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MUON POLARIZATION IN THE NEUTRINO-INDUCED INCLUSIVE REACTION*

— A Test of V-A Weak Interaction at High Energy

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ABSTRACT

A possible modification of the V-A weak interaction at high energy is studied through the introduction of an intermediate vector boson with derivative couplings or of a phenomenological momentum dependent leptonic vector current. It will be shown that a small contamination of a certain derivative coupling changes the sign of the longitudinal polarization of the muon in the neutrino nucleon inelastic scattering in the scaling region and let the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ increase as the neutrino energy gets larger, while a different momentum dependent vector current leaves the sign of the polarization and the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ unchanged.

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I. INTRODUCTION

The conventional V-A theory of the weak interaction[1] has been able to describe low energy leptonic interactions remarkably well, in spite of the fact that higher order contributions in perturbation theory badly diverge. But this seemingly satisfactory consistancy of V-A theory with experiment cannot be regarded as proof of the validity of the theory, rather it implies that present data is not sufficient to fix uniquely all the general nonderivative coupling constants. In fact Jarskog[2] claims that up to 30% scalar and tensor couplings would not be contradictory to the experimental muon decay data. Moreover, even at the phenomenological level, there is no fundamental reason why the current-current theory should prevail at higher momentum transfers. The idea of a mediating particle for the weak interaction analogous to the photon in quantum electrodynamics or the Yukawa-type pion field in the strong interactions has long been attractive, because this nonlocal theory reduces to the local current-current interaction at low energies. Though intermediate vector boson theory is no less renormalizable, theoretical consequences were examined extensively in the literature. But this line of effort has always been embarrassed by the fact that intermediate vector bosons are not observed experimentally. This is also the case for the renormalizable theories of spontaneous symmetry breakdown, which employ unobserved gauge bosons and/or heavy leptons.

On the other hand, it would be very interesting to know what kind of theory even with phenomenological Hamiltonian would explain the experimental data far above the unitary limit.[3] Recently a model[4] of weak interactions at high energy by Appelquist, Bjorken, and Chanowitz extending the conventional current-current theory to incorporate possible higher order effects has been proposed. In this paper, we investigate some theoretical consequences of

derivatively coupled intermediate vector boson (IVB), or equivalently phenomenological momentum dependent vector current, in the Bjorken-Johnson-Low scaling limit. Previously, IVB theorists excluded the possibility of a derivative coupling on the grounds that: (a) in the observed leptonic weak interactions, momentum transfer q is so small as to be neglected, (b) introduction of a derivative coupling provides a momentum q at each vertex in higher orders making already divergent Feynman diagrams more so. Nevertheless high energy accelerators available at present or in the future may be able to detect the momentum dependence of the leptonic weak vector current directly, if it exists at all. And the argument of divergent graphs may be bypassed in the meanwhile because we are dealing with the phenomenological Lagrangian anyway. In fact it turns out that a certain derivative coupling is not worse than the conventional V-A intermediate vector boson theory of weak interactions as we will see later. In Section II we present a model Lagrangian and vector current. This model is used to calculate the polarization of the muon in the deep inelastic neutrino nucleon scattering in Section III.

II. PHENOMENOLOGICAL LAGRANGIAN AND VECTOR CURRENT

Consider a general coupling of lepton and charged vector boson fields,[5]

$$\mathscr{L} = g_W^S \bar{\psi}_\ell(x)\,(a_1 - b_1\gamma_5)\,\psi_\nu(x)\,\partial_\lambda \overline{W}_\lambda(x) - i g_W^V \bar{\psi}_\ell(x)\,\gamma_\lambda(1-\gamma_5)\,\psi_\nu(x)\,\overline{W}_\lambda(x)$$
$$+ \frac{i}{2} g_W^T \bar{\psi}_\ell(x)\,\sigma_{\lambda\mu}(a_2 - b_2\gamma_5)\,\psi_\nu(x)\left(\partial_\lambda \overline{W}_\mu(x) - \partial_\mu \overline{W}_\lambda(x)\right) + g_W^V J_\lambda(x)\overline{W}_\lambda(x) \tag{1}$$

where $J_\lambda(x)$ is the hadronic current. We can write this in another form, viz.:

$$\mathscr{L} = g_W^V\left(j_\lambda(x) + J_\lambda(x)\right)\overline{W}_\lambda(x) + \text{h.c.} \tag{2}$$

where

$$j_\lambda(x) = \bar{\psi}_\ell(x)\left[i\frac{\eta}{m}(a_1-b_1\gamma_5)\,q_\lambda - i\gamma_\lambda(1-\gamma_5) - i\frac{\xi}{m}\sigma_{\lambda\mu}q_\mu(a_2-b_2\gamma_5)\right]\psi_\nu(x) \quad (3)$$

$$\bar{\psi}_\ell(x)q_\lambda\,\psi_\nu(x) \equiv -i\frac{\partial}{\partial X_\lambda}\left(\bar{\psi}_\ell(x)\,\psi_\nu(x)\right)$$

and

$$\frac{\eta}{m} = \frac{g_W^S}{g_W^V}\ , \qquad \frac{\xi}{m} = \frac{g_W^T}{g_W^V} \quad (4)$$

where m is the muon mass. We introduce the muon mass in (3) to make ξ and η dimensionless. In general, a_i and b_i are invariant functions of the momentum transfer q. We may regard (3) as a natural modification of the conventional V-A leptonic weak current. Notice that these derivative coupling terms when applied to hadronic physics with strongly interacting intermediate fields yield Weinberg's form factors[6] with a specific choice of a_i and b_i. Derivative couplings similar to (1) in principle have been used by Igarashi et al. in their strong meson dominance model[7] to get V-αA theory of β decay even though they used V-A leptonic current. Also it is well known that the decay $\pi^- \to \mu^-\bar{\nu}_\mu$ can be described phenomenologically by the derivative coupling of the pion directly to the lepton current,[8] i.e.,

$$\mathscr{L} = g_\pi\left[\bar{\mu}\gamma_\lambda(1-\gamma_5)\nu_\mu + \bar{e}\gamma_\lambda(1-\gamma_5)\nu_\ell\right]\partial_\lambda\Phi_{\pi^-} + \text{h.c.} \quad (5)$$

Therefore, it may be interesting to see the consequences of a current-current theory with $j_\lambda(x)$ defined as in (3). This would not change the well established low energy predictions of the V-A theory for sufficiently small ξ and η. Then this result can readily be translated to that of intermediate vector boson theory, Lagrangian (1).

Cheng and Tung[9] have discussed tests of V-A theory in neutrino scattering processes assuming the most general form of local (nonderivative V-A and S-T) interaction in the helicity formalism. Here we are interested in the nonlocal deviation from the V-A theory, which do not originate from the second order radiative corrections. As for the scalar and tensor hadronic currents, their commutators, hence those hadronic structure functions, are relatively unknown to us simply because only those of the vector and axial vector hadronic currents have been the focus of attention up to now. Therefore we will restrict ourselves to the vector and axial vector hadronic currents.

III. POLARIZATION OF THE MUON IN $\nu N \to \mu X$

The phenomenological current (3) discussed in the previous section shall be used to compute the polarization of the muon in the neutrino induced inclusive reaction. To simplify the calculation we invoke the following assumptions.

(1) Only left-handed neutrino takes part in the leptonic current, i.e., $a_1 = b_1$, $a_2 = b_2$. Choose $a_1 = a_2 = 1$ without loss of generality.

(2) Time reversal invariance, i.e., ξ and η are real.

Then we have,[10] if we put

$$\frac{\left(g_W^V\right)^2}{m_W^2} = \frac{G}{\sqrt{2}}$$

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{G^2}{4\ ME^2}\ \mathfrak{m}_{\mu\lambda} W_{\mu\lambda}$$

where

$$\mathfrak{m}_{\mu\lambda} = \overline{\sum_{\nu\ \mathrm{spin}}} m\, m_\nu \langle \mu(k') | j_\mu(0) | \nu(k) \rangle \langle \nu(k) | j_\lambda^\dagger(0) | \mu(k') \rangle \qquad (6)$$

and

$$W_{\mu\lambda} = \int \frac{d^4x}{4\pi} \; e^{iq\cdot x} \; \langle N(p) | \left[J_\mu(x), J_\nu^\dagger(0) \right] | X(p') \rangle$$

$$= - \left(g_{\mu\nu} - \frac{q_\mu q_\lambda}{q^2} \right) W_1 + \frac{1}{M^2} \left(p_\mu - \frac{\nu}{q^2} q_\mu \right) \left(p_\lambda - \frac{\nu}{q^2} q_\lambda \right) W_2$$

$$- i \frac{\epsilon_{\mu\lambda\alpha\beta} p^\alpha q^\beta}{2M^2} W_3 + \frac{q_\mu q_\lambda}{M^2} W_4 + \frac{(p_\mu q_\lambda + p_\lambda q_\mu)}{2M^2} W_5 \qquad (7)$$

The average over the proton spin is taken in $W_{\mu\lambda}$ and $q = k - k' = p' - p$, $\nu = q\cdot p = M(E-E')$, $q^2 = -Q^2 < 0$, where E and E' are the laboratory energies of the neutrino and the muon respectively. For the sake of illustration, we use the quark model version of the structure functions, which exhibits a scaling behavior. A more general case can be worked out, leading to similar conclusions. Thus we assume that[11, 12]:

$$\lim \text{bj} \; W_1(q^2, \nu) = F_1(x)$$

$$\lim \text{bj} \; \left(\frac{\nu}{M^2} \right) W_{2,3}(q^2, \nu) = F_{2,3}(x) \qquad (8)$$

$$\lim \text{bj} \; \left(\frac{\nu}{M^2} \right)^2 W_{4,5}(q^2, \nu) = F_{4,5}(x)$$

where $x = -(q^2/2\nu)$. On the other hand, from (3) to (6), it follows[10]:

$$\mathfrak{m}_{\mu\nu} = \mathfrak{m}_{\mu\nu}^{(0)} + \mathfrak{m}_{\mu\nu}^{(s)}$$

where

$$\mathfrak{m}_{\mu\nu}^{(0)} = (k'_\mu k_\nu + k'_\mu k_\nu - \delta_{\mu\nu} k\cdot k')(1+\xi)^2 \mp i\epsilon_{\mu\nu\sigma\rho} k'^\sigma k^\rho (1+\xi)^2$$

$$+ \frac{1}{m^2}(k\cdot k') q_\mu q_\nu \eta^2 \mp \frac{1}{m^2}(k\cdot k') \left\{ q_\mu (k+k')_\nu + q_\nu (k+k')_\mu \right\} \eta\,\xi + \frac{1}{m^2}(k\cdot k')(k+k')_\mu (k+k')_\nu \xi^2$$

$$\pm (k_\nu q_\mu + k_\mu q_\nu)(\eta + \eta\xi) - \left\{ k_\mu (k+k')_\nu + k_\nu (k+k')_\mu \right\} (\xi + \xi^2)$$

and

$$
\begin{aligned}
\pm \mathfrak{m}^{(s)}_{\mu\nu} = & -m\left[s_\mu k_\nu + s_\nu k_\mu - \delta_{\mu\nu}\, s\cdot k\right](1+\xi)^2 \pm im\,\epsilon_{\mu\nu\sigma\rho}\, s^\sigma k^\rho (1+\xi)^2 \\
& + \frac{\eta^2}{m}(s\cdot k) q_\mu q_\nu + \frac{i}{m}\epsilon_{\sigma\rho\lambda\nu} k'^\sigma s^\rho k^\lambda \left[-q_\mu(\eta+\eta\xi) \pm (k+k')_\mu(\xi+\xi^2)\right] \\
& - \frac{i}{m}\epsilon_{\sigma\rho\lambda\mu} k'^\sigma s^\rho k^\lambda \left[-q_\nu(\eta+\eta\xi) \pm (k+k')_\nu(\xi+\xi^2)\right] \\
& \pm \frac{1}{m}\left[(s\cdot k)k'_\nu q_\mu - (k\cdot k')s_\nu q_\mu\right](\eta+\eta\xi) \pm \frac{1}{m}\left[(s\cdot k)k'_\mu q_\nu - (k\cdot k')s_\mu q_\nu\right](\eta+\eta\xi) \\
& \mp \frac{1}{m}(s\cdot k)\left[q_\mu(k+k')_\nu + q_\nu(k+k')_\mu\right]\eta\xi + \frac{1}{m}(s\cdot k)(k+k')_\mu(k+k')_\nu\,\xi^2 \\
& + \frac{1}{m}\left[(k\cdot k')\, s_\mu (k+k')_\nu - k'_\mu (k+k')_\nu (s\cdot k)\right](\xi+\xi^2) \\
& + \frac{1}{m}\left[(k\cdot k')\, s_\nu (k+k')_\mu - k'_\nu (k+k')_\mu (s\cdot k)\right](\xi+\xi^2)
\end{aligned}
\tag{9}
$$

where the upper and lower signs refer to the neutrino and antineutrino respectively from here and after. Furthermore, from now on, the following expressions will be used.

$$
\begin{aligned}
& E = -\frac{q^2}{2Mxy}, \qquad E' = -\frac{(1-y)}{2Mxy}\, q^2 \\
& s\cdot k \cong \frac{1}{2m}\left(-q^2 - m^2\frac{1+y}{1-y} - m^2\frac{2M^2x^2y^2}{(1-y)^2q^2}\right) \\
& s\cdot p \cong -\frac{(1-y)}{2m\,xy}\, q^2 + m\frac{M^2xy}{(1-y)q^2}
\end{aligned}
\tag{10}
$$

where $y = \frac{\nu}{ME} = 1 - \frac{E'}{E}$ and s is longitudinal polarization vector of the muon. Now averaging over the muon spin, we get with the Callan Gross relation

$F_1 = F_2/2x$,

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{G^2 M x^2 y^2}{\pi q^4} \sum\sum \mathfrak{m}_{\mu\nu} W_{\mu\nu}$$

$$= \frac{G^2 M x}{\pi} \left[(1-y) \frac{\xi^2}{m^2} F_2(x) + \frac{1}{2Q^2} \left\{ F_2(x) \left(y^2 + 2(1-y) + 2y^2 \xi - y(2-y)\xi^2 \right) \right.\right.$$

$$\left.\left. \mp x F_3(x) \left\{ \frac{y(2-y)}{2} \right\} (1+\xi)^2 + \frac{M^2 \gamma \eta^2 y^2}{m^2} F_2(x) \pm \frac{2M^2 x^2 y(2-y)}{m^2} F_5 \eta \xi \right\} \right] \quad (11)$$

Thus we see that

$$\frac{d^2\sigma}{dQ^2 d\nu} \underset{\substack{Q^2 \to \infty \\ \lim bj}}{\longrightarrow} (1-y) \frac{G^2 M \xi^2}{\pi m^2} x F_2(x) + O\left(\frac{1}{Q^2}\right) \quad (12)$$

while the V-A theory yields the term of the order of $1/Q^2$.

Next we turn to the computation of the muon polarization. We define the polarization P and the longitudinal muon polarization vector S such that

$$P = \frac{d\sigma(\leftarrow) - d\sigma(\rightarrow)}{d\sigma(\leftarrow) + d\sigma(\rightarrow)} , \qquad s = \left(\frac{E'}{m} \hat{k}', \frac{|k'|}{m} \right) \quad (13)$$

Then a straightforward calculation yields the longitudinal polarization P^L.

$$P^L = \pm \frac{\frac{q^4}{2m^2} \xi^2 \left[-F_1 + \frac{F_2}{2x} \frac{(2-y)^2}{y^2} \right] + q^2 \left[F_1 a_1 + \frac{F_2}{2x} a_2 \mp F_3 a_3 + \left(F_1 - \frac{F_2}{2x} \right) a_4 - \frac{F_2 M^2 \gamma \eta^2}{2xm^2} \mp \frac{F_5 2M^2 x^2}{2xm^2} \frac{(2-y)}{y} \eta \xi \right]}{\frac{q^4}{2m^2} \xi^2 \left[-F_1 + \frac{F_2}{2x} \frac{(2-y)^2}{y^2} \right] - q^2 \left[F_1 b_1 + \frac{F_2}{2x} b_2 \mp F_3 b_3 + \left(F_1 - \frac{F_2}{2x} \right) b_4 + \frac{F_2 M^2 \gamma \eta^2}{2xm^2} \pm \frac{F_5 2M^2 x^2}{2xm^2} \frac{(2-y)}{y} \eta \xi \right]}$$

(14)

where

$$a_1 = 1+\xi+\xi^2 \qquad b_1 = 1+2\xi$$

$$a_2 = \frac{2(1-y)}{y^2} - \frac{(4-y)}{y}\xi + \frac{2}{y}\xi^2 \qquad b_2 = \frac{2(1-y)}{y^2} - \frac{(2-y)}{y}\xi^2$$

$$a_3 = \frac{1}{2}\left\{\frac{(2-y)}{y} + 2\frac{(4-y)}{y}\xi + \frac{(6-y)}{y}\xi^2\right\} \qquad b_3 = \frac{(2-y)}{2y}(1+\xi)^2$$

$$a_4 = -\frac{(1+y)}{1-y}\xi^2 \qquad b_4 = \mp\eta\mp\eta\xi+\xi+\frac{1}{2}(\eta^2+\xi^2)$$

$$\gamma = \frac{4x^3F_4-2x^2F_5}{F_2} \tag{15}$$

Furthermore, if we adopt the spin 1/2 parton model, we have the Callan Gross relation $F_1 = F_2/2x$. Then,

$$P^L = \pm\frac{\frac{(1-y)}{y^2}\xi^2 - \frac{m^2}{2Q^2}\left[(1+\xi)^2 + \frac{2(1-y)}{y^2} - \frac{4}{y}\xi + \frac{2}{y}\xi^2 \mp \frac{xF_3}{F_2}\left\{\frac{(2-y)}{y}(1+\xi)^2 + \frac{4}{y}\xi + \frac{4}{y}\xi^2\right\} - \frac{M^2\gamma\eta^2}{m^2} \mp \frac{2M^2x^2}{m^2}\frac{F_5}{F_2}\frac{(2-y)}{y}\eta\xi\right]}{\frac{(1-y)}{y^2}\xi^2 + \frac{m^2}{2Q^2}\left[(1+\xi)^2 + \frac{2(1-y)}{y^2} - \frac{2}{y}\xi^2 \mp \frac{xF_3}{F_3}\frac{2-y}{y}(1+\xi)^2 + \frac{M^2\gamma\eta^2}{m^2} \pm \frac{2M^2x^2}{m^2}\frac{F_5}{F_2}\frac{(2-y)}{y}\eta\xi\right]} \tag{16}$$

We note that in vector gluon model[12] the parameter γ in Eq. (15) is a constant given by $\gamma = (m_\rho/M)^2$, where m_ρ is the bare proton quark mass. The constants $\xi/m < 0.1\ \mathrm{GeV}^{-1}$, $\eta/m < 0.1\ \mathrm{GeV}^{-1}$, i.e., $\xi < 10^{-2}$, $\eta < 10^{-2}$ seem to be reasonable from the low energy experiments which agree with the V-A theory. Moreover, as is well known, the positivity condition gives $0 \le -xF_3 \le F_2$. Therefore for practical purposes we may write, if $y \neq 1$,

$$P^L = \mp 1 \pm \frac{m^2xy^2\left[2F_1 \mp F_3 - (1-y)\left(F_1 - \frac{F_2}{2x}\right)\right]}{Q^2(1-y)\left[xy^2F_1 + (1-y)F_2 \mp xy\left(\frac{2-y}{2}\right)F_3\right]}, \text{ for pure V-A} \tag{17a}$$

$$P^L = \pm 1 \mp \frac{m^2}{Q^2 \xi^2}\left[\frac{y^2}{1-y} + 2 \mp \frac{xF_3}{F_2}\,\frac{y(2-y)}{(1-y)}\right] \quad , \text{ for } \eta,\ \xi, \text{ and V-A} \tag{17b}$$

$$P^L = \mp \frac{1 + \frac{2(1-y)}{y^2} \mp \frac{xF_3}{F_2}\left(\frac{2-y}{y}\right) - \frac{M^2\gamma\eta^2}{m^2}}{1 + \frac{2(1-y)}{y^2} \mp \frac{xF_3}{F_2}\left(\frac{2-y}{y}\right) + \frac{M^2\gamma\eta^2}{m^2}} \quad , \text{for } \xi=0, \text{ V-A} \tag{17c}$$

These results indicate that P^L approaches 1 instead of -1 for large Q^2 and $0 < y < 1$ if $\xi \neq 0$. We illustrate P^L for y=1/2 in Fig. 1. But P^L will not change sign if $\xi = 0$ and $\left(M^2\gamma\eta^2/m^2\right) \ll \left(2/y^2\right)$, as long as the assumption $F_2 = -xF_3$ is used. This corresponds to $\gamma \ll 8 \times 10^2$ which means that $m_\rho \ll 28M$ in the vector gluon model, when $\eta = 10^{-2}$ and y=1/2. An estimate[12] is that $\gamma = 1.6 \times 10^{-2}$, i.e., $m_\rho = 120$ MeV, which leads to $P^L = -1$.

An important feature in the estimate described above is that the momentum transfer at which the change of the muon polarization occurs is very sensitive to the value of ξ and η. Because complete right-handed polarization needs extremely large Q^2, it would be interesting to see the value of Q^2 at which P goes to zero before it changes sign. In the Fig. 1, we see that, for $\xi/m = 0.1\ \text{GeV}^{-1}$, P^L vanishes at $Q^2 = 200\ (\text{GeV})^2$, while for $\xi/m = 0.05\ \text{GeV}^{-1}$ that occurs at $Q^2 = 800\ (\text{GeV})^2$. The perpendicular polarization would not give any insight about the ξ or η term, because it is proportional to m sin θ, which is very small at high energy.

If we do the above calculation in the intermediate vector boson theory, i.e., Lagrangian (1), we can easily obtain the result by simply replacing ξ, η, and G in Eqs. (11) - (17) by

$$\eta \to \frac{m^2}{m_W^2} + \eta\left(\frac{q^2}{m_W^2} - 1\right) , \qquad \xi \to \xi , \qquad G^2 \to \frac{G^2}{\left(1 + \frac{2MxyE}{m_W^2}\right)^2} \tag{18}$$

Therefore we conclude that unless $Q^2/m_W^2 >> 1$, the previous results would not be modified. Notice that as far as the ξ term and polarization are concerned we cannot distinguish between the intermediate vector boson theory, or a momentum dependent weak current. In fact the merit of the polarization measurement rather than that of the cross section in view of testing the V-A theory as formulated here, is that the change of the polarization is not as sensitive to the validity of the Callan-Gross relation or the breakdown of scaling as the cross section is. Nevertheless one might expect that the general IVB theory given by the Lagrangian (1) would be appropriate to describe high energy phenomena rather than the current-current theory with $j_\lambda(x)$ in (3). In that case, if we assume $F_2 = 2xF_1 = -xF_3$ and $F_4 = F_5 = 0$, the violation of scaling in the expression $d^2\sigma/dxdy$ may be parametrized by

$$F_2^\nu(x) \approx F_2^\nu(x) \frac{\left[2 + \frac{\xi^2 Q^2}{m^2}(1-y)\right]}{\left(1 + \frac{Q^2}{m_W^2}\right)^2} \tag{19a}$$

and

$$F_2^{\bar{\nu}}(x) \approx F_2^{\bar{\nu}}(x) \frac{\left[2(1-y)^2 + \frac{\xi^2 Q^2}{m^2}(1-y)\right]}{\left(1 + \frac{Q^2}{m_W^2}\right)^2} \tag{19b}$$

As far as power counting in Feynman diagrams is concerned, the ξ term is not worse than the V-A theory with an IVB, because $\gamma_\mu q_\mu q_\alpha \approx m q_\alpha$, $\sigma_{\mu\nu} q_\nu q_\mu q_\alpha = 0$. We summarize all the results in Table 1. Obviously the realization of the limits in Table 1 depends upon the value of $y = 1 - \frac{E'}{E}$. A larger y requires higher Q^2.

In the recent analysis of momentum independent S-T-P weak currents, Cheng and Tung[9] state that "to the extent that the lepton mass can be neglected, a purely left-handed outgoing lepton indicates V interaction, a purely right-handed one indicates S-T interaction, and the coexistance of both indicates a mixture of the two." Accepting the premise that no matter what the additional term is it must be small compared to the V-A, Cheng and Tung predicts $P^L = -1 \pm \alpha$ when there exists a small contamination of S-T interaction. Here α is a small positive constant. However, our result is that, as can be seen in Table 1, a purely right-handed outgoing muon at high energy indicates the existence of the ξ term whether the intereaction is via current-current theory or IVB theory. It is interesting to note that even though the η term contains momentum transfer q which makes us anticipate that it might dominate over the V-A term at large momentum transfer, it turns out that all the higher momentum dependent part involving η cancel out so that it does not dominate over the V-A term. But a small contamination of the η term would mean $P^L = -1 \pm \alpha$ as in the Cheng and Tung's case with momentum independent S-T interaction. Therefore when a future experiment indicates that $P^L = -1 \pm \alpha$, a very elaborate analysis seems to be necessary to decide whether that is due to the Cheng and Tung's momentum independent S-T interaction or our momentum dependent η type coupling. However, we notice that the result described above concerning the η term is a consequence of the assumption (8). If we assume a different scaling law, such as

$$\lim bj \left(\frac{\nu}{M^2}\right) W_{4,5} = F_{4,5}(x)$$

the role of the η term would be similar to that of the ξ term.

Another interesting prediction of the momentum dependent vector current (ξ type) is that the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ increases as the neutrino energy gets larger, i.e.,

$$R = \frac{\sigma(\bar{\nu})}{\sigma(\nu)} = \frac{\dfrac{ME\xi^2}{6m^2} + \left\{\dfrac{4}{3} - \dfrac{2}{3}\,\dfrac{|\langle xF_3\rangle|}{\langle F_2\rangle} + \dfrac{M^2\gamma\eta^2}{2m^2} - \dfrac{2M^2\eta\xi\langle x^2F_5\rangle}{3m^2\langle F_2\rangle}\right\}}{\dfrac{ME\xi^2}{6m^2} + \left\{\dfrac{4}{3} + \dfrac{2}{3}\,\dfrac{|\langle xF_3\rangle|}{\langle F_2\rangle} + \dfrac{M^2\gamma\eta^2}{2m^2} + \dfrac{2M^2\eta\,\xi\langle x^2F_5\rangle}{3m^2\langle F_2\rangle}\right\}}$$

and

$$\underset{E\to\infty}{\lim\ bj}\ R \longrightarrow 1 \tag{20}$$

where

$$\langle x^nF_m\rangle \equiv \frac{\int_0^1 dx \int_0^1 dy\, x^nF_m\, \dfrac{d^2\sigma}{dxdy}}{\int_0^1 dx \int_0^1 dy\, \dfrac{d^2\sigma}{dxdy}}$$

The above conclusion will not be modified as long as the ratios F_3/F_2, F_4/F_2, and F_5/F_2 scale, as in the IVB theory. The CERN result[13, 14] $R=0.377\pm0.023$ may indicate the deviation from $R=1/3$, although it is obviously too early to draw any definite conclusion. However, various models predict different values of R. Spin 1/2 parton model yields $R=1/3$, while other constituents would give $1/3 < R < 3$. In general, from the positive semidefinite property of $W_{\mu\nu}$ and the scaling behavior of structure functions given in (8), it follows that $F_2 \geq 2xF_1 \geq -xF_3$, which leads to $R \geq 1/3$. However, the above ratio is constant unless scaling breaks down to the extent $F_2(x) = xF_3(x)G(E) = xF_3(x)\,G\left(\frac{Q^2}{2Mxy}\right)$, where G(E) is some function of E. Therefore in order to detect the ξ or η term, we have to look at the energy dependence of the ratio R.

One might expect that the ξ term (or η term) could be obtained as a result of the radiative corrections in the framework of the V-A theory as in the eeA vertex correction in quantum electrodynamics. However, that this is not the case can be seen by the following argument. First, the radiative corrections[15] to the $\mu\nu W$ coupling (which do not have vertex correction in the second order) fail to induce the ξ type effective coupling. Similarly in the Weinberg theory,[16] the $\mu\nu W$ vertex correction[17] due to a neutral vector boson Z_μ rules out this possibility because of $1\pm\gamma_5$ structure of interactions. Thus we cannot as of now tell where those extra derivative coupling may come from. Nevertheless the other higher-order corrections might as well lead to the same conclusions about the polarization of the muon deduced from our formalism.[4] On the phenomenological level, however, we conclude that the polarization measurement of the muon in the inclusive reaction $\nu N \rightarrow \mu X$ at high energy will provide a good criteria for the existence or nonexistence of a phenomenological momentum dependent weak vector current or of the intermediate vector boson theory with Lagrangian (1).

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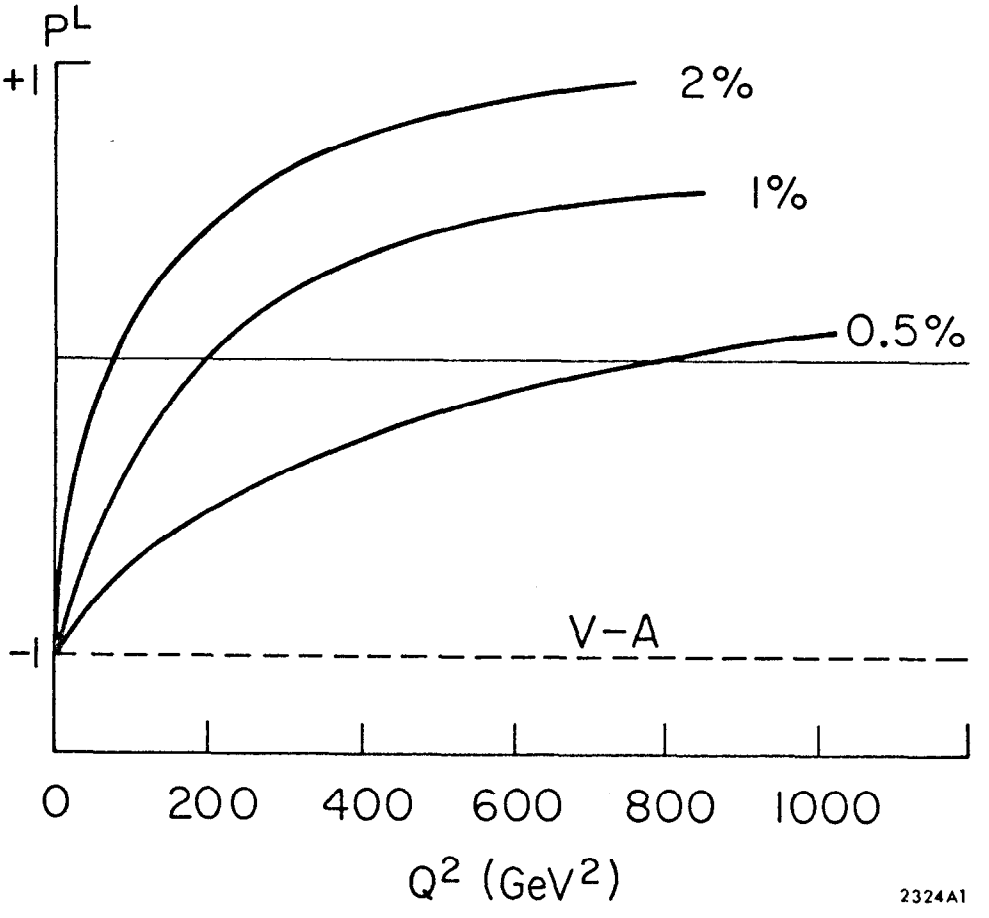
TABLE 1

Longitudinal polarization of the muon in the $\nu N \rightarrow \mu X$ at the scaling limit. MDV refers to the momentum dependent weak vector current. The values of P^L given above are the limit of P^L as $Q^2 \rightarrow \infty$. These values seem to be realized, because of the smallness of ξ and η, at the much larger Q^2 than the one at which structure functions begin to scale in the Bjorken-Johnson-Low limit.

V-A		$P = \mp 1$	IVB	V-A	$P = \mp 1$
MDV	ξ, V-A	± 1		ξ, V-A	± 1
	ξ, η, V-A	± 1		ξ, η, V-A	± 1
	η, V-A	depend upon γ		η, V-A	± 1 if $\frac{Q^2}{m_W^2} \gg 1$
					∓ 1 if $\frac{Q^2}{m_W^2} \ll 1$

FIGURE CAPTION

1. Longitudinal polarization of the muon in the $\nu N \rightarrow \mu X$ with the assumptions $F_4 = F_5 = 0$, $-xF_3 = F_2$, $F_1 = F_2/2x$, only for illustration. Percentage on the curve refers to that of mg_W^T with respect to the vector coupling constant g_W^V. We put y=1/2 which is the average value of y calculated in Ref. 13.


P^L
+1
-1
2%
1%
0.5%
V−A
0
200
400
600
800
1000
Q^2 (GeV^2)
2324A1