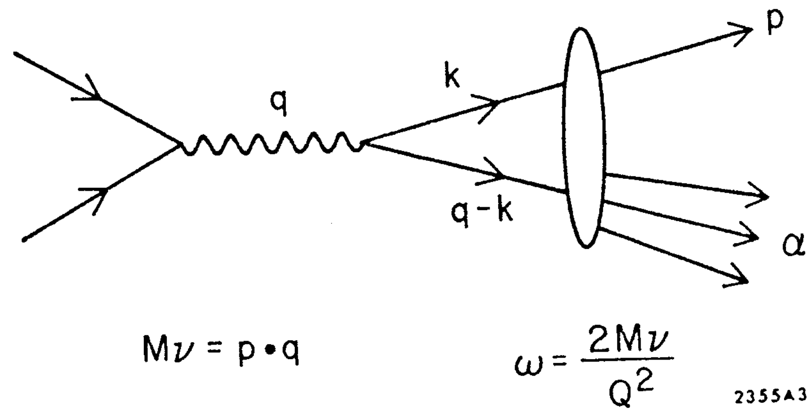

FIG. 3

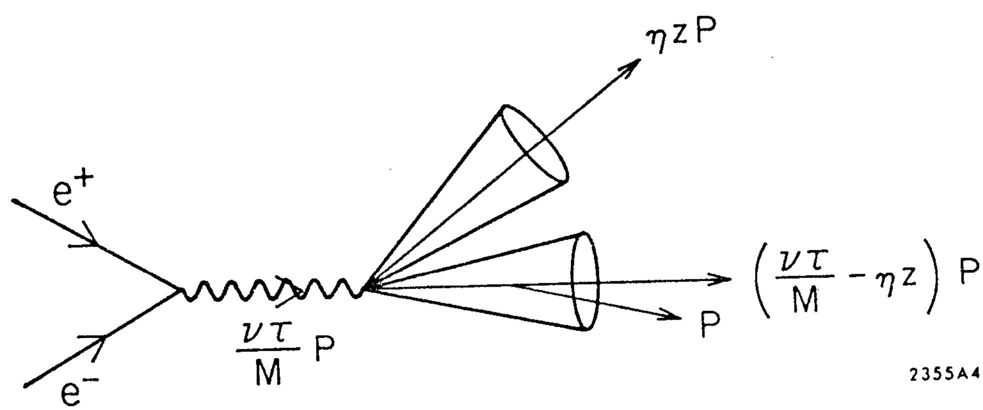

FIG. 4