PAIR CREATION AT LARGE INHERENT ANGLES*

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1. Introduction

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In the next-generation linear colliders, the lowenergy e^+e^- pairs created during the collision of high-energy e^+e^- beams would cause potential deleterious background problems to the detectors. At low collider energies, the pairs are made essentially by the *incoherent* process, where the pair is created by the interaction of beamstrahlung photons on the individual particles in the oncoming beam. This problem was first identified by Zolotarev, et al[1]. At energies where the beamstrahlung parameter Υ lies approximately in the range $0.6 \leq \Upsilon \leq 100$, pair creation from the beamstrahlung photons is dominated by a *coherent* process, first noted by Chen[2].

The seriousness of this pair creation problem lies in the transverse momenta that the pair particles carry when leaving the interaction point (IP) with large angles. One source of transverse momentum is from the kick by the field of the oncoming beam which results in an outcoming angle $\theta \propto 1/\sqrt{x}$, where x is the fractional energy of the particle relative to the initial beam particle energy[2,3]. As was shown in Ref. [3], there in fact exists an energy threshold for the coherent pairs, where $x_{th} \gtrsim 1/2\Upsilon$. Thus within a tolerable exiting angle, there exists an upper limit for Υ where all coherent pairs would leave the detector through the exhaust port[4]. A somewhat different analysis has been done by Schroeder[5]. In the next generation of linear colliders, as it occurs, the coherent pairs can be exponentially suppressed[2] by properly choosing the $\Upsilon(\lesssim 0.6)$. When this is achieved, the incoherent pairs becomes dominant.

Since the central issue is the transverse momentum for particles with large angles, we notice that there is another source for it. Namely, when the pair particles are created at low energies, the intrinsic angles of these pairs when produced may already be large. This issue was first studied in Ref. [1]. In this paper we reinvestigate the problem, following essentially the same equivalent photon approach, but with changes in specific details including the virtual photon spectrum. In addition, various assumptions are made more explicit. The formulas derived are then applied to the collider parameters designed by Palmer[6].

2. The Equivalent Photon Approximation

We will be considering three different incoherent processes:

- 1. Breit-Wheeler process: $\gamma \gamma \rightarrow e^+ e^-$;
- 2. Bethe-Heitler process: $e\gamma \rightarrow ee^+e^-$;

3. Landau-Lifshits process: $e^+e^- \rightarrow e^+e^-e^+e^-$. The basic kernel of these processes is the same. For the BW process both photons are real beamstrahlung photons; for the BH process one is real and one is virtual; and for the LL process both photons are virtual.

When invoking the equivalent photon picture with both photons on-shell, the relativistic kinematics relates the fractional energy x of the outcoming positron (or electron) to its angle θ (relative to the initial particle trajectory) and the fractional energies of the two photons, y_1, y_2 , as

$$x = \frac{2y_1y_2}{y_1(1-\beta c) + y_2(1+\beta c)}$$
, (1)

where $c \equiv \cos \theta$ and β is the speed of the particle. In the following, we shall assume $\beta \simeq 1$, as the pair particles of concern are still relativistic. Since the transverse momentum $p_{\perp} = x\sqrt{1-c^2}$, with any choice of p_{\perp} and c, the initial photon energies are limited by

$$y_{\pm} = \frac{x}{2}(1 \pm c) = \frac{p_{\perp}}{2} \sqrt{\frac{1 \pm c}{1 \mp c}} \quad . \tag{2}$$

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Furthermore, for any given value of $y_2 \ge y_-$, the lower bound for y_1 is

$$y_b = \frac{y_2 y_+}{y_2 - y_-} \quad . \tag{3}$$

From the virtual photon propagator, one can establish the spectrum of the equivalent photons[7]:

$$dn_v = \frac{\alpha}{\pi} \frac{dy}{y} \frac{y_{\perp}^2 dy_{\perp}^2}{(y_{\perp}^2 + y^2)^2} \quad , \tag{4}$$

where y_{\perp} is the normalized transverse momentum q of the photon: $y_{\perp} \equiv q/\gamma m$. Integrating over y_{\perp}^2 , we get

$$n_{\nu}(y) = \frac{\alpha}{\pi} \frac{1}{y} \ell n \left(\frac{y_{\perp max}^2 + y^2}{y_{\perp min}^2 + y^2} \right) \quad , \qquad (5)$$

where $y_{\perp max}$ and $y_{\perp min}$ are the maximum and minimum momentum transfer, respectively. In the equivalent photon approximation, it is assumed [7] that the transverse momentum of the virtual photon satisfies the condition

$$\frac{y}{\gamma^2} \ll y_\perp \ll \frac{1}{\gamma} \quad . \tag{6}$$

Since n_v depends on $y_{\perp max}$ and $y_{\perp min}$ only logarithmically, it is customary to take the bounds in Eq. (6) as their values. We then have

$$n_{v}(y) = \frac{2\alpha}{\pi} \frac{1}{y} \ell n\left(\frac{1}{y}\right). \tag{7}$$

3. The Partial Cross Sections

The partial cross section for all positrons (or, equivalently, all electrons) with transverse momentum $p_{\perp} \ge p_{\perp 0}$ and outcoming angle $-c_0 \le c \le c_0$ is

$$\sigma_{ab \to e^{+}}(p_{\perp 0}, c_{0}) = g \int_{-c_{0}}^{c_{0}} dc \int_{y_{-}}^{\infty} dy_{2} \int_{y_{b}}^{\infty} dy_{1} n_{a}(y_{1}) n_{b}(y_{2}) \sigma_{\gamma\gamma}(y_{1}, y_{2}),$$
(8)

where g = 1/4 for the BW process, and 1 for both

BH and LL processes, respectively. For the BW process, the effective collision time for the beamstrahlung photons, which are emmitted during the collision itself, is only 1/4 of that of the primary beam particles. For the BH process, the factor 1/2 that arises from the effective collision time is compensated by the matching between the real and the virtual photons from both beams. The upper limits of the spectral integrals are set at infinity since the contributions are essentially dominated by the lower bounds. The photon spectra n_a and n_b depend on whether the photons are real or virtual. For virtual photons, it is the spectrum derived in the previous section. For real beamstrahlung photons, we define an effective Υ for the entire beam as

$$\Upsilon = \frac{5}{6} \frac{\gamma r_c^2 N}{\alpha \sigma_z \sigma_y (1+R)} \quad , \tag{9}$$

where σ_z, σ_y are the beam sizes and $R = \sigma_x/\sigma_y$ is the aspect ratio. The beamstrahlung spectrum is then

$$n_b(y) = \frac{1}{\pi} \Gamma(2/3) \left(\frac{\alpha \sigma_z}{\gamma \lambda_c}\right) (3\Upsilon)^{2/3} y^{-2/3}$$

$$\equiv A y^{-2/3} , \qquad (10)$$

where $\Gamma(2/3) \simeq 1.3541$.

In Eq. (8), $\sigma_{\gamma\gamma}(y_1, y_2)$ is the differential cross section for $\gamma\gamma \to e^+e^-$:

$$d\sigma_{\gamma\gamma} = 8\pi r_e^2 \frac{m^2 ds}{t(t-4m^2)} \\ \left\{ -\frac{1}{4} \left(\frac{s-m^2}{u-m^2} + \frac{u-m^2}{s-m^2} \right) \\ + \left(\frac{m^2}{s-m^2} + \frac{m^2}{u-m^2} \right) \\ + \left(\frac{m^2}{s-m^2} + \frac{m^2}{u-m^2} \right)^2 \right\},$$
(11)

where s, t, u are the Mandelstam variables. In terms of our variables, we have

$$s = 4\gamma^2 m^2 y_1 y_2 ,$$

$$t - m^2 = -2\gamma^2 m^2 y_1 x (1 - c) , \qquad (12)$$

$$u - m^2 = -2\gamma^2 m^2 y_2 (1 + c) .$$

With the help of Eq. (1), we find

$$d\sigma_{\gamma\gamma} = \frac{\pi}{4} \frac{r_e^2}{\gamma^2 y_1 y_2 - 1} \frac{dc}{y_2} \times \left\{ 2 \frac{y_1^2 (1 - c)^2 + y_2^2 (1 + c)^2}{(1 - c^2) [y_1 (1 - c) + y_2 (1 + c)]} + \frac{2}{\gamma^3} \frac{y_1 (1 - c) + y_2 (1 + c)}{y_1 y_2 (1 - c^2)} - \frac{1}{2\gamma^4} \frac{[y_1 (1 - c) + y_2 (1 + c)]^3}{y_1 y_2 [y_1 y_2 (1 - c^2)]^2} \right\}.$$
(13)

The first term obviously dominates. Furthermore, we expect that the major contribution comes from $c \leq c_0 \leq 1$. Thus $y_1(1-c) + y_2(1+c) \approx 2y_2$. With $\gamma^2 y_1 y_2 \gg 1$, we arrive at

$$\sigma_{\gamma\gamma}(y_1, y_2) \approx \frac{\pi r_e^2}{\gamma^2 y_1 y_2} \frac{1}{1 - c^2} \\ \times \left\{ \frac{y_1^2 (1 - c)^2 + y_2^2 (1 + c)^2}{[y_1 (1 - c) + y_2 (1 + c)]^2} \right\} . \\ \approx \frac{\pi r_e^2}{\gamma^2 y_1 y_2} \frac{1}{1 - c^2}$$
(14)

The last approximation is made due to the fact that the factor in the parenthesis is a slow varying function ranges from 1/2 to 1. In so doing our estimates are upper bounds which is too big by less than a factor of 2.

Inserting the photon spectra and Eq. (14) into Eq. (8), we can calculate the partial cross section for positrons with momentum larger than $p_{\perp 0}$ in all 4π solid angle, excluding the forward and backward cones of half-angle θ_0 . After lengthy derivations, the integrals over photon energies y_1 and y_2 are carried out exactly:

1. Breit-Wheeler process:

$$\sigma_{BW}(p_{\perp 0}, \theta_0) = \frac{9}{16} \Gamma^{-1}(1/3) \Gamma^2(2/3) \frac{\pi r_e^2}{\gamma^2} A^2 \\ \times \left\{ \int_{-c_0}^{c_0} \frac{dc}{1 - c^2} \frac{1}{y_+^{2/3} y_-^{2/3}} \right\};$$
(15)

2. Bethe-Heitler process:

$$\sigma_{BH}(p_{\perp 0},\theta_0) = \frac{9}{5} \frac{\alpha r_e^2}{\gamma^2} A \int_{-c_0}^{c_0} \left[\frac{dc}{(1-c^2)} \frac{1}{y_- y_+^{2/3}} \times \left\{ -\ell n y_- - \psi(8/3) + \psi(1) \right\} \right];$$
(16)

3. Landau-Lifshitz process:

$$\sigma_{LL}(p_{\perp 0}, \theta_0) = \frac{2\alpha^2 r_e^2}{\pi \gamma^2} \int_{-c_0}^{c_0} \left[\frac{dc}{1 - c^2} \frac{1}{y_+ y_-} \times \left\{ \ell n y_+ \ell n y_- + \frac{3}{2} \ell n (y_+ y_-) + \frac{73}{12} - \frac{\pi^2}{6} \right\} \right].$$
(17)

Here $\Gamma(1/3) \simeq 2.6789$, $\psi(8/3) \simeq 0.7818$, and $\psi(1) \simeq 0.5772$. We note that in the final integrations over the angle, both BW and LL processes are forward-backward symmetric, while BH is asymmetric in c. This is the result of the matching between the two different photon spectra in the BH case.

Since $y_+y_- = p_{\perp 0}^2/4$, and is independent of c, the integral in the BW process is straight forward. When ignoring the logarithmic angular dependences through y_+ and y_- , the BH and LL processes can also be integrated. The results are

$$\sigma_{BW}(p_{\perp 0},\theta_{0}) = \frac{9}{16\pi} \Gamma^{-1}(1/3) \Gamma^{4}(2/3) \frac{r_{e}^{2}}{\gamma^{2}} \times \left(\frac{\alpha \sigma_{z}}{\gamma \lambda_{c}}\right)^{2} \left(\frac{6\Upsilon}{p_{\perp 0}}\right)^{4/3} \ell n \left(\frac{1+c_{0}}{1-c_{0}}\right);$$
(18)

$$\sigma_{BH}(p_{\perp 0}, \theta_0) = \frac{54}{5\pi} \Gamma(2/3) \frac{\alpha r_e^2}{\gamma^2} \left(\frac{\alpha \sigma_z}{\gamma \lambda_c}\right) \left(\frac{36 \Upsilon^2}{p_{\perp 0}^5}\right)^{1/3} \\ \times \left[\left(\frac{1+c_0}{1-c_0}\right)^{1/6} - \left(\frac{1-c_0}{1+c_0}\right)^{1/6} \right] \\ \times \left[-\ell n \left(\frac{p_{\perp 0}}{2} \sqrt{\frac{1-c_0}{1+c_0}}\right) - \psi(8/3) + \psi(1) \right];$$
(19)

$$\sigma_{LL}(p_{\perp 0}, \theta_0) = \frac{8}{\pi} \frac{\alpha^2 r_e^2}{\gamma^2} \frac{1}{p_{\perp 0}^2} \ell n \left(\frac{1+c_0}{1-c_0}\right) \times \left\{ \ell n \left(\frac{p_{\perp 0}}{2} \sqrt{\frac{1+c_0}{1-c_0}}\right) \ell n \left(\frac{p_{\perp 0}}{2} \sqrt{\frac{1-c_0}{1+c_0}}\right) + 3\ell n(p_{\perp 0}/2) + \frac{73}{12} - \frac{\pi^2}{6} \right\}.$$
(20)

The above expressions account for only one of the two particles (say positron) in the e^+e^- pairs. The number is twice if both low energy e^+ and $e^$ in a pair are to be estimated.

4. Numerical Examples

We now apply the above formulas to specific examples from Palmer's designs in Ref. [6]. Previously, in considering the transverse momentum acquired from the deflections by the beam, the attention[2,3,5] has been on the low energy particles moving against the primary beam which has the same sign of charge. In that situation these particles will be deflected unbound by the collective field of the beam, and receive the maximum transverse momentum and angle. In this paper, our concern is the already large inherent transverse momentum and angle. Further deflections on either species in the pair by the beam should not effectively alter the ultimate outcoming transverse momenta and angles. Therefore both electrons and positrons, irrespect of the directions of flight, should be counted. So the contribution should be twice of what would be given from the partial cross sections in Eqs. (18)-(20).

First we estimate the yields from a 1/2 TeV Intermediate Linear Collider (ILC) (Palmer's Machine I), where $\gamma = 5 \times 10^5$, $\sigma_z = 0.11$ mm, $\Upsilon = 0.17$, and luminosity $\mathcal{L} = 1.95 \times 10^{31}$ /cm²/bunch train (10 bunches per train) and 130 Hz collision repetetion rate. We plot the partial cross sections in Eqs. (18)-(20) and their sum as a function of $p_{\perp 0}$ in Fig. 1, with the cut-off angle fixed at $\theta_0 = 0.1$. The dominant scalings of $p_{\perp 0}^{-4/3}$, $p_{\perp 0}^{-5/3}$, and $p_{\perp 0}^{-2}$, for the BW, BH, and LL processes, respectively, are clearly seen. On the other hand, the dependence of the partial cross sections on the cut-off angles is much milder, as expected. Figure 2 shows such a plot, again with the ILC parameters and $p_{\perp 0} = 10$ MeV/c.

The choice of the cut-offs depends on the practical considerations in the design of the detector masking[8]. From Ref. [8], it should be reasonable to assume $\theta_0 = 100$ mrad = 0.1 and the transverse momentum cut-off at 10MeV/c, or $p_{\perp 0} = 4 \times 10^{-5}$. The total number of large inherent angle e^+ and e^- is obtained simply by doubling the partial cross sections in Eqs. (18)-(20) and multiplying by the luminosity. The corresponding events per bunch train are

$$\begin{split} N_{BW} &= 2 \times \sigma_{BW} \mathcal{L} \approx 200 \quad ; \\ N_{BH} &= 2 \times \sigma_{BH} \mathcal{L} \approx 1200 \quad ; \\ N_{LL} &= 2 \times \sigma_{LL} \mathcal{L} \approx 500 \quad . \end{split}$$

So the total yield per bunch train is ≈ 1900 .



Fig. 1. The partial cross section as a function of the cut-off transverse momentum, at a fixed cut-off angle $\theta_0 = 0.1$, for the BW, BH, and LL processes, shown in dotted curves. The sum of these three processes (TOTAL = BW + BH + LL), is shown in solid curve. The parameters are based on the ILC in Ref. [6].



Fig. 2. The partial cross section as a function of the cut-off angle. The traverse momentum cut-off is fixed at $p_{\perp 0} = 10 \text{MeV/c}$. The same ILC parameters are assumed.

Next we turn to a 1-TeV Linear Collider (TLC) (Palmer's Machine J), where $\gamma = 1 \times 10^6, \sigma_z = 0.12 \text{mm}, \Upsilon = 0.60$, and luminosity $\mathcal{L} = 8.04 \times 10^{31}/\text{cm}^2/\text{bunch train}$ (17 bunches per train) and 128 Hz collision repetetion rate. With the same cut-offs at 0.1 rad and 10 MeV/c ($p_{\perp 0} = 2 \times 10^{-5}$), we find, for every bunch train

$$egin{aligned} &N_{BW} pprox 900 &; \ &N_{BH} pprox 5300 &; \ &N_{LL} pprox 2400 &. \end{aligned}$$

The total yield is ≈ 8600 . Finally, the dependence of the total events on the transverse momentum cut-off for ILC and TLC is shown in Fig. 3, where the angular cut-off is fixed at $\theta_0 = 0.1$.



Fig. 3. The total yield of pair particles in ILC and TLC per bunch train, as a function of the transverse momentum cut-off.

5. Geometric Reduction

In Chapter 2 we saw that for a given equivalent photon energy y, the dominant contribution to the cross section comes from the region of small transverse momentum transfer y_{\perp} . Quantum mechanically, this corresponds to the region of large impact parameters: $\rho \sim 1/q_{\perp} = 1/(\gamma m y_{\perp})$, up to a typical value of $\rho_m \sim 1/my = \lambda_c/y$, where λ_c is the Compton wavelength. If ρ_m turns out to be larger than the beam transverse size, these equivalent photons would extend physically to the outside of the oncoming beam. Since equivalent photons with impact parameters larger than the beam size cannot participate in interactions, the effective cross sections will be smaller than those computed above. This geometric reduction effect was first observed at Novosibirsk[9, 10], and subsequently developed theoretically by several authors[11-13].

For the next generation of linear colliders, such as the ILC and TLC that we discussed above, the e^+e^- colliding beams are typically very flat, i.e., $\sigma_x \gg \sigma_y$. Thus the geometric reduction is dominated by the minor dimension. The typical beam height is $2\sigma_y$, so the corresponding "cut-off" impact parameter is $\rho_c \sim 2\sigma_y$. Thus the region of transverse momentum transfer $1/\rho_c \gtrsim q_\perp \gtrsim 1/\rho_m$ is suppressed. Let us denote the effective cross section by $\bar{\sigma} = \sigma - \sigma'$; then the cut-off cross section σ' is associated with the equivalent photon spectrum in Eq. (5) where $y_{\perp max}$ and $y_{\perp min}$ are related to ρ_m and ρ_c , respectively, i.e.,

$$n'_{v}(y) \simeq \frac{2\alpha}{\pi} \frac{dy}{y} \ell n\left(\frac{\lambda_{c}/2\sigma_{y}}{y}\right) \quad .$$
 (23)

The cut-off cross section σ' for the BH and LL processes can be derived by inserting n'_v into Eq. (8). By construction, the above spectrum is applicable for $y \leq \lambda_c/2\sigma_y$. Thus the upper bounds of y-integrations must be replaced by $\lambda_c/2\sigma_y$. But since the dominant contribution comes from the lower bounds, this change does not affect the leading logarithmic behavior. One could therefore in principle repeat the calculations in Chapter 3 for the geometric reductions. However, as a rough estimate, we shall simply look for the effect on the total cross sections (with lower bounds defined by the threshold condition: $y_1y_2 = 1/\gamma^2$). We find the reduced effective total cross sections to be^{*}

$$\bar{\sigma}_{BH} \sim \alpha r_e^2 (\ell n 4 + 1) \ell n (2\sigma_y / \lambda_c) \quad ,$$

$$\bar{\sigma}_{LL} \sim \frac{1}{3\pi} \alpha^2 r_e^2 (\ell n 4 + 1) \qquad (24)$$

$$\times \left[\ell n \ 3(\gamma^2) - \ell n \ 3(\gamma^2 \lambda_c / 2\sigma_y) \right] \quad .$$

For ILC, the beam height is as miniscule as $2\sigma_y = 8$ nm. The geometric reduction turns out to be $\bar{\sigma}_{BH}/\sigma_{BH} \sim 0.36$, and $\bar{\sigma}_{LL}/\sigma_{LL} \sim 0.74$. For TLC, $2\sigma_y = 6.2$ nm. The corresponding reductions are 0.33 and 0.70 for the BH and LL processes, respectively. We expect a similar amount of reduction for the large inherent angle events. The geometric reduction is therefore a welcome effect in the context of e^+e^- backgrounds.

^{*} Our formulas give a numerical factor ln4 + 1 for both processes, which is smaller than the standard result of 28/9. But this will not affect the relative geometric reduction.

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