SLAC-PUB-1271
(T)
July 1973

# TEST OF AMPLITUDE SIGNS IN PION PHOTOPRODUCTION PREDICTED BY THE TRANSFORMATION FROM CURRENT TO CONSTITUENT QUARKS*

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## ABSTRACT

The transformation from current to constituent quark basis states is discussed as it applies to relating amplitudes for photon-nucleon and pion-nucleon decays of baryon resonances. The predictions for the relative signs of pion photoproduction amplitudes proceeding through baryon resonances in the $\underline{70}$ L=1 and $\underline{56}$ L=2 multiplets are presented and compared with experiment. Theory and experiment are found to be completely consistent, with the pion-nucleon decay amplitudes of resonances in the $\underline{70}$ L=1 having the signs characteristic of the $(3,\bar{3}) - (\bar{3},3)$ rather than $(8,1) - (1,8)$ term in the transformed axial-vector charge.

(Submitted for publication.)

*Work supported by the U. S. Atomic Energy Commission.

A considerable advance in the theory of weak and electromagnetic transitions of hadrons has been achieved recently through formulation and application of the idea of a unitary transformation from current to constituent quark states.[1] By abstracting certain algebraic properties of the transformed vector and axial-vector currents found in the free quark model by Melosh,[2] and identifying the observed hadrons with simple constituent quark model states, one is able to relate many different hadron' $\rightarrow$ hadron + current transition amplitudes to each other.[3]

In particular, with the assumption of PCAC, pion transitions fall within the domain of applicability of the theory. As a result, both $\pi N$ and $\pi\Delta$ decay widths and the relative signs of the decay amplitudes for different $N^*$'s provide tests of the theory.[3,4] Although pionic decay widths, particularly of mesons, provide some experimental support of the theory, the relative signs of partial wave amplitudes in $\pi N \rightarrow \pi\Delta$ provide a crucial test which the theory seems to fail[4,5] when compared to the present analysis[6,7] of the reaction $\pi N \rightarrow \pi\pi N$.

The reaction $\gamma N \rightarrow \pi N$ provides another test of the theory. In this case, both the $N^* \rightarrow \pi N$ and $N^* \rightarrow \gamma N$ amplitudes are involved, so that in principle information about each vertex is obtainable for comparison with the theory. In addition, this reaction is considerably cleaner from the standpoint of experimental analysis, as no broad overlapping resonances are present in the final state. Of course, if the theory is correct, the information on the $N^* \rightarrow \pi N$ vertex obtained from $\gamma N \rightarrow \pi N$ must agree with that on the $N^* \rightarrow \pi N$ and $N^* \rightarrow \pi\Delta$ vertices obtained from $\pi N \rightarrow \pi\Delta$.

In this paper we will present the predictions for the relative signs of amplitudes for $\gamma N \rightarrow \pi N$ following from the algebraic properties of the transformed vector and axial-vector currents, as abstracted from the free quark model

calculation of Melosh.[2] Comparison with experiment yields $\pi N$ and $\gamma N$ coupling signs which are consistent with the theory and which imply that for N*'s in the $\underline{70}$ L=1 multiplet, $N^* \to \pi N$ amplitudes have the signs characteristic of the $(3,\bar{3}) - (\bar{3},3)$ rather than $(8,1) - (1,8)$ term in the transformed axial-vector charge.

The specific matrix elements we need to consider are of the form $\langle \text{hadron}' | Q_5^\alpha | \text{hadron} \rangle$ and $\langle \text{hadron}' | D_\pm^\alpha | \text{hadron} \rangle$. The operators $Q_5^\alpha$ and $D_\pm^\alpha$ are the axial-vector charge and the first moment of the vector current density, respectively:

$$Q_5^\alpha = \int d^3x \, A_0^\alpha(\vec{x}, t) \tag{1}$$

$$D_\pm^\alpha = \int d^3x \left[ \mp \frac{(x \pm iy)}{\sqrt{2}} \right] V_0^\alpha(\vec{x}, t) \quad . \tag{2}$$

$Q_5^\alpha$ is one of sixteen vector and axial-vector charges, $Q^\alpha$ and $Q_5^\alpha$ for $\alpha = 1, \ldots, 8$, which form the chiral SU(3) × SU(3) current algebra of Gell-Mann.[8] Taken between states at infinite momentum, this is a subalgebra of the still larger $SU(6)_W$ algebra of currents.[9]

We label an irreducible representation (IR) of chiral SU(3) × SU(3) as $\left\{ (A,B)_{S_z}, L_z \right\}$ where A and B are the representations of $Q^\alpha + Q_5^\alpha$ and $Q^\alpha - Q_5^\alpha$ respectively, and $S_z$ is the eigenvalue of $Q_5^0$, the singlet axial-vector charge. The quantity $L_z$ is then defined as $L_z = J_z - S_z$. With such a labeling, $Q_5^\alpha$ transforms as $\left\{ (8,1)_0 - (1,8)_0, \; 0 \right\}$ and $D_\pm^\alpha$ as $\left\{ (8,1)_0 + (1,8)_0, \; \pm 1 \right\}$.

Representations of SU(3) × SU(3) can then be built up from the representations $(3,1)_{\frac{1}{2}}$, $(1,3)_{-\frac{1}{2}}$, $(1,\bar{3})_{+\frac{1}{2}}$, and $(\bar{3},1)_{-\frac{1}{2}}$, which we define to be the current quark and current antiquark states with z spin projection $\pm\frac{1}{2}$. As such, we might attempt to form baryons as qqq and mesons as $q\bar{q}$. Such simple combinations

of quark states lead to irreducible representations of $SU(3) \times SU(3)$ characterizing the transformation properties of hadrons under the algebra of currents.[10] However, it has long been known that the physically observed hadrons, like the nucleon, do not transform as simple IR's of the algebra of currents. For example, if the nucleon were built of only three current quarks, $g_A$ would equal 5/3 and $\mu_A$ would vanish. Phenomenological analysis[11] shows that the nucleon and other hadrons are quite complicated mixtures of IR's of the algebra of currents.

Let us assume that there exists a unitary operator, V, which transforms the IR characteristic of simple combinations of current quarks into the corresponding physical state:

$$|\,\text{hadron}\rangle = V\,|\,\text{IR, currents}\rangle \quad . \tag{3}$$

Then a matrix element we wish to consider may be rewritten

$$\langle\text{hadron}'\,|\,Q_5^\alpha\,|\,\text{hadron}\rangle = \langle\text{IR}',\ \text{currents}\,|\,V^{-1}Q_5^\alpha V\,|\,\text{IR, currents}\rangle \quad , \tag{4}$$

and similarly for matrix elements of $D_\pm^\alpha$.

The operators $V^{-1}Q_5^\alpha V$ and $V^{-1}D_\pm^\alpha V$ may be studied as independent quantities, and then applied to calculating the matrix element formed by taking $Q_5^\alpha$ or $D_\pm^\alpha$ between any two hadron states.[12] Following the work of Melosh,[2] we abstract the algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_+^\alpha V$ from the free quark model, and assume that:

$$\begin{aligned} V^{-1}Q_5^\alpha V \text{ transforms as a sum of } & \left\{(8,1)_0 - (1,8)_0,\ 0\right\} \\ \text{and } & \left\{(3,\bar{3})_1,\ -1\right\} - \left\{(\bar{3},3)_{-1},\ 1\right\} \end{aligned} \tag{5}$$

$$\begin{aligned} V^{-1}D_+^\alpha V \text{ transforms as a sum of } & \left\{(8,1)_0 + (1,8)_0,\ +1\right\} \\ \text{and } & \left\{(3,\bar{3})_1,\ 0\right\} \\ \text{and } & \left\{(\bar{3},3)_{-1},\ 2\right\} \quad , \end{aligned} \tag{6}$$

and all terms behave as components of the $\underline{35}$ representation of the full $SU(6)_W$ of currents.

With the assumption that the IR in Eq. (3) is that of the simple quark model, where baryons are qqq and mesons $q\bar{q}$ with internal angular momentum L between the quarks, we know the transformation properties of all quantities appearing on the right-hand side of Eq. (4). One matrix element may be related to another by the Clebsch-Gordan coefficients of the $SU(6)_W$ of currents. In particular, for given initial and final $SU(6)_W$ multiplets, Eqs. (5) and (6) show that[4] there are at most two independent reduced matrix elements for $Q_5^\alpha$, and at most three for $D_+^\alpha$.

Invoking the PCAC hypothesis, we find a proportionality between matrix elements of $Q_5^\alpha$ (between states at infinite momentum), and those of the pion field, $\phi_\pi^\alpha$. The theory then applies to all pion transitions between hadrons,[3] and in particular to the decays of the $\underline{70}$ L=1 and $\underline{56}$ L=2 baryon resonances by pion emission into the ground state $\underline{56}$ L=0. For each of these sets of decays we have two independent reduced matrix elements, corresponding to the $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ terms in $V^{-1}Q_5^\alpha V$, and all helicity amplitudes for the respective decays may be expressed in terms of them.[4] Linear combinations of these reduced matrix elements have definite orbital angular momentum ($\ell$) properties (between the final hadron and pion), and these are most convenient for our purposes here.

Therefore, for $\underline{70}$ L=1 $\rightarrow$ $\underline{56}$ L=0 and $\underline{56}$ L=2 $\rightarrow$ $\underline{56}$ L=0 pionic decays we express the $N^* \rightarrow \pi N$ amplitudes in terms of amplitudes[13] S and D, corresponding to $\ell$=0 and 2, and in terms of amplitudes P and F corresponding to $\ell$=1 and 3,

respectively. The normalization is chosen such that[4, 13]

$$\begin{aligned} &\langle L{=}1 \| (8,1)_0 - (1,8)_0 \| L{=}0 \rangle = \frac{1}{3}(S{+}2D) \\ &\langle L{=}1 \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| L{=}0 \rangle = \frac{1}{3}(S{-}D) \end{aligned} \quad , \tag{7}$$

and

$$\begin{aligned} &\langle L{=}2 \| (8,1)_0 - (1,8)_0 \| L{=}0 \rangle = \frac{1}{5}(2P{+}3F) \\ &\langle L{=}2 \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| L{=}0 \rangle = \frac{\sqrt{3}}{5}(P{-}F) \quad . \end{aligned} \tag{8}$$

Then $S=+D$ and $P=+F$ if only the $(8,1)_0 - (1,8)_0$ term in $V^{-1}Q_5V$ is present, and $S=-2D$ and $2P=-3F$ if only $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ is present. Note that for either the $\underline{70}$ L=1 or $\underline{56}$ L=2 pionic decays the $\ell$-amplitudes have the same (opposite) sign if they originate from the $(8,1)_0 - (1,8)_0$ $\left((3,\bar{3})_1 - (\bar{3},3)_{-1}\right)$ term in $V^{-1}Q_5V$. Thus their relative sign is a direct indication of which term is dominant in a particular set of decays from one SU(6) multiplet to another.

For the $N^* \to \gamma N$ vertex we need no additional assumption: matrix elements of $D_{\pm}^{\alpha}$ are proportional to helicity $\pm 1$ real photon transition amplitudes between states at infinite momentum. While we have made an extensive analysis[14] of both meson and baryon photon transitions within the context of the present theory, we only employ here the results for $\gamma N$ decays of baryon resonances in the $\underline{70}$ L=1 and $\underline{56}$ L=2 multiplets. For the $\underline{70}$ L=1 decays to $\gamma N$, only the $\left\{(8,1)_0 + (1,8)_0,\ 1\right\}$ and $\left\{(3,\bar{3})_1,\ 0\right\}$ terms in $V^{-1}D_{+}^{\alpha}V$ can contribute, as $|\Delta L_z| \leq 1$, and we label the reduced matrix elements as $R_1$ and $R_2$ respectively. For $\underline{56}$ L=2 decays the $\left\{(8,1) + (1,8)_0,\ 1\right\}$, $\left\{(3,\bar{3})_1, 0\right\}$, and $\left\{(\bar{3},3)_{-1},\ 2\right\}$ terms in $V^{-1}D_{+}^{\alpha}V$ can all contribute, and their reduced matrix elements are defined as $R_1'$, $R_2'$, $R_3'$.

To construct the predictions for relative signs in $\gamma N \to \pi N$, we consider the usual expansion of helicity amplitudes[15]

$$f_{\mu\lambda}(s,\theta) = \sum_J (2J+1)\, d^J_{\lambda\mu}(\theta)\, f^J_{\mu\lambda}(s) \quad , \tag{7}$$

where $\lambda$ and $\mu$ are the net helicities in the initial and final state, respectively. A given resonance with spin J contributes a pole to $f^J_{\mu\lambda}(s)$ whose residue is proportional to the product of the resonance couplings to $\gamma N$ (with net helicity $\lambda$) and to $\pi N$ (with net helicity $\mu$). For each resonance with given $J^P$, there are two independent[16] photon couplings and therefore two independent partial wave helicity amplitudes $f^J_{\mu\lambda}$, which we choose as $f^J_{1/2,1/2}$ and $f^J_{1/2,3/2}$. In Table I we list the product of the theoretically predicted $\gamma N$ and $\pi N$ couplings,[17] as they contribute to $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$ in the amplitudes $f^J_{1/2,1/2}$ and $f^J_{1/2,3/2}$ for resonances in the $\underline{70}$ L=1 and $\underline{56}$ L=2. Also given are the experimental values[17] for the resonant amplitudes, as analyzed by Moorhouse and Oberlack.[18,19]

First consider states in the $\underline{70}$ L=1, and in particular the $D_{13}(1520)$, as observed in $\gamma p \to \pi^+ n$, which we take as input. The nearly zero $\lambda=1/2$ amplitude (compared to that for $\lambda=3/2$) must arise from a cancellation between the $R_1$ and $R_2$ reduced matrix elements ($R_1 \simeq +2R_2$), which establishes both their relative sign and magnitude.[20] From $f^{3/2}_{1/2,3/2}$ on protons, we may fix $F_1D$ to be positive, using the free overall sign we have at our disposal. Taking D/S to be negative, all other amplitudes for resonances in the $\underline{70}$ L=1 are now fixed in sign, with the predicted signs listed in the table.[21] All 12 remaining signs agree with experiment, with the possible exception of the poorly determined amplitudes for production on neutrons of the $S_{11}(1715)$ and $D_{15}(1670)$ with $\lambda=1/2$ and the $D_{13}(1700)$ with $\lambda=3/2$, which are zero within errors.[22] In particular, we note

that the sign of D/S can be determined from the rather well determined $S_{11}(1535)$ and $D_{13}(1520)$ amplitudes on a proton target. Possible mixing of the quark spin 1/2 and 3/2 $S_{11}$ and $D_{13}$ states does not change this result, since the $N^* \to \gamma p$ vertex vanishes for the S=3/2 states.

For the 56 L=2, we fix our attention on the $F_{15}(1688)$. Again $R_1'$ and $R_2'$ must cancel each other ($R_1' \simeq +\sqrt{3}\, R_2'$) to make the $\lambda=1/2$ amplitude small in $\gamma p \to \pi^+ n$. Furthermore, $R_3'$ must also be negligible to yield the small $\lambda=3/2$ amplitude in $\gamma n \to \pi^- p$. Choosing $R_1'F$ to be positive to agree with the large $\lambda=3/2$ amplitude in $\gamma p \to \pi^+ n$, we predict all(5) other experimentally determined resonance signs correctly. Unfortunately, no photoproduction amplitudes for p-wave $\pi N$ resonances in the 56 L=2 are known, so we can say nothing yet about the sign of P/F.

In summary, there is agreement of the experimentally determined signs of resonant amplitudes in $\gamma N \to \pi N$ with the theory. Furthermore S/D negative for $\pi N$ decays of baryons in the 70 L=1 is strongly indicated. This is the sign of S/D given by the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term in $V^{-1}Q_5^{\alpha}V$. It furthermore is the sign suggested[3-5] in $\pi N \to \pi\Delta$, if the resonant amplitudes (of the $D_{13}(1520)$) below the energy gap in the analyzed data for $\pi N \to \pi\pi N$ could be reversed relative to those above the gap. In as much as there is no such gap in the photoproduction data, the agreement found in the present paper lends strong support to the theory for both $\gamma N$ and $\pi N$ decays, and to the dominance of the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term in $V^{-1}Q_5^{\alpha}V$ in the $\pi N$ decays of resonances in the 70 L=1. We will present more quantitative tests of this theory of photon transitions in the near future.[14]

We thank M. Kugler, S. Meshkov, R. G. Moorhouse, and H. Oberlack for informative discussions.

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9. R. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965).
10. We always consider the classification of hadron states at infinite momentum, or, what is equivalent for our purposes here, the classification of states at rest under light-like charges.
11. For a review, see H. Harari in Spectroscopic and Group Theoretical Methods in Physics (North-Holland, Amsterdam, 1968); p. 363.

12. In the following we need only consider the operator $D_+^\alpha$ with $J_z=+1$, since all matrix elements of $D_-^\alpha$ are related to those of $D_+^\alpha$ by parity.

13. See the discussion in Ref. 4. Closely related amplitudes were originally defined in the $\ell$-broken $SU(6)_W$ scheme by W. P. Petersen and J. L. Rosner, Phys. Rev. D 6, 820 (1972).

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15. M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959).

16. For a given resonance, parity relates the other two partial wave helicity amplitudes to those considered.

17. For the theoretical couplings of $\gamma N$ and $\pi N$ we take the matrix elements of $D_+$ and $Q_5$, respectively, as discussed in Refs. 4 and 14. For the experimental values of photoproduction amplitudes we take the results of Moorhouse and Oberlack, Ref. 18, Table I, multiplied by the correct signs of isotopic spin Clebsch-Gordan coefficients necessary to have their amplitudes correspond to the processes $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$. The resulting experimental amplitudes are all related by known, positive, kinematic factors to the theoretical values in our table.

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19. R. G. Moorhouse and H. Oberlack, Ref. 18, also present theoretical values for the photoproduction amplitudes, based on the quark model of R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971). The theoretical signs of the $\lambda=1/2$ photoproduction amplitudes on neutrons for the $S_{11}(1715)$, $D_{13}(1700)$, and $D_{15}(1670)$ resonances in

Ref. 18, which all disagree with those presented in Table I of this paper, are in error. We thank Professor Moorhouse for a private communication on this matter, and a discussion of amplitude signs in quark models.

20. This corresponds to the well-known cancellation in quark models between the terms arising from the convection current and the quark magnetic moments in the $\lambda=1/2$ amplitude for $\gamma p \rightarrow D_{13}(1520)$.

21. We use the relation $R_1 \simeq +2R_2$ to obtain the signs of amplitudes where both $R_1$ and $R_2$ occur with opposite signs.

22. Photoproduction amplitudes of the $S_{11}(1715)$ and $D_{13}(1700)$, assumed here to correspond mostly with the quark spin 3/2 states, are also somewhat sensitive to mixing within the $\underline{70}$ L=1 between the strongly coupled quark spin 1/2 and weakly coupled quark spin 3/2 states.

TABLE I

Comparison of predictions for signs of resonant amplitudes in $\gamma N \rightarrow \pi N$ with experimental values. Resonant states are labeled by their corresponding quark model quantum numbers $J^P$ and $[\text{SU(3) multiplet}]^{2S+1}$, where S is the net quark spin. The labels p and n stand for the processes $\gamma p \rightarrow \pi^+ n$ and $\gamma n \rightarrow \pi^- p$ respectively, while $\lambda=1/2$ and $\lambda=3/2$ refer to the partial wave helicity amplitudes $f^J_{1/2,1/2}$ and $f^J_{1/2,3/2}$. In the $\underline{70}$ L=1, S/D is taken to be negative to obtain the predicted signs given in the table.

| Resonance | | Product of Theoretical Couplings[17] | Predicted Sign | Experimental Value[17, 18] |
|---|---|---|---|---|
| | | 70 L=1 | | |
| $D_{13}(1520)$ | $p, \lambda=1/2$ | $(1/54)(R_1-2R_2)D$ | $\approx 0$ (input) | - 26±15 |
| $3/2^-$, $[8]^2$ | $p, \lambda=3/2$ | $(\sqrt{3}/54)\ R_1 D$ | + (input) | +194±31 |
| | $n, \lambda=1/2$ | $(1/162)(3R_1-2R_2)D$ | + | + 85±14 |
| | $n, \lambda=3/2$ | $(\sqrt{3}/54)\ R_1 D$ | + | +124±13 |
| $S_{11}(1535)$ | $p, \lambda=1/2$ | $(-1/54)(R_1+R_2)S$ | + | + 53±20 |
| $1/2^-$, $[8]^2$ | $n, \lambda=1/2$ | $(-1/162)(3R_1+R_2)S$ | + | + 48±21 |
| $S_{31}(1640)$ | $p, \lambda=1/2$ | $(-1/648)\ (3R_1-R_2)S$ | + | + 90±76 |
| $1/2^-$, $[10]^2$ | | | | |
| $D_{33}(1690)$ | $p, \lambda=1/2$ | $(1/648)(3R_1+2R_2)D$ | + | + 68±42 |
| $3/2^-$, $[10]^2$ | $p, \lambda=3/2$ | $(\sqrt{3}/216)\ R_1 D$ | + | + 22±52 |
| $D_{15}(1670)$ | $p, \lambda=1/2$ | 0 | 0 | + 11±12 |
| $3/2^-$, $[8]^4$ | $p, \lambda=3/2$ | 0 | 0 | + 21±20 |
| | $n, \lambda=1/2$ | $(1/180)\ R_2 D$ | + | - 10±40 |
| | $n, \lambda=3/2$ | $(\sqrt{2}/180)\ R_2 D$ | + | + 35±14 |
| $D_{13}(1700)$ | $p, \lambda=1/2$ | 0 | 0 | + 3±? |
| $3/2^-$, $[8]^4$ | $p, \lambda=3/2$ | 0 | 0 | + 20±? |
| | $n, \lambda=1/2$ | $(1/1620)\ R_2 D$ | + | + 28±? |
| | $n, \lambda=3/2$ | $(\sqrt{3}/540)\ R_2 D$ | + | - 27±? |
| $S_{11}(1715)$ | $p, \lambda=1/2$ | 0 | 0 | + 66±42 |
| $1/2^-$, $[8]^4$ | $n, \lambda=1/2$ | $(1/324)\ R_2 S$ | - | + 72±66 |

| Resonance | | Product of Theoretical Couplings[17] | Predicted Sign | Experimental Value[17, 18] |
|---|---|---|---|---|
| | | 56 L=2 | | |
| $F_{15}(1690)$ | $p, \lambda=1/2$ | $(2\sqrt{3}/135)(R_1'-\sqrt{3}R_2')F$ | ≈0 (input) | - 8±4 |
| $5/2^+$, $[8]^2$ | $p, \lambda=3/2$ | $(2\sqrt{6}/135)(\sqrt{2}R_1'+R_3')F$ | + (input) | +100±12 |
| | $n, \lambda=1/2$ | $(-4/135)\ R_2'\ F$ | - | - 17±14 |
| | $n, \lambda=3/2$ | $(+4\sqrt{3}/405)\ R_3'\ F$ | ≈0 (input) | + 5±18 |
| $P_{13}(1860)$ | $p, \lambda=1/2$ | $(-2/135)(\sqrt{3}R_1'+2R_2')P$ | | |
| $3/2^+$, $[8]^2$ | $p, \lambda=3/2$ | $(2/135)(R_1'-2\sqrt{2}R_3')P$ | | |
| | $n, \lambda=1/2$ | $(-8/405)\ R_2'P$ | | |
| | $n, \lambda=3/2$ | $(-8\sqrt{2}/405)R_3'P$ | | |
| $F_{37}(1950)$ | $p, \lambda=1/2$ | $(-8/4725)(6R_2'+\sqrt{6}R_3')F$ | - | -133±46 |
| $7/2^+$, $[10]^4$ | $p, \lambda=3/2$ | $(-8/4725)(2\sqrt{15}R_2'+\sqrt{10}R_3')F$ | - | -100±41 |
| $F_{35}(1890)$ | $p, \lambda=1/2$ | $(-24/13175)(R_2'-\sqrt{6}R_3')F$ | - | - 60±? |
| $5/2^+$, $[10]^4$ | $p, \lambda=3/2$ | $(-8\sqrt{6}/13175)(3\sqrt{3}R_2'-2\sqrt{2}R_3')F$ | - | -100±? |
| $P_{33}(\quad)$ | $p, \lambda=1/2$ | $(-8/2025)(R_2'-\sqrt{6}R_3')P$ | | |
| $3/2^+$, $[10]^4$ | $p, \lambda=3/2$ | $(+8/2025)(\sqrt{3}R_2'+\sqrt{2}R_3')P$ | | |
| $P_{31}(1860)$ | $p, \lambda=1/2$ | $(8/2025)(R_2'+\sqrt{6}R_3')P$ | | |
| $1/2^+$, $[10]^4$ | | | | |