SLAC-PUB-1275
(T)
July 1973

$K_{\ell 3}$ DECAYS IN A TWO COMPONENT THEORY OF PCAC*

J. L. Newmeyer and S. D. Drell
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

$K_{\ell 3}$ decays are discussed in an explicit model of PCAC involving the pion and a heavier particle, the π'. This model has recently been shown to be capable of yielding the correct $\pi^0 \to 2\gamma$ decay rate from the Adler PCAC anomaly using one triplet of fractionally charged Gell-Mann/Zweig quarks, as well as reproducing all the "good" results of strong PCAC. In this paper we extend the model to treat $K_{\ell 3}$ decays and compare it to the earlier Brandt-Preparata theory of weak PCAC.

(Submitted to Phys. Rev.)

*Work supported by the U. S. Atomic Energy Commission.

I. INTRODUCTION

A. Two Models of Weak PCAC

Models of weak PCAC[1] have been proposed in which the pion pole dominance assumption is applied only to those matrix elements of the divergence of the axial current that are taken with composite hadronic states.[2,3] The motivation for these models is inter alia to reproduce the "good" results of PCAC (Goldberger-Treiman relation,[4] Adler consistency condition,[5] and Adler-Weisberger sum rule[6]), while at the same time providing a framework yielding the correct $\pi^o \to 2\gamma$ decay rate from the Bell-Jackiw-Adler theory of the PCAC anomaly[7] without simultaneously requiring that we abandon the original Gell-Mann/Zweig quark model[8] of an elementary triplet of fractionally charged quarks. In weak PCAC schemes the equivalence between the divergence of the weak axial vector current, D(x), and the pion field, $\phi(x)$, is expressed as an approximate equality between hadronic matrix elements in a restricted momentum transfer range

$$\langle a|D|b\rangle \approx \frac{f_\pi \mu^2}{\sqrt{2}\,(\mu^2 - p^2)} \langle a\pi|b\rangle \qquad (0 < p^2 < \mu^2) \qquad \text{(I. 1)}$$

where $p = p_b - p_a$ is the four-momentum carried by the operator D, and a and b are hadrons. Here μ is the pion mass and f_π the $\pi_{\mu 2}$ decay constant. Equation (I. 1) is essentially the statement that the matrix element of D^i $(i = 1, 2, 3)$ between hadronic states is dominated (for small p^2) by the pion pole. In "strong PCAC" (I. 1) is replaced by an operator statement, $D^i = \mu^2 f_\pi \phi_i^\dagger$, which implies weak PCAC, but not vice versa. In application to the theory of the $\pi^o \to 2\gamma$ decay anomaly, strong PCAC requires that a less simple quark scheme — (viz., colored quarks[9] or the Han-Hambu[10] model of three integrally-charged triplets)— be adopted in order to explain the $\pi^o \to 2\gamma$ decay rate from the theory of the anomaly. The calculated matrix element is too small by a factor of ≈ 3 on the basis of the Gell-Mann/Zweig triplet model of quarks.

In this paper we want to apply a recent[2] model of weak PCAC (hereafter called Model A) to a discussion of $K_{\ell 3}$ decay, and of the Callan-Treiman[11] relation in particular. We also compare it with an earlier version of weak PCAC put forth several years ago by Brandt and Preparata[3] (called Model BP). As will be discussed shortly, the original aim of Model BP was to furnish a framework in which a possible "breakdown" of the Callan-Treiman relation for $K_{\ell 3}$ could be understood. However, the most recent experiment[12] is consistent with a smooth extrapolation of the $K_{\ell 3}$ form factors to the Callan-Treiman point, and, based on this beautiful new precision result, the experimental motivation for introducing Model BP no longer exists. We shall delineate the differences between Models A and BP.

B. Model A

Model A is a weak PCAC scheme in which specific statements are made about the matrix elements of the axial divergence between physical states. To epitomize it, consider a matrix element between nuclear states $\langle a|D|b\rangle$, suitably normalized, which we shall denote by a scalar function $Q(p^2)$ multiplied by a suitable tensor, where $p^2 \equiv (p_a - p_b)^2$. $Q(p^2)$ is assumed to satisfy an unsubtracted dispersion relation (USDR)

$$Q(p^2) = \frac{f_\pi \mu^2}{\mu^2 - p^2 - i\epsilon} \sqrt{2}\, g_{\pi ab}(\mu^2) + \frac{1}{\pi} \int_{9\mu^2}^{\infty} \frac{\rho(\sigma^2)\, d\sigma^2}{\sigma^2 - p^2 - i\epsilon} \tag{I.2}$$

where we have separated out the pion pole term. In the simplest form of Model A we assume that for $p^2 < \mu^2$ we can effectively replace the integral along the cut $\geq 9\mu^2$ by another pole term representing a (fictitious) particle, the π',

$$Q(p^2) \approx \frac{f_\pi \mu^2}{\mu^2 - p^2 - i\epsilon} \sqrt{2}\, g_{\pi ab}(\mu^2) + \frac{f'_\pi \mu'^2}{\mu'^2 - p^2 - i\epsilon} \sqrt{2}\, g'_{\pi' ab}(\mu'^2) \qquad (I.3)$$

whose mass μ' ($>3\mu$), decay constant f'_π, and coupling $g'_{\pi' ab}(\mu'^2)$ are parameters determined in Ref. 2 from the model's prediction for the Goldberger-Treiman relation and $\pi^0 \to 2\gamma$ decay.

The coupling constants in (I.3) are related to physical vertices as follows:

$$\langle a|D|b\rangle \approx \frac{f_\pi \mu^2}{\sqrt{2}(\mu^2 - p^2 - i\epsilon)} \langle a\pi|b\rangle + \frac{f'_\pi \mu'^2}{\sqrt{2}(\mu'^2 - p^2 - i\epsilon)} \langle a\pi'|b\rangle$$

$$(0 < p^2 < \mu^2) \qquad (I.4)$$

Now, if a and b are <u>extended composite structures</u>, then as shown in Ref. 2 we can expect the matrix elements to satisfy $\langle a\pi'|b\rangle \ll \langle a\pi|b\rangle$. It follows that with a and b hadronic states, (I.4) can be approximated by (I.1), the canonical statement of weak PCAC.

In Ref. 2 relevant parameters in Model A were fixed by experimental comparison to be

$$\frac{f'_\pi}{f_\pi} \approx -0.7 \left(\frac{g_a}{g'_a}\right)$$

$$\mu' \approx 1.65 \text{ GeV} \qquad (I.5)$$

where g_a and g'_a are the coupling of π and π' to elementary constituents. As in Ref. 2 we shall assume, for simplicity, that in the range $p^2 < \mu^2$ all the physics in the 0^- channel can be attributed to the π and π'.

C. <u>Model BP</u>

As in Eq. (I.1), the working hypothesis of Model BP is that the matrix element $\langle a|D^i|b\rangle$ of D^i ($i = 1, 2, 3$) between hadronic states a and b is dominated for small p^2 by the pion pole. In contrast to Model A, which accomplishes this by noting that the form factors in (I. 4) should satisfy $<a\pi'|b> \ll <a\pi|b>$, Model BP accomplished (I. 1) by postulating Regge asymptotic behavior for the absorptive parts of all relevant forward scattering amplitudes, which provides a criterion as to whether or not a highly convergent USDR is likely to exist for certain matrix elements. For $K_{\ell 3}$ in particular the analysis was made by studying the asymptotic behavior of the amplitudes in terms of the dimensionality of the operators and their corresponding light cone structure.[13] The matrix element $<a|D|b>$ is one of those for which a highly convergent USDR is asserted to exist, suggesting dominance by the pion. However, the matrix elements of the current operators are assumed to be less smooth than their divergences since they are of higher dimensionality and this leads to the possibility of large corrections to PCAC. The importance of these corrections depends on the way in which $SU_3 \times SU_3$ is broken as an exact symmetry. We will outline in this paper the relation of Model A to BP and show that the former leads naturally to the Callan-Treiman relation with small corrections comparable to those found in the other familiar successes of PCAC.

II. $K_{\ell 3}$ DECAYS

The $K_{\ell 3}$ experimental form factors $f_\pm(q^2)$ are defined by the physical matrix element[14]

$$F_\mu\left(q^2;p_\mu,q_\mu\right) = <\pi^0|V_\mu^{K^-}|K^+> \equiv \frac{i}{\sqrt{2}}\left[(k+p)_\mu f_+(q^2) + q_\mu f_-(q^2)\right] \qquad \text{(II. 1)}$$

where k(p) is the momentum of the $K^+(\pi^0)$, $q=k-p$ is the momentum of the lepton pair, and $V_\mu^{K^-}$ is the strangeness changing vector current. We shall also have occasion to discuss the physical matrix element of the divergence $\mathscr{D} \equiv \partial_\mu V^\mu$

of the vector current:

$$E(q^2) = \langle \pi^o | \mathscr{D}^{K^-} | K^+ \rangle \equiv -iq^\mu F_\mu(q^2) = \frac{1}{\sqrt{2}} \left[\left(m_K^2 - \mu^2 \right) f_+(q^2) + q^2 f_-(q^2) \right] \tag{II. 2}$$

The form factors $f_\pm(q^2)$ are usually parametrized $f_\pm(q^2) = f_\pm(0) \left\{ 1 + \lambda_\pm (q^2/\mu^2) \right\}$. K_{e3} experiments[15] readily give $f_+(0) \approx 1$ and $\lambda_+ \approx 0.03$, but the experimental status of f_- has been ambiguous. It is customary to take $\lambda_- \approx 0$ and report on the ratio $\xi(q^2) \equiv f_-(q^2)/f_+(q^2)$ for which polarization and Dalitz plot measurements have typically yielded $\xi(0) \sim -1$ while $K_{e3}/K_{\mu 3}$ branching measurements have yielded $\xi(0) \sim 0$, both results always quoted with large uncertainties.[15] In this regard, results from a recent SLAC-Santa Cruz collaboration[12] measuring the Dalitz plot are distinguished by very small error bars. They obtain $\xi(q^2) \approx \xi(0) = 0.01 \pm 0.04$, consistent with the original Callan-Treiman prediction. Consequences of this on models of weak PCAC will be summarized later.

Applying PCAC to (II. 1) in the soft pion limit and using current algebra to evaluate the equal time commutator of the axial charge with a strangeness-changing vector current gives directly the Callan-Treiman relation[11]

$$\left[\hat{f}_+(p^2, q^2) + \hat{f}_-(p^2, q^2) \right] \Big|_{p \to 0} = \frac{f_K}{f_\pi} \qquad \left(\text{with } \hat{f}_\pm(\mu^2, q^2) \equiv f_\pm(q^2) \text{ in (II. 1)} \right) \tag{II. 3}$$

However if, following Brandt and Preparata,[3] we choose to work with the "smoother" matrix element (II. 2) in making the PCAC extrapolation we must evaluate an equal time commutator of the axial charge with the divergence of the strangeness changing current, $\mathscr{D}^{K^-}$, and this requires the introduction of specific parameters for describing the breaking of $SU_3 \times SU_3$ invariance. The formal derivation following this path has been described elsewhere[16] and will be recapitulated briefly in the appendix.

Turning to Model A we construct the off-shell extrapolation of (II. 1) to the soft pion ($p \to 0$) point from

$$F_\mu^A\left(p^2, q^2; p_\mu, q_\mu\right) = \frac{\sqrt{2}\,(\mu^2 - p^2)}{f_\pi \mu^2} \int d^4x\, e^{ip\cdot x} \langle 0 | T\left(V_\mu^{K^-}(0) D^3(x)\right) | K\rangle \qquad \text{(II. 4)}$$

where D^3 is the divergence of the axial current. The right-hand side is evaluated by writing an unsubtracted dispersion relation in p^2, inserting the pion pole at $p^2 = \mu^2$, and replacing the continuum with a π' pole at μ'^2 as in (I. 3). This gives

$$F_\mu^A\left(p^2, q^2; p_\mu, q_\mu\right) = \frac{\sqrt{2}\,(\mu^2 - p^2)}{f_\pi \mu^2} \frac{1}{\pi} \int \frac{dp'^2}{p'^2 - p^2} \operatorname{abs}\left[F_\mu\left(p'^2, q^2; p_\mu, q_\mu\right)\right]$$

$$= \frac{i}{\sqrt{2}}\left\{\left[(k+p)_\mu f_+(q^2) + q_\mu f_-(q^2)\right] + \left(\frac{\mu^2 - p^2}{\mu'^2 - p^2}\right)\left(\frac{\mu'^2 f'_\pi}{\mu^2 f_\pi}\right)\left[(k+p)_\mu f'_+(q^2) + q_\mu f'_-(q^2)\right]\right\}$$

(II. 5)

where f'_π is the decay constant of the π' and $f'_\pm(q^2)$ are its couplings to the K and the lepton pair as illustrated in Fig. 1. The π' term in the above is that part of the off-shell matrix element of V^{K^-} which is due to the process of Fig. 1. Extrapolating to the soft pion point $p \to 0$ and applying current algebra as before gives

$$\left[f_+\left(m_K^2\right) + f_-\left(m_K^2\right)\right]\left\{1 + \left(\frac{f'_\pi}{f_\pi}\right) \frac{f'_+\left(m_K^2\right) + f'_-\left(m_K^2\right)}{f_+\left(m_K^2\right) + f_-\left(m_K^2\right)}\right\} = \frac{f_K}{f_\pi} \qquad \text{(II. 6)}$$

where the size of the "correction term" to pion dominance depends on the product of ratios shown. Equation (II. 6) is precisely the statement of the Callan-Treiman relation (II. 3) in the language of Model A.

Concerning the magnitude of the correction term we expect $|f'_\pi / f_\pi|$ to be not greatly different from unity, to the extent that the π' and π have elementary coupling constant that are comparable — i.e., $g'_a/g_a \sim 1$ in (I. 5).[2] We use the following considerations to estimate the size of the rest of the correction term. In Model A the form factors $f_{\pm}(q^2)$ describe how the K and π, members of the same pseudoscalar octet, are connected by the vector current V^K. The π' represents that contribution to the 0^- channel arising from states which lie outside this octet; $f'_{\pm}(q^2)$ describe the coupling of K and π' through V^K. If SU_3 were a perfect symmetry, we would expect $\langle K|V_0^K(q=0)|\pi\rangle = 1/\sqrt{2}$ and $\langle K|V_0^K(q=0)|\pi'\rangle = 0$. In the real world SU_3 is not perfect; we assume the ratio $\langle K|V_0^K(q=0)|\pi'\rangle / \langle K|V_0^K(q=0)|\pi\rangle$ to be a number on the order of SU_3 breaking. Normally we characterize SU_3 breaking as a 10-20% effect; here it can be measured by the ratio of the mass splittings <u>within</u> the pseudoscalar octet to the mass interval <u>between</u> the pseudoscalar octet and the excited spectrum containing the π', which gives the same result — i.e., $\left(m_K^2-\mu^2\right)/\left(\mu'^2-m_8^2\right) \approx 0.1$ where m_8 is the mean π-K octet mass. Extrapolating from $q^2=0$ to $q^2=m_K^2$ as needed in (II. 6) should not alter the above ratio significantly: the lowest mass state contributing in a dispersion analysis of the vertex $\langle K|V_0^K|\pi'\rangle$ is the K*(890) and $m_K^2 \approx \frac{1}{4} m_{K*}^2$. On the basis of this argument we expect the ratio $\left[f'_+\left(m_K^2\right)+f'_-\left(m_K^2\right)\right]/\left[f_+\left(m_K^2\right)+f_-\left(m_K^2\right)\right]$ — and therefore the entire correction term in (II. 6) — to be small, i.e., pion dominance to be good.

We now turn briefly to look at Model BP and compare it with Model A. The expression corresponding to (II. 5) (the off-shell extrapolation of (II. 1)) in the framework of Model BP is

$$F_\mu^{BP}\left(p^2,q^2;p_\mu,q_\mu\right) = \frac{i}{\sqrt{2}}\left[(k+p)_\mu\, \hat{f}_+(p^2,q^2) + q_\mu \hat{f}_-(p^2,q^2)\right] \qquad \text{(II. 7)}$$

with $\hat{f}_{\pm}(\mu^2, q^2) = f_{\pm}(q^2)$ of (II.5). The off-shell extrapolation of matrix element (II.2) (which is the "smooth" one in the BP hierarchy) is

$$E^{BP}(p^2, q^2) = \frac{1}{\sqrt{2}} \left[\left(m_K^2 - p^2 \right) \hat{f}_+(p^2, q^2) + q^2 \hat{f}_-(p^2, q^2) \right] + \frac{(\mu^2 - p^2) m_K^2 f_K}{\sqrt{2} \mu^2 f_\pi} \left(\frac{\frac{2}{3}\epsilon_2}{\epsilon_3 - \frac{2}{3}\epsilon_2} \right) \tag{II.8}$$

(The additional term in (II.8) comes from the equal-time commutator obtained in moving the divergence outside the time-ordered product, as indicated in the appendix.) Whereas the p^2 dependence of Model A in F_μ^A and E^A is explicitly exhibited — at the cost of extra free parameters $f'_{\pm}(q^2)$ — it is hidden inside the form factors $f_{\pm}(p^2, q^2)$ in the treatment of Model BP. In this sense Model BP is a more general formulation (parametrization) than Model A, being so far arbitrary regarding the number and strength of the various 0^- contributions.

On the basis of the light cone arguments mentioned in the introduction, Brandt and Preparata[3, 13] conclude that (II.8) will extrapolate smoothly from $p^2 = 0$ to $p^2 = \mu^2$ and point out that no similar claim can be made for (II.7). We write the low energy theorem in terms of (II.8), using (see appendix)

$$E\left(0, m_K^2\right) = \frac{m_K^2 f_K}{\sqrt{2} f_\pi} \frac{\epsilon_3}{\epsilon_3 - \frac{2}{3}\epsilon_2} \tag{II.9}$$

where ϵ_2 and ϵ_3 are the symmetry-breaking parameters used by Brandt and Preparata, defined in the appendix. Combining (II.9) with (II.8) gives the Callan-Treiman relation as derived by BP. Assuming a smooth extrapolation to the pion pole at $p^2 = \mu^2$ based on (II.8), BP obtained corrections to the Callan-Treiman relation that depend strongly on the ratio of symmetry breaking parameters ϵ_2/ϵ_3. In this way they were able to accommodate the then existing experimental indications of $\xi(0) \equiv \hat{\xi}(\mu^2, 0) \approx -1$, i.e., sizeable curvature in the extrapolations from the physical region for $K_{\ell 3}$ decay to the Callan-Treiman point, by choosing a large ratio of ϵ_2/ϵ_3, in contrast to the Gell-Mann, Oakes, and Renner (GOR)[17] result of approximate $SU_2 \times SU_2$ symmetry.

In Model A we do not have this freedom: changes in ϵ_2/ϵ_3 are reflected only in $f'_+\left(m_K^2\right)$ as in Eq. (A.12) of the appendix. In Model A the assumption of 10-20% absolute goodness of SU_3 automatically requires $\xi \sim 0$ from (II.6) regardless of the goodness of $SU_2 \times SU_2$; if the data showed $\xi \sim -1$, Model A would be ruled out.

III. SUMMARY

We have considered the treatment of $K_{\ell 3}$ decays in a recently proposed model[2] (Model A) of weak PCAC. In this model the 0^- channel is represented (for $0 < p^2 < \mu^2$) by a pion and another (fictitious) particle, the π', which approximates the contributions of the dispersion cut $p^2 > 9\mu^2$. The fact that the π' is not a member of the K-π octet, combined with some validity of classification under SU_3, leads naturally to a small value for the $K_{\ell 3}$ parameter ξ. Such a value for ξ is now being seen by experimentalists.[12] The generality of an earlier approach (Model BP), whose main virtue was that it provided a framework in which a larger value for $|\xi|$ could be accommodated, is thus no longer necessary.

Acknowledgements

We thank many of our colleagues at SLAC, especially L. F. Li and M. Weinstein, for several helpful discussions.

APPENDIX

In this appendix we define our notation and derive the low energy theorems (A. 11) (which is the same as (II. 6)) and (A. 12).

It is conventional to discuss models of PCAC in terms of a symmetry-breaking Hamiltonian density which is assumed to belong to a $(\bar{3}, 3) + (3, \bar{3})$ representation[18] of $SU_3 \times SU_3$:

$$H' = \frac{\epsilon_2 - \epsilon_3}{\sqrt{2}} s_0 + \epsilon_3 s_8 \tag{A. 1}$$

where s_0 and s_8 are the usual nonet scalar densities, and ϵ_2 and ϵ_3 are the parameters used by Brandt and Preparata. These last are related to the α_0 and α_8 of Gell-Mann, Oakes, and Renner[17] by $\epsilon_2 = \sqrt{2}\,\alpha_0 + \alpha_8$ and $\epsilon_3 = \alpha_8$. It has been customary to identify ϵ_2 with the $SU_2 \times SU_2$ symmetry-breaking strength ($\epsilon_2 = 0$ would indicate perfect chiral symmetry), but the identification of ϵ_3 with the SU_3 symmetry-breaking strength is ambiguous.[19] On the basis of mass formulae and using the symmetry-breaking Hamiltonian density (A. 1), Gell-Mann, Oakes, and Renner (GOR) estimate $\epsilon_2/\epsilon_3 \simeq -0.13$, which does imply that $SU_2 \times SU_2$ is a better symmetry than SU_3.

Formally, the off-shell extrapolations of both (II. 1) and (II. 2) are constructed from

$$F_\mu\left(p^2, q^2; p_\mu, q_\mu\right) \equiv \frac{\sqrt{2}(\mu^2 - p^2)}{f_\pi \mu^2} \tilde{F}_\mu\left(p^2, q^2; p_\mu, q_\mu\right)$$

$$\tilde{F}_\mu\left(p^2, q^2; p_\mu, q_\mu\right) = \int d^4x\, e^{ip\cdot x} < 0 \left| T\left(V_\mu^{K^-}(0)\, D^3(x)\right)\right| K > \tag{A. 2}$$

$$E(p^2, q^2) \equiv \frac{\sqrt{2}\,(\mu^2 - p^2)}{f_\pi \mu^2}\, \widetilde{E}(p^2, q^2)$$

$$\widetilde{E}(p^2, q^2) = \int d^4x\, e^{ip\cdot x} \langle 0 | T\left(\mathscr{D}^{K^-}(0)\; D^3(x)\right) | K \rangle \qquad \text{(A. 3)}$$

where D^3 is the axial vector current divergence and $\mathscr{D}^K$ is the vector current divergence. We can relate (A. 2) and (A. 3) using

$$T\left(\mathscr{D}^{K^-}(y) D^3(0)\right) = \frac{\partial}{\partial y_\mu}\, T\left(V_\mu^{K^-}(y) D^3(0)\right) - \delta(y_0)\left[V_0^{K^-}(y), D^3(0)\right]$$

$$\left[V_0^K(0),\, D^3(x)\right] = \frac{\frac{2}{3}\epsilon_2}{\epsilon_3 - \frac{2}{3}\epsilon_2}\, D^{K^-}(0)\, \delta(\vec{x}) \qquad \text{(A. 4)}$$

where we have evaluated the equal-time commutator in terms of the structure of the symmetry-breaking Hamiltonian density (A. 1) using $\partial_\mu J_\mu = -i[H', Q]$. The relation between (A. 2) and (A. 3) is therefore

$$E(p^2, q^2) = -iq^\mu F_\mu\left(p^2, q^2; p_\mu, q_\mu\right) + \frac{(\mu^2 - p^2) m_K^2 f_K}{\sqrt{2}\,\mu^2 f_\pi}\left(\frac{\frac{2}{3}\epsilon_2}{\epsilon_3 - \frac{2}{3}\epsilon_2}\right) \qquad \text{(A. 5)}$$

This equation is an identity on the mass shell; off the mass shell it describes the extrapolation in terms of the symmetry-breaking model.

We can now write down two low energy ($p \to 0$) theorems:

$$F_\mu\left(0, m_K^2; 0, k_\mu\right) = \frac{ik_\mu f_K}{\sqrt{2}\, f_\pi} \qquad \text{(A. 6)}$$

$$E\left(0, m_K^2\right) = \frac{m_K^2 f_K}{\sqrt{2}\, f_\pi}\; \frac{\epsilon_3}{\epsilon_3 - \frac{2}{3}\epsilon_2} \qquad \text{(A. 7)}$$

Here (A. 6) follows directly from (A. 2) and is the familiar model- and symmetry-independent result derived by Callan and Treiman. The low energy theorem (A. 7) which follows from (A. 6) using (A. 5), _is_ dependent on the symmetry breaking.

If we wish to extract predictions on the form factors $f_+(q^2)$ and $f_-(q^2)$ in (II. 1) or (II. 2) at the unphysical point $q^2 = m_K^2$ we must invoke the above low energy theorems and make assumptions regarding the p^2-extrapolation characteristics of $F_\mu\left(p^2, q^2; p_\mu, q_\mu\right)$ and $E(p^2, q^2)$ in (A. 2) and (A. 3).

To express the low energy theorems (A. 6) and (A. 7) in the language of Model A, we first write a dispersion relation in p^2 for $F_\mu\left(p^2, q^2; p_\mu q_\mu\right)$ of (A. 2):

$$F_\mu\left(p^2, q^2; p_\mu, q_\mu\right) = \frac{\sqrt{2}\,(\mu^2 - p^2)}{f_\pi \mu^2} \frac{1}{\pi} \int \frac{dp'^2}{p'^2 - p^2} \operatorname{abs} \widetilde{F}_\mu\left(p'^2, q^2; p_\mu q_\mu\right)$$

with

$$\begin{aligned}\operatorname{abs} \widetilde{F}_\mu\left(p'^2, q^2; p_\mu, q_\mu\right) &= \int d^4x\, e^{ip'\cdot x} \langle 0 | \left[D^3(x), V_\mu^K(0)\right] | K\rangle \\ &= (2\pi)^4 \sum_n \delta^4(p' - p_n) \langle 0 | D^3(0) | n\rangle \langle n | V_\mu^K(0) | K\rangle \qquad \text{(A. 8)}\end{aligned}$$

In Model A the sum contributes a term with $\delta(p'^2 - \mu^2)$ and one with $\delta(p'^2 - \mu'^2)$, so we find

$$\begin{aligned}F_\mu^A\left(p^2, q^2; p_\mu q_\mu\right) &= \frac{i}{\sqrt{2}} \left[(k+p)_\mu f_+(q^2) + q_\mu f_-(q^2)\right] + \frac{\mu^2 - p^2}{\mu'^2 - p^2} \frac{\mu'^2 f'_\pi}{\mu^2 f_\pi} \frac{i}{\sqrt{2}} \left[(k+p)_\mu f'_+(q^2) + q_\mu f'_-(q^2)\right] \\ &= \Pi_\mu + \Pi'_\mu \qquad \text{(A. 9)}\end{aligned}$$

A similar analysis for $E(p^2, q^2)$ in (A. 3) yields

$$\begin{aligned}E^A(p^2, q^2) &= \frac{1}{\sqrt{2}} \left[\left(m_K^2 - \mu^2\right) f_+(q^2) + q^2 f_-(q^2)\right] + \frac{\mu^2 - p^2}{\mu'^2 - p^2} \frac{\mu'^2 f'_\pi}{\mu^2 f_\pi} \frac{1}{\sqrt{2}} \left[\left(m_K^2 - \mu'^2\right) f'_+(q^2) + q^2 f'_-(q^2)\right] \\ &= \Pi + \Pi' \qquad \text{(A. 10)}\end{aligned}$$

Thus in this model all extrapolation properties in p^2 for $F_\mu(p^2, q^2; p_\mu, q_\mu)$ and $E(p^2, q^2)$ are explicit and relate only to the π' term in each: the extrapolation properties are fixed once values are assigned to f'_+ and f'_-.

Taking $p \to 0$ in both the above and combining with the exact low energy theorems (A.6) and (A.7) as before, we deduce the following exact low energy theorems for $K_{\ell 3}$ in Model A:

$$F^A_\mu: \left[f_+\left(m_K^2\right) + f_-\left(m_K^2\right)\right] + \frac{f'_\pi}{f_\pi}\left[f'_+\left(m_K^2\right) + f'_-\left(m_K^2\right)\right] = \frac{f_K}{f_\pi} \tag{A.11}$$

$$E^A: \left[\left(1 - \frac{\mu^2}{m_K^2}\right) f_+\left(m_K^2\right) + f_-\left(m_K^2\right)\right] + \frac{f'_\pi}{f_\pi}\left[\left(1 - \frac{\mu'^2}{m_K^2}\right) f'_+\left(m_K^2\right) + f'_-\left(m_K^2\right)\right] = \frac{f_K}{f_\pi}\left(\frac{\epsilon_3}{\epsilon_3 - \frac{2}{3}\epsilon_2}\right) \tag{A.12}$$

Equation (A.11) is the same as (II.6).

Consistency between (A.11) and (A.12) requires

$$f'_+\left(m_K^2\right) = \frac{f_K}{f'_\pi}\,\frac{m_K^2}{\mu^2}\left(\frac{-\frac{2}{3}\epsilon_2}{\epsilon_3 - \frac{2}{3}\epsilon_2}\right) - \frac{\mu^2 f_\pi}{\mu'^2 f'_\pi}\, f_+\left(m_K^2\right)$$

$$= -0.16\left(\frac{-\frac{2}{3}\epsilon_2}{\epsilon_3 - \frac{2}{3}\epsilon_2}\right) + 0.01\ (1.44) \tag{A.13}$$

where we have used $f_K/f_\pi \approx 1.3$ with (I.5) and $f_+(q^2) = 1 + 0.03\ q^2/\mu^2$.[12]

If the pion accounted for all the physics in the 0^- channel, then the π' terms in (A.11) and (A.12) would be absent and those two equations could be solved to yield

$$f_+\left(m_K^2\right) = \frac{m_K^2}{\mu^2}\,\frac{f_K}{f_\pi}\left(\frac{-\frac{2}{3}\epsilon_2}{\epsilon_3 - \frac{2}{3}\epsilon_2}\right) \tag{A.14}$$

which can be thought of as determining the "symmetry hierarchy" ϵ_2/ϵ_3 in terms of other experimentally known factors. (Although (A.14) looks different from the GOR[17] relationship between meson octet masses and the symmetry hierarchy (from which they obtain $\epsilon_2/\epsilon_3 \approx -0.13$), it has followed from the same assumption they used; namely that s_0 and s_8 in the symmetry-breaking Hamiltonian (A.1) belong to the same $(3,\bar{3}) + (\bar{3},3)$ representation of $SU_3 \times SU_3$. That relation (A.14) is satisfied experimentally to within ~20% (using the GOR value for ϵ_2/ϵ_3) is an indication that in Model A the π' will not be very important in the decay, in agreement with the expectation that, to the extent that the physical K and π lie in the same SU_3 octet and that the π' lies outside, the linking of K and π' by the vector current should be much smaller than the linking of K and π.

REFERENCES

1. For a general review of weak and strong PCAC, see S. L. Adler and R. F. Dashen, Current Algebras (W. A. Benjamin, New York, 1968).
2. S. D. Drell, Phys. Rev. D7, 2190 (1973).
3. R. A. Brandt and G. Preparata, Ann. Phys. (N.Y.) 61, 119 (1970).
4. M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).
5. S. L. Adler, Phys. Rev. 139, B1638 (1965).
6. S. L. Adler, Phys. Rev. 140, B736 (1965). W. I. Weisberger, Phys. Rev. 143, 1302 (1966).
7. J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969). S. L. Adler, Phys. Rev. 177, 2426 (1969). For a general discussion with extensive references, see S. Adler, 1970 Brandeis Summer Institute Lectures (MIT Press, Cambridge, 1971). Without the anomaly, the $\pi^0 \to 2\gamma$ decay rate vanishes in the $q^2 \to 0$ limit with PCAC. D. G. Sutherland, Nucl. Phys. B2, 433 (1967). M. Veltman, Proc. Roy. Soc. A301, 107 (1967).
8. M. Gell-Mann, Phys. Letters 8, 214 (1964). G. Zweig, CERN Report Numbers 8182/Th.401 and 8419/Th.412 (1964) (unpublished).
9. M. Gell-Mann, Lectures on "Quarks," XI Internationale Universitätswochen für Kernphysik, Schladming (1972).
10. M. Han and Y. Nambu, Phys. Rev. 139, 1006 (1965).
11. C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966).
12. G. Donaldson, D. Fryberger, D. Hitlin, J. Liu, B. Meyer, R. Piccioni, A. Rothenberg, D. Uggla, S. Wojcicki, and D. Dorfan, Report No. SLAC-PUB-1254, Stanford Linear Accelerator Center (1973) (to be published).
13. R. A. Brandt and G. Preparata, Phys. Rev. Letters 26, 1605 (1971).

14. L.-M. Chounet, J.-M. Gaillard, and M. K. Gaillard, Phys. Reports 4C, 201 (1972).

15. See a summary in Ref. 14 and references contained therein.

16. M. K. Gaillard, Nuovo Cimento 61, 499 (1969). M. Weinstein, Phys. Rev. D3, 481 (1971).

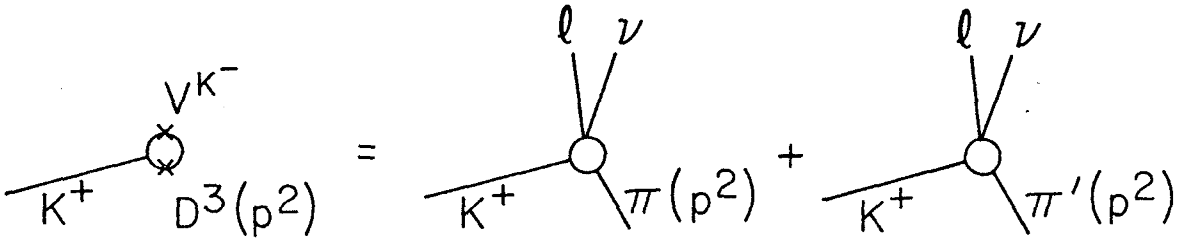
17. M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

18. It should be noted that this assumption has been criticized as being unduly restrictive. See A. Sirlin and M. Weinstein, Phys. Rev. D6, 3588 (1972).

19. In fact, $\epsilon_3 \rightarrow 0$ corresponds in some philosophies to a large violation of SU_3. See M. Weinstein, Ref. 16.

FIGURE CAPTION

1. This figure illustrates the decomposition of the off-shell matrix element (II.4) into a π and π' term as in (II.5). On the mass shell $p^2 = \mu^2$, only the first (π) term in (II.5) (i.e., the first diagram on RHS above) is present.



2326A1