STATUS OF SM CALCULATIONS OF $B \rightarrow S$ TRANSITIONS

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We report recent progress in SM calculations of $b \rightarrow s$ transitions. We discuss the first NNLL prediction of the $\bar{B} \rightarrow X_s \gamma$ branching ratio, including important additional subtleties due to non-perturbative corrections and logarithmically-enhanced cut effects, and also recent results on the inclusive mode $\bar{B} \rightarrow X_s \ell^+ \ell^-$. Moreover, new results on the corresponding exclusive modes are reviewed. Finally, we comment on the present status of the so-called $B \rightarrow K\pi$ puzzle in hadronic $b \rightarrow s$ transitions.

1. Introduction

In any viable new physics model we have to understand the important flavour problem, namely why flavour-changing neutral currents are suppressed. Rare decays and CP violating observables exclusively allow an analysis of this problem. However, if new physics does not show up in flavour physics through large deviations, as recent experimental data indicate, the focus on theoretically clean observables within the indirect search for new physics is mandatory. This also calls for more precise SM calculations in the first place.

A crucial problem in the new physics search within flavour physics is the optimal separation of new physics effects from hadronic uncertainties. It is well known that inclusive decay modes are dominated by partonic contributions; non-perturbative corrections are in general rather small [1]. Also ratios of exclusive decay modes such as asymmetries are well suited for the new-physics search. Here large parts of the hadronic uncertainties partially cancel out; for example, there are CP asymmetries that are governed by one weak phase only; thus the hadronic matrix elements cancel out completely.

Data from K and B_d physics show that new sources of flavour violation in $s \rightarrow d$ and $b \rightarrow d$ are strongly constrained, while the possibility of sizable new contributions to $b \rightarrow s$ remains open [2,3]. We also have hints from model building: flavour models are not very effective in constraining the $b \rightarrow s$ sector [4]. Moreover, in SUSY-GUTs the large mixing angle in the neutrino sector relates to large mixing in the right-handed b-s sector [5,6,7].

In the following we discuss recent progress on several $b \rightarrow s$ observables.

2. $\bar{B} \rightarrow X_s \gamma$

Among the flavour-changing current processes, the inclusive $b \rightarrow s\gamma$ mode is still the most prominent. The stringent bounds obtained from this mode on various non-standard scenarios are a clear example of the importance of clean FCNC observables in discriminating new-physics models. Its branching ratio has already been measured by several independent experiments using semi-inclusive or fully inclusive methods [8,9,10,11,12]. The world average of those five measurements (performed by the Heavy Flavour Averaging Group (HFAG) [13]) for a photon energy cut $E_\gamma > 1.6$ GeV reads

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\exp} =$$
$$= \left(3.55 \pm 0.24 \, ^{+0.09}_{-0.10} \pm 0.03\right) \times 10^{-4} \quad (1)$$

NNLO SM Prediction
3.15 ± 0.23 x10⁻⁴
hep-ph/0609232

CLEO Phys. Rev. Lett. 87, 251807 (2001)

BELLE Phys.Lett. B 511, 151 (2001)

BELLE Phys.Rev.Lett.93:061803,2004

BABAR PRD 72, 052004 (2005)

BABAR hep-ex/0507001

HFAG Average
3.55 ± 0.26 x10⁻⁴
Extrapolation to Eγ > 1.6 GeV
from PRD73:073008,2006

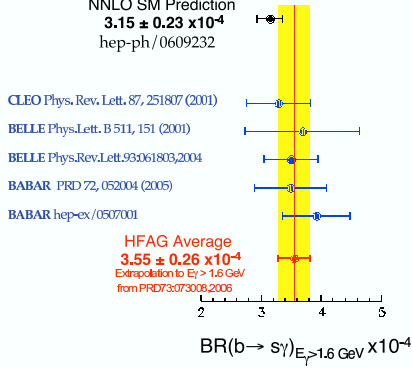BR(b→ sγ)$_{E_\gamma > 1.6\,\mathrm{GeV}}$ x10⁻⁴

Fig. 1.   New NNLL prediction versus HFAG average.

where the errors are combined statistical and systematic, systematic due to the extrapolation, and due to the $b \to d\gamma$ fraction.

On the theory side, perturbative QCD contributions to the decay rate are dominant and lead to large logarithms $\alpha_s(M_W) \times \log(m_b^2/M_W^2)$, which have to be resummed in order to get a reasonable result. Resumming all the terms of the form $(\alpha_s(m_b))^p \, \alpha_s^n(m_b) \log^n(m_b/M)$ (with $M = m_t$ or $M = m_W$, $n = 0, 1, 2, \ldots$) for fixed p corresponds for $p = 0$ to leading-log (LL), for $p = 1$ to next-to-leading-log (NLL), and for $p = 2$ to next-to-next-to-leading-log (NNLL) precision. The previous NLL prediction, based on the original QCD calculations of several groups [14,15,16,17,18,19,20,21,22,23,24], had an additional charm mass renormalization scheme ambiguity, first analysed in Ref.[25]. For an energy cut $E_\gamma > 1.6$ GeV it reads [27]

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\mathrm{NLL}} =$$
$$= (3.61^{+0.24}_{-0.40} \pm 0.02 \pm 0.25 \pm 0.15) \times 10^{-4} \quad (2)$$

where the errors are due to the charm scheme dependence, CKM input, further parametric dependences, and to perturbative scale dependence. The dominant uncertainty related to the definition of m_c was taken into account by varying m_c/m_b in the conservative range $0.18 \le m_c/m_b \le 0.31$, which covers

both, the pole mass value (with its numerical error) and the running mass value $\bar{m}_c(\mu_c)$ with $\mu_c \in [m_c, m_b]$; for the central value $m_c/m_b = 0.23$ was used [27]. However, the renormalization scheme for m_c is an NNLL issue. It was shown that a complete NNLL calculation reduces this large uncertainty at least by a factor of 2 [28].

Within a global effort, such a NNLL calculation was quite recently finalized [29]. The calculational steps were performed by various groups [31,32,33,38,35,34,36,37,30,39]. One crucial piece is the calculation of the three-loop matrix elements of the four-quark operators, which was first made within the so-called large-β_0 approximation [30]. A calculation that goes beyond this approximation by employing an interpolation in the charm quark mass m_c from $m_c > m_b$ to the physical m_c value has just been completed [39]. It is that part of the NNLL calculation where there is still space for improvement.

All those results lead to the first estimate of the $\bar{B} \to X_s\gamma$ branching ratio to NNLL precision. It reads for a photon energy cut $E_\gamma > 1.0$ GeV [29]:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\mathrm{NNLL}} =$$
$$(3.27 \pm 0.23) \times 10^{-4}. \quad (3)$$

The overall uncertainty consists of non-perturbative (5%), parametric (3%), higher-order (3%) and m_c-interpolation ambiguity (3%), which have been added in quadrature. For higher photon energy cut we have the following numerical fit:

$$\left(\frac{\mathcal{B}(E_\gamma > E_0)}{\mathcal{B}(E_\gamma > 1.0\,\mathrm{GeV})} \right) \simeq 1 - 0.031y - 0.047y^2, \quad (4)$$

where $y = E_0/(1.0\,\mathrm{GeV}) - 1$. This formula coincides with the NNLL results up to $\pm 0.1\%$ for $E_0 \in [1.0,\ 1.6]$ GeV. The error is practically E_0-independent in this range. For $E_\gamma > 1.6$ GeV the NNLL prediction
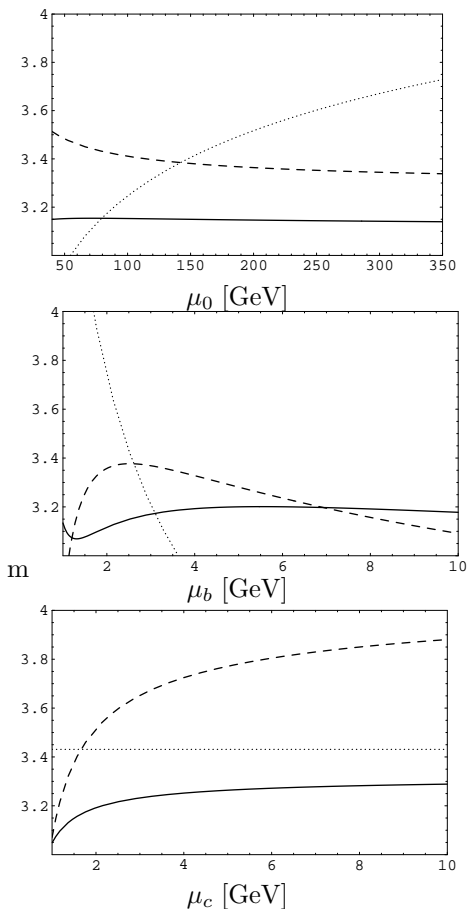
Fig. 2. Renormalization-scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in units 10^{-4} at the LL (dotted lines), NLL (dashed lines) and NNLL (solid lines). The plots describe the dependence on the matching scale μ_0, the low-energy scale μ_b, and the charm mass renormalization scale μ_c.

reads [29]:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\text{NNLL}} =$$
$$(3.15 \pm 0.23) \times 10^{-4}. \qquad (5)$$

Compared with the HFAG average, given in Eq.(1), the NNLL prediction is 1.2σ below the experimental data (see Fig. 1 [26]).

The reduction of the renormalization-scale dependence at the NNLL is clearly seen in Fig. 2. The most important effect occurs for the charm mass $\overline{\text{MS}}$ renormalization scale μ_c that was the main source of uncertainty

at the NLL. The current uncertainty of $\pm 3\%$ due to higher-order ($\mathcal{O}(\alpha_s^3)$) effects is estimated from the NNLL curves in Fig. 2. The reduction factor of the perturbative error is more than a factor 3. The central value of the NNLL prediction is based on the choices $\mu_b = 2.5\text{GeV}$ and $\mu_c = 1.5$ GeV.

There are some perturbative NNLL corrections which are not included yet in the present NNLL estimate, but are expected to be smaller than the current uncertainty: the virtual- and bremsstrahlung contributions to the $(\mathcal{O}_7, \mathcal{O}_8)$ and $(\mathcal{O}_8, \mathcal{O}_8)$ interferences at order α_s^2, the NNLL bremsstrahlung contributions in the large β_0-approximation beyond the $(\mathcal{O}_7, \mathcal{O}_7)$ interference term (which are already available [40]), the four-loop mixing of the four-quark operators into the operator $\mathcal{O}_8$ (see recent work [38]), the exact m_c dependence of the various matrix elements beyond the large β_0 approximation (see [41]) and perturbative logarithmically-enhanced cut effects (see discussion below and [60]).

Nevertheless, the final result includes subdominant contributions such as the perturbative electroweak two-loop corrections of order -3.7% [42,24,43,44] and the non-perturbative corrections scaling with $1/m_b^2$ or $1/m_c^2$ of order $+1\%$ and $+3\%$ respectively [45,48,49,50,51].

It is well known, that the local operator product expansion (OPE) for the decay $\bar{B} \to X_s\gamma$ has certain limitations if one takes into account other operators than the leading $\mathcal{O}_7$, as was already shown within the analysis of $1/m_c^2$ power corrections. The additional error of 5% in the NNLL prediction corresponds to non-perturbative corrections, which scale with $\alpha_s\Lambda/m_b$. Quite recently, a specific piece of the additional non-perturbative corrections was estimated [52]. Because the overall sign of the whole effect is still unknown, this partial estimate is not included in the central value of the present NNLL prediction [29].

However, there are more subtleties. There is an additional sensitivity to non-perturbative physics, due to necessary cuts in the photon energy spectrum to suppress the background from other B decays (see Fig. 3). This leads to a breakdown of the local OPE, which can be cured by partial resummation of these effects to all orders into a non-perturbative shape-function [53,54,55].
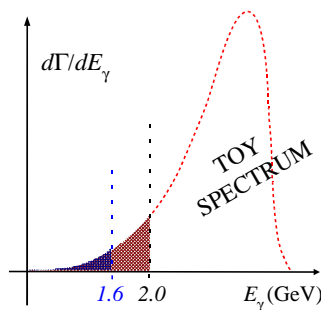

Fig. 3.   Cut in the photon energy spectrum

Those shape-function effects are taken into account in the experimental value by the HFAG [13] and the corresponding theoretical uncertainties due to this model dependence is reflected in the extrapolation error in the experimental number quoted above in Eq. (1). Here, one should keep in mind that the experimental energy cuts in the last experiments are at $1.8\,\mathrm{GeV}$ or $1.9\,\mathrm{GeV}$ or even higher. The extrapolation down to $1.6\,\mathrm{GeV}$ is done using three different theoretical schemes to calculate the extrapolation factor [56,24,57,58] and averaging those results.

Moreover, it was argued that a cut around $1.6\,\mathrm{GeV}$ might not guarantee that a theoretical description in terms of a local OPE is sufficient because of the sensitivity to the scale $\Delta = m_b - 2E_\gamma$ [59]. A multiscale OPE with three short-distance scales m_b, $\sqrt{m_b\Delta}$, and Δ was proposed to connect the shape function and the local OPE region. Quite recently, such addi-

tional cutoff-related effects were numerically estimated using (model-independent) SCET methods [60,61,62]. Those perturbative effects due to the additional scale are negligible for $1.0\,\mathrm{GeV}$ but lead to an effect of order 3% at $1.6\,\mathrm{GeV}$ [60]. The size of these effects at $1.6\,\mathrm{GeV}$ is at the same level as the 3% higher-order uncertainty in the present NNLL prediction. It is suggestive that in the future those additional perturbative cut effects get analysed and combined together with those already included in the experimental average of the HFAG.

There are also other claims for non-negligible cut effects at $1.6\,\mathrm{GeV}$ [63] which, however, are based on models of the non-perturbative shape function. Moreover, there is an alternative approach to the cut effects in the photon energy spectrum based on dressed gluon exponentiation and incorporating Sudakov and renormalon resummations [64,65]. It should be emphasized that the higher predictive power of this approach is related in part to the assumption that non-perturbative power corrections associated with the shape function follow the pattern of ambiguities present in the perturbative calculation [66].

3. $\bar{B} \to X_s \ell^+ \ell^-$

In comparison to the $\bar{B} \to X_s\gamma$, the inclusive $\bar{B} \to X_s\ell^+\ell^-$ decay presents a complementary and also more complex test of the SM. The decay $\bar{B} \to X_s\ell^+\ell^-$ is particularly attractive because of kinematic observables such as the invariant dilepton mass spectrum and the forward–backward (FB) asymmetry. These observables are dominated by perturbative contributions if the $c\bar{c}$ resonances that show up as large peaks in the dilepton invariant mass spectrum are removed by appropriate kinematic cuts (see Fig. 4). In the 'perturbative $s = q^2/m_b^2$-windows', namely in the low-dilepton-mass region $1\,\mathrm{GeV} < q^2 < 6\,\mathrm{GeV}$, and also in the high-dilepton-

mass region with $q^2 > 14.4\,\text{GeV}$, theoretical predictions for the invariant mass spectrum are dominated by the perturbative contributions; a theoretical precision of order 10% is in principle possible. Regarding the choice of precise cuts in the dilepton mass spectrum, it is important that theory and experiment can be compared using the same energy cuts and any kind of extrapolation is avoided.
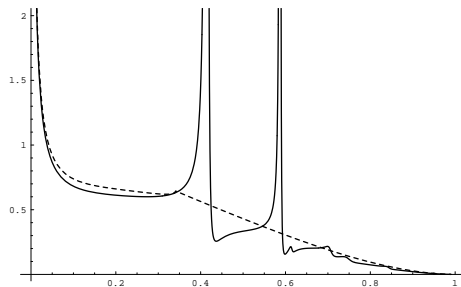

Fig. 4. Schematic dilepton-mass dependence of the differential branching ratio $d/ds\,BR(\bar{B} \to X_s\ell^+\ell^-)$ in units 10^{-5} (the dashed line corresponds to the perturbative contribution).

The recently calculated NNLL contributions [67,68,69,70,71,72] have significantly improved the sensitivity of the inclusive $\bar{B} \to X_s\ell^+\ell^-$ decay in testing extensions of the SM in the sector of flavour dynamics, in particular the value of the dilepton invariant mass q_0^2, for which the differential forward–backward asymmetry vanishes, is one of the most precise predictions in flavour physics with a theoretical uncertainty of order 5%.

Also non-perturbative corrections scaling with $1/m_b^2$ or $1/m_c^2$ [47,75,51,76,77,78] are taken into account in the present NNLL predictions. The unknown non-perturbative corrections which scale with $\alpha_s\Lambda/m_b$ are less important than in the case of the decay $\bar{B} \to X_s\gamma$.

Recently, further refinements were presented such as the NLL QED two-loop corrections to the Wilson coefficients of order 2% [72]. Furthermore, it was shown that in the QED two-loop corrections to matrix elements large collinear logarithms of the form $log(m_b/m_{\text{lepton}})$ survive integration if only a restricted part of the dilepton mass spectrum is considered. This adds another contribution of order +2% in the low-q^2 region [73].

A recent update of the dilepton mass spectrum, integrated over the low dilepton invariant mass region in the muonic case, leads to [73]

$$\mathcal{B}(\bar{B} \to X_s\mu^+\mu^-) = (1.59 \pm 0.11) \times 10^{-6}\,, \tag{6}$$

where the error includes the parametric and perturbative uncertainties only. For $\mathcal{B}(B \to X_s e^+e^-)$, in the current BaBar and Belle setups, the logarithm of the lepton mass gets replaced by angular-cut parameters and the integrated branching ratio for the electrons is expected to be close to that for the muons. The analogous update of the other NNLL predictions will be presented in a forthcoming paper [74].

There are further subtleties, which again lead to larger theoretical uncertainties. In the high-s region, one encounters the breakdown of the heavy-mass expansion at the endpoint; while the partonic contribution vanishes in the end-point, the $1/m_b^2$ corrections tend towards a non-zero value. In contrast to the endpoint region of the photon energy spectrum in the $\bar{B} \to X_s\gamma$ decay, no partial all-orders resummation into a shape function is possible here. However, for an integrated high-s spectrum $R(s)$ an effective expansion is found in inverse powers of

$$m_b^{\text{eff}} = m_b \times (1 - \sqrt{s_{\text{min}}}) \tag{7}$$

rather than m_b. The expansion converges less rapidly, depending on the lower dilepton-mass cut s_{min} [69].

A hadronic invariant-mass cut is imposed in the present experiments (Babar: $m_X < 1.8\,\text{GeV}$, Belle: $m_X < 2.0\,\text{GeV}$) in order to eliminate the background such as $b \to c\,(\to se^+\nu)e^-\bar{\nu} = b \to se^+e^- +$

missing energy. The high-dilepton mass region is not affected by this cut and in the low-dilepton mass region the kinematics with a jet-like X_s and $m_X^2 \leq m_b \Lambda_{\mathrm{QCD}}$ implies the relevance of the shape function. A recent SCET analysis shows that using the universality of jet and shape functions the $10-30\%$ reduction of the dilepton mass spectrum can be accurately computed using the $\bar{B} \to X_s \gamma$ shape function. Nevertheless effects of subleading shape functions lead to an additional uncertainty of 5% [79,80].

It is well-known, that the measurement of the dilepton mass spectrum, the zero of the forward-bachward asymmetry, and the $\bar{B} \to X_s \gamma$ branching ratio allows to fix the magnitude and sign of all relevant Wilson coefficients within the SM. In view of the fact that at present only restricted data sets are available, it was recently proposed to focus on quantities which are integrated (over q^2); besides the total rate and the integrated forward-bachward asymmetry it was shown that the third angular decomposition within the $b \to \ell^+ \ell^-$ mode is sensitive to a different combination of Wilson coefficients [81].

4. Exclusive $b \to s$ transitions

The corresponding rare exclusive decays, such as $B \to K^* \gamma$, $B \to K^* \mu^+ \mu^-$ or also $B_s \to \phi\gamma$, $B_s \to \phi\mu^+\mu^-$, are well-accessible at the forthcoming LHCb experiment. In contrast to the measurement of the branching ratios, measurements of CP, forward-backward, and isospin asymmetries are less sensitive to hadronic uncertainties.

For example, the value of the dilepton invariant mass in $B \to K^* \mu^+ \mu^-$, q_0^2, for which the differential forward–backward asymmetry vanishes, can be predicted in quite a clean way. In the QCD factorization approach, at leading order in $\Lambda_{\mathrm{QCD}}/m_b$, the value of q_0^2 is free from hadronic uncertainties at order α_s^0, a dependence on the soft form factor $\xi_\perp$ and the light-cone wave functions of the B and K^* mesons appear at order α_s^1. The latter contribution, calculated within the QCD factorization approach, leads to a large shift (see [82,83,84]). Nevertheless, there is the well-known issue of power corrections (Λ_{QCD}/m_b) within the QCD factorization approach which increases the theoretical uncertainty.

An extension of the QCD factorization formula to the non-resonant decay $B \to K\pi\ell^+\ell^-$ with an energetic $K\pi$ pair and also with an energetic kaon and a soft pion was presented [85]. Here one relies on the fact that the the forward-backward asymmetry is due to the interference of the helicity $J = 1^+, 1^-$ amplitudes induced by the $b \to s$ current. So it seems that no angular analysis is necessary to disentangle vector and tensor final states; however, the dependence of the non-perturbative input functions on the kinematic variables might differ for the two cases. This suggests that a restriction to the resonant states only is still the theoretically cleanest option.

There are also certain transversity amplitudes in $B \to K^* \mu^+ \mu^-$, in which the hadronic formfactors also cancel out at leading order. Thus, such observables are rather insensitive to hadronic uncertainties, but highly sensitive to non-standard chiral structures of the $b \to s$ current [86,87].

Quite recently, branching ratios, isospin and CP asymmetries in exclusive radiative decays like $B \to K^* \gamma$ and $B_s \to \phi\gamma$ were estimated combining QCD factorization results with QCD sum rule estimates of power corrections, namely long-distance contributions due to photon and soft-gluon emisson from quark loops [88]. Particulary, this leads to an estimate of the time-dependent CP asymmetry in $B^0 \to K^{*0}\gamma$ of $S = -0.022 \pm 0.015^{+0}_{-0.01}$ [88]. The contribution due to soft gluon emssion is estimated within the QCD sum rule approach to be very small, $S^{sgluon} = -0.005 \pm 0.01$, while a conservative

dimensional estimate of this contribution due to a nonlocal SCET operator series leads to $|S^{sgluon}| \approx 0.06$ [89,90]. Furthermore. one finds a larger time-dependent CP asymmetry of around 10% within the inclusive mode [89]; however, the SCET estimate shows that the expansion parameter is Λ_{QCD}/Q where Q is the kinetic energy of the hadronic part, while there is no contribution at leading order. Therefore, the effect is expected to be larger for larger invariant hadronic mass, thus, the K^* mode has to have the smallest effect, below the 'average' 10% [89].

5. Comments on the so-called $B \to K\pi$ puzzle

The $B \to K\pi$ modes are well known for being sensitive to new electroweak $b \to s$ penguins beyond the SM [91,92]. The data on CP-averaged $K\pi$ branching ratios can be expressed in terms of three ratios:

$$R = \frac{\tau_{B^+}}{\tau_{B^0}} \frac{\mathcal{B}[B^0 \to \pi^- K^+] + \mathcal{B}[\bar{B}^0 \to \pi^+ K^-]}{\mathcal{B}[B_d^+ \to \pi^+ K^0] + \mathcal{B}[B_d^- \to \pi^- \bar{K}^0]}$$

$$R_n = \frac{1}{2} \frac{\mathcal{B}[B^0 \to \pi^- K^+] + \mathcal{B}[\bar{B}^0 \to \pi^+ K^-]}{\mathcal{B}[B^0 \to \pi^0 K^0] + \mathcal{B}[\bar{B}^0 \to \pi^0 \bar{K}^0]}$$

$$R_c = 2 \frac{\mathcal{B}[B_d^+ \to \pi^0 K^+] + \mathcal{B}[B_d^- \to \pi^0 K^-]}{\mathcal{B}[B_d^+ \to \pi^+ K^0] + \mathcal{B}[B_d^- \to \pi^- \bar{K}^0]}$$

The actual data presented at ICHEP06 read [94,95,96]

$$R = 0.92^{+0.05}_{-0.05} R_n = 1.00^{+0.07}_{-0.07} R_c = 1.10^{+0.07}_{-0.07}$$

One should emphasize that in the previous analyses the radiative electromagnetic corrections to charged particles in the final state were not taken into account, as was emphasized in the past (see for example Ref. [104]). These corrections, worked out in Ref. [105], are now properly included in the analysis of both experiments.

The present data is compatible with the approximate sum rule proposed in Refs. [97,98,99], which leads to the prediction $R_c \approx R_n$. The data is also in agreement with
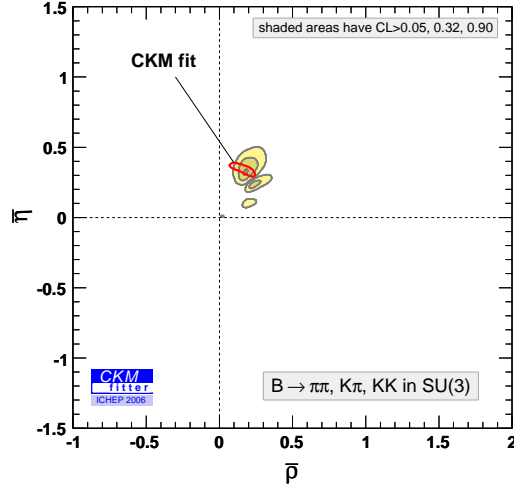

Fig. 5. Constraint in the $(\bar{\rho}, \bar{\eta})$ plane induced by the $\pi\pi, K\pi, K\bar{K}$ data compared with the standard CKM fit.

the available SM approaches to these data based on QCD factorization techniques and on $SU(3)_F$ symmetry assumptions [102,103]. For example, the BBNS predictions, based on the QCD factorization approach [100,101], are

$$R = 0.91^{+0.13}_{-0.11} R_n = 1.16^{+0.22}_{-0.19} R_c = 1.15^{+0.19}_{-0.17}$$

The latest $SU(3)_F$ analysis of the CKM fitter group [106], includes all available $\pi\pi, K\pi, K\bar{K}$ modes; also so-called annihilation/exchange topologies and factorizable $SU(3)_F$ breaking are taken into account.

As shown in Fig. 5 [106] the constraint in the $(\bar{\rho}, \bar{\eta})$ plane induced by these data implies that the compatibility with the $SU(3)$ and SM hypothesis is very good (the so-called pValue of that SM analysis is of order $30 - 40\%$). But the $\chi^2_{\min}$ is not always the best measure of the compatibility of the data with the theory. Among the main contributions to the χ^2 there is the ratio R_n and the CP asymmetry $S(K_S^0\pi^0)$. The latter quantity is defined via

$$a_{CP}[K_S^0\pi^0](t) = \tag{8}$$
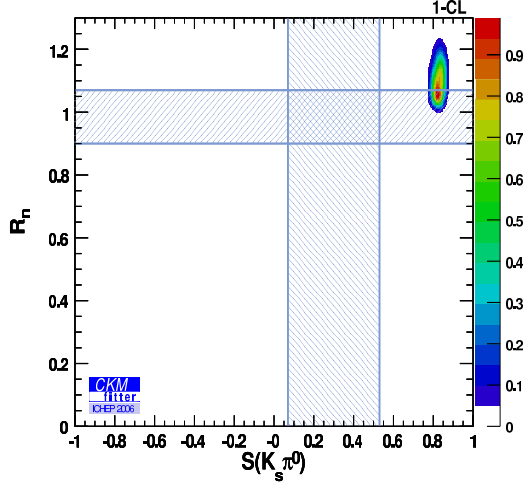$$S(K_S^0\pi^0)\sin(\Delta m_d t) - C(K_S^0\pi^0)\cos(\Delta m_d t).$$

Fig. 6.   Comparison of direct measurements of R_n and $S(K_S^0 \pi^0)$ (1σ band) with the indirect fit (2σ contour).

Both observables are very sensitive to new electroweak penguins. After removing them from the global fit, Fig. 6 [106] shows the comparison of the indirect fit (2σ contour), with $\bar{\rho}, \bar{\eta}$ from the CKM fit and all other available modes, with the direct measurements (1σ band) using the new data. The indirect prediction for R_n is now in good agreement with the direct measurement. There is still a small tension in the case of the observable $S(K_S^0 \pi^0)$ which is at present not really 'puzzling' from the statistical point of view. Fig. 6 shows that these two quantities are not correlated, so that possible deviations from the SM values could have a completely different origin. These findings were just recently confirmed in a similar approach [107].

Because of the large non-factorizable contributions identified in the $B \rightarrow \pi\pi$ channel, large non-factorizable $SU(3)_F$ - or isospin-violating QCD and QED effects within the SM cannot be ruled out at present [108]. Future data from the B factories and LHCb will clarify the situation completely. There will be up to 38 measured observables depending on the same 13+2 theoretical parameters. This will allow for the study of $SU(3)$ breaking and new-physics effects.

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