# $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{0} \gamma$ DECAYS IN A CURRENT-CURRENT QUARK MODEL* 

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#### Abstract

The decay rates for $\mathrm{K}^{ \pm} \rightarrow \pi^{ \pm} \pi^{\circ} \gamma$ are calculated in a zero parameter modified fermion-loop model first proposed by Rockmore and Wong. The weak Hamiltonian is phenomenologically constructed from one-baryon octet matrix elements. The predicted branching ratio $r=R\left(K^{ \pm} \rightarrow \pi^{ \pm} \pi \pi^{0} \gamma\right.$; $\left.55 \mathrm{MeV} \leq \mathrm{T}_{\pi^{ \pm}} \leq 90 \mathrm{MeV}\right) / \mathrm{R}\left(\mathrm{K}^{ \pm} \rightarrow \mathrm{all}\right)=1.56 \times 10^{-5}$ is in excellent agreement with the recent experimental result of Abrams et al.


[^0]Recently two of us ${ }^{1}$ have shown that when the baryon-loop model, first introduced by Steinberger ${ }^{2}$ to explain the decay $\pi^{0} \rightarrow \gamma \gamma$, is suitably modified for weak interactions, ${ }^{1}$ it unexpectedly provides a qualitative explanation for the decay $K_{2}^{o} \rightarrow \gamma \gamma$. In a subsequent paper, ${ }^{3}$ the same authors calculated the decay rate for

$$
\begin{equation*}
\mathrm{K}_{2}^{\mathrm{o}} \rightarrow \pi^{+} \pi^{-} \gamma \tag{1}
\end{equation*}
$$

and found that the same zero parameter model gives a result which is just below the experimental upper limit. ${ }^{4}$

In a recent publication, Abrams et al. ${ }^{5}$ reported the observation of a direct emission amplitude in the decays

$$
\begin{equation*}
\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{0} \gamma \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{K}^{-} \rightarrow \pi^{-} \pi^{\mathrm{o}} \gamma \tag{3}
\end{equation*}
$$

The experimental branching ratio is

$$
\begin{equation*}
\frac{\mathrm{R}\left(\mathrm{~K}^{\perp} \rightarrow \pi^{\perp} \pi^{\mathrm{O}} \gamma\right)}{\mathrm{R}\left(\mathrm{~K}^{ \pm} \rightarrow \text { all }\right)}=(1.56 \pm 0.35) \times 10^{-5} \tag{4}
\end{equation*}
$$

with $55 \mathrm{MeV} \leq \mathrm{T}_{\pi^{ \pm}} \leq 90 \mathrm{MeV}$. This number presents a direct challenge to our model.

In this note, we give the result of a calculation of the decay rates for reactions (2) and (3). The calculation is very similar to-the one for the decay (1) and we refer to Ref. 3 for the details. As in Ref. 3, we describe the decays in terms of the two possible mechanisms graphically illustrated in Figs. (1) and (2). Their contributions to the decay amplitudes are denoted by
$A^{( \pm)}$and $A_{\rho}^{( \pm)}$respectively, where

$$
\begin{gather*}
\epsilon\left(\epsilon(\mathrm{q}, \lambda) \mathrm{p}_{\mathrm{K}^{\mathrm{p}} \pi^{+}} \mathrm{p} \pi^{\mathrm{o}}\right)\left[\mathrm{A}^{(+)}+\mathrm{A}_{\rho}^{(+)}\right]\left(\mathrm{p}_{\pi^{+}}^{2}, \mathrm{p}_{\pi^{\mathrm{o}}}^{2}, \mathrm{p}_{\mathrm{K}}^{2}, \mathrm{p}_{\mathrm{K}} \cdot \mathrm{p}_{\pi^{+}}, \mathrm{p}_{\mathrm{K}^{\prime}} \cdot \mathrm{p}_{\pi^{\mathrm{o}}}, \mathrm{p}_{\pi^{+}} \cdot \mathrm{p}_{\pi^{\mathrm{o}}}\right) \\
\quad=\left(16 \mathrm{~m}_{\mathrm{K}^{2}} \mathrm{E}_{\pi^{+}} \mathrm{E}_{\pi^{\mathrm{o}}} \mathrm{E}_{\gamma}\right)^{1 / 2}\left\langle\gamma(\mathrm{q}) \pi^{+}\left(\mathrm{p}_{\pi^{+}}\right) \pi^{\mathrm{o}}\left(\mathrm{p}_{\pi^{\mathrm{o}}}\right) \text { out }\right| \mathscr{H}_{\mathrm{W}}(0)\left|\mathrm{K}^{+}\left(\mathrm{p}_{\mathrm{K}}\right)\right\rangle \tag{5}
\end{gather*}
$$

The baryons travelling around the loop can be p, $\Sigma^{+}$, etc. with the appropriate charge and $\operatorname{SU}(3)$ index. A straightforward calculation gives

$$
\begin{equation*}
A^{(-1)}=\frac{\sqrt{2} e^{3}}{(4 \pi)^{2} m^{4}} 2\left\{f F\left(3 f^{2}-\frac{73}{3} d^{2}\right)+d D\left(13 f^{2}+3 d^{2}\right)\right\} \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
A_{\rho}^{(+)}=\frac{\sqrt{2} \operatorname{egg}_{\rho} \phi}{(4 \pi)^{2} \mathrm{~m}^{2}} \frac{64}{9}\left\{\mathrm{dD}+\frac{\delta}{\phi}(\mathrm{fD}-2 \mathrm{dF})\right\} \times \frac{1}{\left(\mathrm{p}_{\pi^{+}}+\mathrm{p}_{\pi^{0}}\right)^{2}-\mathrm{m}_{\rho}^{2}} \tag{7}
\end{equation*}
$$

The definitions of the various quantities can be found in Ref. 3.
We remark that Eqs. (6) and (7) are the result of complicated sums of many terms, and they can not be obtained from Eqs. (6) and (8) of Ref. 3 by a simple isospin argument. ${ }^{6}$ On the other hand, we do have

$$
\begin{equation*}
A^{(+)}=-A^{(-)}, \quad A_{\rho}^{(+)}=-A_{\rho}^{(-)} \tag{8}
\end{equation*}
$$

as can be seen from the following observation. Consider, for example, the diagrams in Fig. 3a. They are identical except for the direction of the loop momenta, which gives rise to a different sign from the tensor structure. In the case of Fig. 3b, however, the direction of loop momenta does not matter, but the $\rho \pi \pi$ vertex changes sign.

Finally, the decay rate is given by

$$
\begin{aligned}
& \mathrm{R}=\frac{1}{64 \pi^{3} \mathrm{~m}_{\mathrm{K}}} \int \mathrm{dE}{\pi^{+}}^{\mathrm{dE}} \pi_{\pi^{\mathrm{o}}} \quad \theta\left\{4\left(\mathrm{E}_{\pi^{+}}^{2}-\mathrm{m}_{\pi^{+}}^{2}\right)\left(\mathrm{E}_{\pi^{\mathrm{o}}}^{2}-\mathrm{m}_{\pi^{\mathrm{o}}}^{2}\right)\right. \\
& \left.+\mathrm{m}_{\mathrm{K}}^{2}-2 \mathrm{~m}_{\mathrm{K}}\left(\mathrm{E}_{\pi^{+}}+\mathrm{E}_{\pi^{\mathrm{o}}}\right)+2 \mathrm{E}_{\pi^{+}} \mathrm{E}_{\pi^{\mathrm{o}}}+\mathrm{m}_{\pi^{+}}^{2}+\mathrm{m}_{\pi^{\mathrm{o}}}^{2}\right\}
\end{aligned}
$$

with $55 \mathrm{MeV} \leq \mathrm{T}_{\pi^{+}} \equiv\left(\mathrm{E}_{\pi^{+}}-\mathrm{m}_{\pi^{+}}\right) \leq 90 \mathrm{MeV}$.
A two dimensional numerical integration of Eq. (9) gives

$$
\begin{equation*}
\mathrm{R}\left(\mathrm{~K}^{ \pm} \rightarrow \pi^{ \pm} \pi^{\mathrm{o}} \gamma\right)=0.832 \times 10^{-12} \mathrm{eV} \tag{10}
\end{equation*}
$$

or

$$
\begin{equation*}
\left[\frac{\mathrm{R}\left(\mathrm{~K}^{ \pm} \rightarrow \pi^{ \pm} \pi^{o} \gamma\right)}{\mathrm{R}\left(\mathrm{~K}^{ \pm} \rightarrow \text { all }\right)}\right]_{\text {theo }}=1.56 \times 10^{-5} \tag{11}
\end{equation*}
$$

which is in excellent agreement with the experimental value in Eq. (4).
As a check on our program we also calculated the inner bremsstrahlung contribution to the decays (2) and (3) in the same energy interval finding the branching ratio $2.43 \times 10^{-4}$. This agrees with the number quoted in Ref. 5 .

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6. A similar situation occurred before in the calculation of $\gamma+\gamma \rightarrow \pi^{0} \pi^{\circ} \pi^{\circ}$ and $\gamma+\gamma \rightarrow \pi^{+} \pi^{-} \pi^{\circ}$. See, for example, T. F. Wong, Phys. Rev. Letters 27, 1617 (1971).

## FIGURE CAPTIONS

1. Baryon-loop graphs for emission of "uncorrelated" pions in $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{\circ} \gamma$ decays.
2. Baryon-loop graphs for emission of "correlated" pions (from virtual $\rho$-decay) in $\mathrm{K}^{ \pm} \rightarrow \pi^{ \pm} \pi^{\mathrm{o}} \gamma$ decays.
3. Examples of diagrams in $\mathrm{K}^{ \pm} \rightarrow \pi^{ \pm} \pi^{0} \gamma$ which are equal to each other and opposite in sign.




+ (Three Sets of Similar Diagrams With The $x$ on Other Sides)

Fig. 1


Fig. 2


Fig. 3


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