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HADRONIC DECAY PROCESSES AND THE TRANSFORMATION FROM CURRENT TO CONSTITUENT QUARKS*

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ABSTRACT

Theoretical work on the connection between the algebras associated with current and constituent quarks is reviewed. Tests of the proposed connection are presented using both the magnitudes and signs of amplitudes for pionic transitions between hadrons.

INTRODUCTION

At the Philadelphia Conference on Meson Spectroscopy last year, I devoted a sizable fraction of my talk[1] to a discussion of chiral $SU(2)\times SU(2)$, or more generally, chiral $SU(3)\times SU(3)$, and the question of what representations of this algebra are realized by the observed hadron states.[2,3] This was because in this case the theoretical issues at stake are very closely associated with experimental data which form the heart of spectroscopy: namely, what states exist and what is the strength of the pion, or more generally, pseudoscalar meson, transitions between them. Unfortunately, at that time there had been little theoretical progress in the subject for several years, and previous applications to actual hadrons suffered from being done in a mostly piecemeal, case-by-case fashion.

However, the last year, even the past few months, have seen some very rapid and important progress. We now have a theory which is (1) simple in its algebraic properties, (2) systematic in treating all mesons and baryons in a unified way, and (3) definite in that the theory has a clear origin and structure, the resulting amplitudes are related by Clebsch-Gordan coefficients, and the decay widths are related to the amplitudes involved in the theory in a nonarbitrary, known way.

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In one's more optimistic moments, it might even be said that these new theoretical developments are the most important for the understanding of meson or baryon transitions since the development of SU(3). For, just as SU(3) relates the decay rates among different members of an SU(3) multiplet, the new theory provides us for the first time with a clear and respectable basis on which to relate decays of different members of an SU(6) multiplet. There is only one small difficulty which might temper one's enthusiasm: the predictions of the theory do not agree with the results of certain experiments presented to this conference — in particular with the signs of amplitudes[4,5] in $\pi N \to N^* \to \pi\Delta$. We shall return to this and other experimental tests of the theory presently, but first let us introduce the theory we have been discussing.

THE $SU(6)_W$ ALGEBRAS CORRESPONDING TO CURRENT AND CONSTITUENT QUARKS

We first consider the algebra formed by the 16-vector and axial-vector charges, $Q^\alpha(t)$ and $Q_5^\alpha(t)$, where $\alpha=1,\ldots,8$. The charges are simply the integrals over all space of the time components of the corresponding weak and/or electromagnetic current densities. At equal times these charges commute to form the algebra proposed by Gell-Mann[6]:

$$\begin{aligned}
\left[Q^\alpha(t),\ Q^\beta(t)\right] &= if^{\alpha\beta\gamma}\, Q^\gamma(t) \\
\left[Q^\alpha(t),\ Q_5^\beta(t)\right] &= if^{\alpha\beta\gamma}\, Q_5^\gamma(t) \\
\left[Q_5^\alpha(t),\ Q_5^\beta(t)\right] &= if^{\alpha\beta\gamma}\, Q^\gamma(t) \quad .
\end{aligned} \tag{1}$$

This is the algebra of chiral $SU(3)\times SU(3)$, for it can be easily shown that (1) is equivalent to the statement that the right-handed charges, $Q^\alpha+Q_5^\alpha$, and the left-handed charges, $Q^\alpha-Q_5^\alpha$, each form an SU(3), and that they commute with each other — hence, chiral $SU(3)\times SU(3)$. For $\alpha=1,2,3$ the Q^α's are the generators of isospin rotations; for $\alpha=1,\ldots,8$, they are the generators of SU(3). The last of Eqs. (1), sandwiched between nucleon states moving at infinite momentum in the z direction, yields the Adler-Weisberger sum rule.[7]

With the addition of integrals over certain (unobservable) tensor current densities, the algebra of Q^α and Q_5^α can be enlarged still further to form an $SU(6)_W$ algebra whose elements commute like the products of SU(3) and Dirac matrices: λ^α, $\lambda^\alpha\beta\sigma_x$, $\lambda^\alpha\beta\sigma_y$ and $\lambda^\alpha\sigma_z$. We refer to this algebra, introduced by Dashen and Gell-Mann[8] in 1965, as the <u>$SU(6)_W$ of currents</u>.

It will be convenient in what follows to label various representations of this algebra of currents. For this purpose we shall use just the $SU(3)\times SU(3)$ subgroup of the $SU(6)_W$ of currents to write

$$(A,\ B)_{S_z} \quad ,$$

where A is the SU(3) representation under $Q^{\alpha}+Q_5^{\alpha}$, B the representation under $Q^{\alpha}-Q_5^{\alpha}$, and S_z is the eigenvalue of Q_5^{o}, the singlet axial-vector charge, which is naively the (current) quark spin component along the z direction. The "ordinary" (Q^{α}) SU(3) content of such a representation is just that of the direct product A×B.

As an example, consider Q_5^{α}. Since we can always write

$$Q_5^{\alpha} = \frac{1}{2}\left(Q^{\alpha} + Q_5^{\alpha}\right) - \frac{1}{2}\left(Q^{\alpha} - Q_5^{\alpha}\right) , \tag{2}$$

it is clear that under the algebra of currents Q_5^{α} transforms as just $(8,1)_0-(1,8)_0$, and its ordinary SU(3) content is that of an octet. Hadron states on the other hand are known to transform as very complicated mixtures of representations of the SU(3)×SU(3) of currents.[9] This is already apparent from the Adler-Weisberger sum rule itself, for it shows that the nucleon is connected by a generator of the algebra, the axial-vector charge Q_5 (in the form of the pion field through the use of PCAC), to many higher mass N*'s. Thus the nucleon and these N*'s must be in the same representation of SU(3)×SU(3). Conversely, the nucleon state must be a sum of many different representations of the SU(3)×SU(3) of currents.

This is not the case for the other $SU(6)_W$ algebra we consider, that of strong interactions.[10] This $SU(6)_W$ is isomorphic to the one considered above, and contains a corresponding SU(3)×SU(3) subalgebra. However, by construction, this algebra acts on the constituent quarks of hadrons, where a (nonexotic) meson is just $q\bar{q}$ and a baryon is qqq. Hadron states are then very simple in terms of this algebra, i.e., they transform as known irreducible representations (IR) of the $SU(6)_W$ of strong interactions or its SU(3)×SU(3) subalgebra.

It is therefore clear from what we have discussed that these algebras are not identical.[11] Hadrons are complicated in terms of currents, but simple in terms of constituents. Furthermore, identifying the two algebras leads immediately to such undesirable results as $g_A = 5/3$, $\mu_A(N) = 0$, $\mu^*(N \to \Delta) = 0$, etc.

THE TRANSFORMATION FROM CURRENT TO CONSTITUENT QUARKS

Even if the two algebras can not be directly identified with each other, there still might be a unitary transformation V, which relates them[11, 12]:

$$\left[SU(6)_{W,\,strong}\right] = V\left[SU(6)_{W,\,currents}\right]V^{-1} \quad . \tag{3}$$

Instead of applying the transformation V to the operators of the algebra, as in Eq. (3), we apply it to the hadron states, which are assumed to be in irreducible representations of the $SU(6)_W$ of strong interactions.

Equation (3) then implies that

$$\begin{aligned} |\text{Hadron}\rangle &= |\text{IR, constituents}\rangle \\ &= V\,|\text{IR, currents}\rangle \quad , \end{aligned} \tag{4}$$

where $|\text{IR, currents}\rangle$ is a state which transforms under the $SU(6)_W$ of currents exactly as $|\text{IR, constituents}\rangle$ transforms under the $SU(6)_W$ of strong interactions. The transformation V then changes the simple $|\text{IR, currents}\rangle$ state into the complicated sum of irreducible representations of the $SU(6)_W$ of currents which comprise an actual hadron state.

Now consider a matrix element $\langle\text{Hadron}'|Q_5^\alpha|\text{Hadron}\rangle$, which through PCAC we will relate the amplitude for Hadron' → Hadron + pion. We may rewrite this using Eq. (4) as

$$\begin{aligned} \langle\text{Hadron}'|Q_5^\alpha|\text{Hadron}\rangle &= \langle\text{IR}', \text{ constituents}|Q_5^\alpha|\text{IR, constituents}\rangle \\ &= \langle\text{IR}', \text{ currents}|V^{-1}Q_5^\alpha V|\text{IR, currents}\rangle \quad . \end{aligned} \tag{5}$$

All the complication of the hadron states under the algebra of currents has now been transferred to $V^{-1}Q_5^\alpha V$, which may be studied as an independent object, and whose properties may then be used to predict matrix elements of Q_5^α between any two hadron states.

In the case of the free quark model, the transformation V has been formulated and studied by Melosh, who was able to find an explicit expression for it.[12, 13] He was then able to study $V^{-1}Q_5^\alpha V$, and found that algebraically it contains only two terms under the $SU(3)\times SU(3)$ of currents: one term transforms as $(8,1)_0 - (1,8)_0$ (algebraically like Q_5^α itself), and the other term transforms as $(3,\bar{3})_1 - (\bar{3},3)_{-1}$. Each of these terms is in a 35 of the $SU(6)_W$ of currents.

It is this remarkably simple algebraic property of $V^{-1}Q_5^\alpha V$, in spite of the complication of V itself, which we now abstract from the free quark model and generalize to hold in the real world. Namely, we take as a basic assumption of the new theory that $V^{-1}Q_5^\alpha V$ transforms as a sum of an $(8,1)_0 - (1,8)_0$ term and a $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term under the algebra of currents. We proceed to apply this hypothesis to the study of pionic transitions between hadrons.[14]

ADDITIONAL PHYSICAL ASSUMPTIONS

For this purpose we make three additional assumptions. First, we assume that matrix elements of Q_5 are related to those of the pion field by the PCAC hypothesis. This relates matrix elements of Q_5 to the corresponding pion couplings. The decay width for Hadron' → Hadron + pion

is then given in narrow resonance approximation by

$$\Gamma = \frac{c}{2J'+1} \frac{p_\pi (M'^2 - M^2)^2}{M'^2} \sum_\lambda |\langle \text{Hadron}', \lambda | Q_5 | \text{Hadron}, \lambda \rangle|^2 , \qquad (6)$$

where c is a constant related to the charged pion decay rate and the isotopic spin of the hadrons, p_π is the pion momentum, and the sum extends over the possible common helicities, λ, of the hadrons. There is no arbitrary choice of phase space factors; the width is fixed directly by the matrix elements of Q_5, up to the validity of PCAC.[15]

Second, we assume that the observed (nonexotic) hadrons are identifiable to good approximation with constituent quark states. For baryons composed of qqq, we have the familiar SU(6) representations $\underline{56}$ $L=0^+$, $\underline{70}$ $L=1^-$, $\underline{56}$ $L=2^+$, etc., where L is the internal quark angular momentum. For mesons we have correspondingly the $q\bar{q}$ states $\underline{35}$ $L=0^-$, $\underline{1}$ $L=0^-$, $\underline{35}$ $L=1^+$, etc.[16]

Third, we assume that constituent quark states with different values of the quark spin are related by the $SU(6)_W$ of strong interactions. Then, after transforming we know the $SU(6)_W$ (of currents) properties of each term in a given Q_5 matrix element, and we may use the Wigner-Eckart theorem and tables of $SU(6)_W$ Clebsch-Gordan coefficients to carry out the calculation from this point onward. Note that we do not assume $SU(6)_W$ invariance — just the transformation properties of the various terms.

For each matrix element of Q_5 we write the initial and final hadron states with $J_z=\lambda$ in terms of states with definite S_z. This involves coupling internal quark L and S to form total J for each hadron. After transforming to an $SU(6)_W$ of currents basis, the matrix element of the $(8,1)_0 - (1,8)_0$ or $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term in $V^{-1}Q_5^\alpha V$ can be written as a reduced matrix element times the product of quark angular momentum, $SU(6)_W$, SU(3), and W-spin Clebsch-Gordan coefficients. The matrix elements of all hadrons states in a given SU(6) multiplet are therefore related and there are at most two independent reduced matrix elements for the pionic transitions between any two SU(6) multiplets, corresponding to the $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ pieces of $V^{-1}Q_5^\alpha V$.

PIONIC TRANSITIONS AS AN APPLICATION OF THE CONNECTION BETWEEN CURRENT AND CONSTITUENT QUARKS

Several authors have applied the ideas discussed above, or variants thereof, to hadron transitions. In his thesis, Melosh[12] already looked at the ratio of axial-vector couplings between nucleon and nucleon (g_A) and nucleon and 3-3 resonance (g^*). Here only the $(8,1)_0 - (1,8)_0$ term contributes, and one recovers the old SU(6) result for g_A/g^*, which agrees with experiment. For vector current transitions he was similarly able to

rederive the SU(6) results for the total moments, $\mu_T(n)/\mu_T(p) = -2/3$ and $\mu^*/\mu_T(p) = 2\sqrt{2}/3$.

In a somewhat different vein, Gilman and Kugler[17] have tried adding an additional, stronger assumption on the $(8,1)_0 - (1,8)_0$ term in $V^{-1}Q_5^\alpha V$: that it not only transforms like Q_5, but is proportional to it. This results in a closed algebra composed of Q^α, Q_5^α, and the terms in $V^{-1}Q_5^\alpha V$ which transform as $(3,\bar{3})_1$ and $(\bar{3},3)_{-1}$. The operator V can then be constructed explicitly out of the generators of this larger algebra. As Q_5 is a generator of the algebra of currents, this additional assumption determines not only the transformation properties of the $(8,1)_0 - (1,8)_0$ term, but its reduced matrix elements as well. The resulting algebraic structure has been applied to hadron decays, particularly those of mesons, with some success. However, neither Gilman and Kugler[17] nor Hey and Weyers,[18] who test just the $SU(3)\times SU(3)$ structure of $V^{-1}Q_5^\alpha V$ with no additional assumptions, employ the $SU(6)_W$ of strong interactions to relate quark spin states. As a result, their equations involve relations only among matrix elements of hadrons with a given helicity λ.

In the remainder of this talk, I will discuss the results obtainable with the three assumptions of the previous section, but <u>no</u> assumption of proportionality between the $(8,1)_0 - (1,8)_0$ term (in $V^{-1}Q_5^\alpha V$) and Q_5^α. As such, I will be reporting results from recent work of Gilman, Kugler and Meshkov.[19,20]

First consider meson decays, and in particular those of <u>35</u> L=1 mesons into <u>35</u> L=0 mesons by pion emission. The two independent reduced matrix elements may be determined by normalizing to $\Gamma(A_2 \to \pi\rho) = 77$ MeV, and requiring that $\Gamma_{\lambda=0}(B \to \pi\omega) = 0$, in agreement with experiments which show a dominantly transverse decay.[21] This latter condition is equivalent to the reduced matrix element of the $(8,1)_0 - (1,8)_0$ term vanishing,[22] and makes all matrix elements proportional to one reduced matrix element, that of the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term. The resulting predictions for the various decays of L=1 mesons are shown in Table I. As can be seen, where comparison of theory and experiment is possible, the agreement is quite good.

The pionic decays of other meson multiplets, e.g., $L=0 \to L=0$, $L=2 \to L=0$, $L=1 \to L=1$, and $L=2 \to L=1$ have also been calculated. While little comparison with experiment is possible at this time, interesting selection rules emerge. For example, for transitions between hadrons with different values of internal (quark) angular momentum, L' and L, the relative orbital angular momentum, ℓ, between the pion and final hadron obeys the rule,[24]

$$\| L-L'|-1| \leq \ell \leq |L+L'+1| \quad . \tag{7}$$

In addition, if L'=L then the $(8,1)_0-(1,8)_0$ term is purely p-wave. In the particular case $L=0 \to L=0$, only the $(8,1)_0 - (1,8)_0$ term can contribute, and the predicted relative couplings for $\rho \to \pi\pi$, $K^* \to \pi K$, and $\omega \to \pi\rho$ are in good agreement with experiment.

TABLE I

Decays of 35 L=1 mesons into 35 L=0 mesons by pion emission.[a,b]

Decay	Γ(predicted) (MeV)	Γ(experimental)[23] (MeV)
$A_2(1310) \to \pi\rho$	77 (input)	77 ± 20
$B(1235) \to \pi\omega$ $\lambda=0$	0 (input)	dominantly λ=1 100 ± 20 total width
$B(1235) \to \pi\omega$ $\lambda=1$	76	
$A_1(1070) \to \pi\rho$ $\lambda=0$	52	?
$A_1(1070) \to \pi\rho$ $\lambda=1$	26	
$A_2(1310) \to \pi\eta$	17	16 ± 4
$\delta(975) \to \pi\eta$	37	~60 total width
$f(1260) \to \pi\pi$	118	125 ± 25
$\sigma(760\,?) \to \pi\pi$	234	Broad?

a. Decay rates of the corresponding K* states may be obtained using SU(3) for the matrix elements of Q_5, but add no significant additional test at present.

b. The ω, f and σ mesons are taken as ideal mixtures of singlets and octets, so as to be purely constituted by nonstrange quarks. The η is assumed to be pure octet.

Encouraged by the meson results, we turn to baryons. Because of the availability of data we consider the decays of nonstrange baryons. For 56 L=0 → 56 L=0 transitions only the $(8,1)_0 - (1,8)_0$ term contributes and, as already noted, the predicted amplitudes are in satisfactory agreement with experiment.

For 70 L=1 → 56 L=0 decays, linear combinations of the two reduced matrix elements correspond to s- and d-wave amplitudes for decay into πN or πΔ. Although quark spin S=1/2 and 3/2 states having the same total quantum numbers within the 70 may be mixed, sums over such mixed states of squares of the Q_5 matrix elements are independent of mixing, and we compare these with experiment. The predictions for widths are given in Table II, where we have used combined widths of the two D_{13} states and two S_{11} states decaying into πN to fix the d- and s-wave amplitudes, respectively.

A similar analysis of 56 L=2 → 56 L=0 decays relates the two independent reduced matrix elements to p- and f-wave πN and πΔ decay amplitudes. In Table II we present the predicted widths, fixing the f- and p-wave amplitudes by the $F_{15}(1688) \to \pi N$ and $P_{31}(1860) \to \pi N$ decay rates, respectively.

A study of Table II shows that while there are many successes, there are also predicted widths which are in disagreement with those determined experimentally by factors of 2 to 3. For example, $\Gamma(D_{15} \to \pi\Delta)/\Gamma(D_{15} \to \pi N)$ is smaller than predicted (by a factor ~ 2.5), and the experimental situation in this case is rather solid. This is one of the worst discrepancies — in most other cases the agreement is better. Some of these may be due to experimental difficulties in determining a partial width; others to the use in the theoretical calculations of the narrow resonance approximation,

TABLE II

Decays of 70 L=1 and 56 L=2 baryons into 56 L=0 baryons by pion emission. All rates are fixed by the D_{13} and S_{11} decays to πN for the 70 L=1 decays, and by the F_{15} and P_{31} decays to πN for the 56 L=2 decays. For two states which may be mixed, a combination of widths which is independent of mixing is used and listed under Γ(predicted).

Decay	Γ(predicted) (MeV)	Γ(experimental)[23,5,25] (MeV)
$D_{13}(1520) \to (\pi N)_d$	$\Gamma(1520) + 0.50\,\Gamma(1700)$	
$D_{13}(1700) \to (\pi N)_d$	= 79 MeV (input)	79 ± 20
$D_{13}(1520) \to (\pi\Delta)_d$	$\Gamma(1520) + 0.243\,\Gamma(1700)$	
$D_{13}(1700) \to (\pi\Delta)_d$	= 30 MeV	10 ± 6
$S_{11}(1535) \to (\pi\Delta)_d$	$\Gamma(1535) + 0.264\,\Gamma(1715)$	
$S_{11}(1715) \to (\pi\Delta)_d$	= 35 MeV	not seen
$D_{15}(1670) \to (\pi N)_d$	21 MeV	56 ± 14
$D_{15}(1670) \to (\pi\Delta)_d$	82 MeV	84 ± 21
$S_{31}(1640) \to (\pi\Delta)_d$	81	52 ± 20
$D_{33}(1690) \to (\pi N)_d$	19	32 ± 9
$D_{33}(1690) \to (\pi\Delta)_d$	55	not seen
$S_{11}(1535) \to (\pi N)_s$	$\Gamma(1535) + 0.505\,\Gamma(1715)$	
$S_{11}(1715) \to (\pi N)_s$	= 116 (input)	116 ± 55
$D_{13}(1520) \to (\pi\Delta)_s$	$\Gamma(1520) + 0.243\,\Gamma(1700)$	
$D_{13}(1700) \to (\pi\Delta)_s$	= 46	19 ± 10
$S_{31}(1640) \to (\pi N)_s$	18	48 ± 9
$D_{33}(1690) \to (\pi\Delta)_s$	61	172 ± 60
$F_{15}(1688) \to (\pi N)_f$	84 (input)	84 ± 25
$F_{37}(1950) \to (\pi N)_f$	74	92 ± 20
$F_{37}(1950) \to (\pi\Delta)_f$	65	37 ± 18
$F_{35}(1880) \to (\pi N)_f$	14	36 ± 18
$F_{35}(1880) \to (\pi\Delta)_f$	77	16 ± 16
$P_{33}(\quad) \to (\pi\Delta)_f$		?
$F_{15}(1688) \to (\pi\Delta)_f$	12	not seen
$P_{13}(1860) \to (\pi\Delta)_f$	57	not seen
$P_{31}(1860) \to (\pi N)_p$	75 (input)	75 ± 25
$P_{31}(1860) \to (\pi\Delta)_p$	8	not seen
$P_{33}(\quad) \to (\pi N)_p$		?
$P_{33}(\quad) \to (\pi\Delta)_p$		?
$F_{35}(1880) \to (\pi\Delta)_p$	44	not seen
$P_{13}(1860) \to (\pi N)_p$	118	75 ± 25
$P_{13}(1860) \to (\pi\Delta)_p$	5	not seen
$F_{15}(1688) \to (\pi\Delta)_p$	15	22 ± 7

to which $\pi\Delta$ decays are notably sensitive. Possible mixing between different SU(6) multiplets has also been neglected.[26] From this standpoint we might regard the predictions in Table II as a reasonable first approximation to the data.

The predicted relative signs of amplitudes in $\pi N \to N^* \to \pi\Delta$ for both $\underline{70}$ L=1 $\to$ $\underline{56}$ L=0 and $\underline{56}$ L=2 $\to$ $\underline{56}$ L=0 decays are compared with experiment in Table III. It poses a stringent test of the theory. There are two

TABLE III

Signs of the amplitudes for $\pi N \to N^* \to \pi\Delta$ for N^*'s in the $\underline{70}$ L=1 and $\underline{56}$ L=2. Products of the theoretical and experimental[4,5] signs for decays through the $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ terms are presented, with the overall phase chosen so that DD13(1520) is positive. Signs which are independent of which term dominates are denoted by "*". Experiment and theory agree within the $\underline{70}$ L=1 or within the $\underline{56}$ L=2 if all the signs in either column are the same.

	Amplitude	$(8,1)_0 - (1,8)_0$	$(3,\bar{3})_1 - (\bar{3},3)_{-1}$
$\underline{70}$ L=1 $\to$ $\underline{56}$ L=0	DD13(1520)	+*	+*
	DS13(1520)	-	+
	SD31(1640)	+	-
	DS33(1690)	+	-
	DS13(1700)	+	-
	DD15(1670)	-*	-*
$\underline{56}$ L=2 $\to$ $\underline{56}$ L=0	FP15(1688)	-	+
	FF35(1880)	-*	-*
	FF37(1950)	-*	-*

kinds of relations in the table: (1) those that involve the same partial wave in both the incoming (πN) and outgoing ($\pi\Delta$) states have definite relative signs independent of what values the reduced matrix elements of the $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ terms have; (2) those that involve different initial and final partial waves depend on these values and may indicate which term is dominant.

As can be seen, the $\underline{56}$ L=2 $\to$ $\underline{56}$ L=0 decays have consistent signs and indicate the $(8,1)_0 - (1,8)_0$ term (first column) dominating. However, the $\underline{70}$ L=1 $\to$ $\underline{56}$ L=0 decays disagree with both kinds of relations. Note that if the signs of the $D_{13}(1520)$ decays into $\pi\Delta$ in both s- and d-waves could be reversed, then the $\underline{70}$ L=1 decays would all be consistent in sign and indicate $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ dominance.[27] Since there is a gap in the data analyzed by the LBL-SLAC group[4,5] from 1540 - 1650 MeV (c.m. energy), one might hope that such a reversal of all the lower energy signs would be possible. Up to this time, however, a continuous solution through the energy gap which does not add new resonances in the gap and which has the "correct" signs has not been obtained.[28] It is obviously of great importance to fill in this energy gap in the experiments, and see if the present solution to

the $\pi N \rightarrow \pi\Delta$ amplitudes is verified. If this is the only solution for the $\pi N \rightarrow \pi\Delta$ phase shifts, the theory faces serious difficulty.

DISCUSSION AND SUMMARY

The algebraic properties of the matrix elements of Q_5 in the theory described here are identical to those for pion coupling constants obtained in certain quark models[29] and some broken-$SU(6)_W$ schemes.[30] However, the results for widths, e.g., Tables I and II, differ from previous calculations in that PCAC is used and it imposes an unambiguous connection between the matrix elements of Q_5 and the widths which does not contain arbitrary ℓ dependent centrifugal barrier factors. Because of the similar algebraic structure, the predictions for the signs of amplitudes coincide with those of the cited models. Thus difficulties stemming from Table III are common to all these approaches.

By considering matrix elements of the vector current, we can extend our considerations to photon transitions. Again, the results[20] turn out to be algebraically identical to explicit quark model calculations.[31] For example, the radiative decays from $\underline{70}$ L=1 $\rightarrow$ $\underline{56}$ L=0 depend on two independent matrix elements, those of $(\overline{8,1})_0 + (1,\overline{8})_0$ and $(3,\bar{3})_1 + (\bar{3},3)_{-1}$ terms. These correspond respectively to the convection current and magnetic moment terms in quark models. The relative signs and magnitudes of the transition amplitudes predicted in this case are in agreement with experiment.[32]

In summary, we now have a simple and elegant theory of the transformation of the axial-vector charge so that its hadronic matrix elements may be computed by taking it between known irreducible representations of the algebra of currents. Supplemented by PCAC, we can analyze all pionic transition amplitudes between hadrons. The results for decay widths, particularly those of mesons, are encouraging. However, the relative signs of the amplitudes in $\pi N \rightarrow \pi\Delta$ are a crucial test, and the theory is in conflict with the results of the present experimental analysis. If this disagreement persists, we will have to face the possibility that either (1) there is large mixing of SU(6) multiplets, invalidating the identification of the observed hadrons with simple quark model states; (2) the use of $SU(6)_W$ to relate different quark spin states is wrong, and only a weaker symmetry holds, or; (3) the simple algebraic properties of $V^{-1}Q_5^{\alpha}V$ abstracted from the free quark model do not hold in Nature.

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