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Measurement of $\sin 2\beta$ in tree-dominated B^0-decays and ambiguity removal

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The most recent results from the B-factories on the time-dependent CP asymmetries measured in B^0-decays mediated by $b \to c\bar{c}s$ quark-transitions are reviewed. The Standard Model interpretation of the results in terms of the parameter $\sin 2\beta$ leads to a four-fold ambiguity on the unitarity triangle β which can be reduced to a two-fold ambiguity by measuring the sign of the parameter $\cos 2\beta$. The results on $\cos 2\beta$ obtained so far are reviewed.

1. INTRODUCTION

Within the Standard Model, quark flavor-mixing is described by the 3×3 unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. The size of CP violation carried by the CKM matrix is quantified by the imaginary part of the apex, $\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$, of the unitarity triangle. Constraints on the parameter $\sin 2\beta$ ($\beta = \arctan(\frac{\bar{\eta}}{1-\bar{\rho}})$) can be obtained from the measurement of time-dependent decay rates of neutral B-mesons into CP eigenstates f_{CP} mediated by $b \to c\bar{c}s$ quark transitions. The time-dependent rate asymmetry between $\bar{B}^0$ and B^0 decays is given by $A_{CP}(t) = \frac{2 \cdot \Im \lambda}{1+|\lambda|^2} \sin(\Delta m_d \cdot t) - \frac{1-|\lambda|^2}{1+|\lambda|^2} \cos(\Delta m_d \cdot t)$, where Δm_d is the mass difference between the heavy ($B_H = pB^0 + q\bar{B}^0$) and the light ($B_L = pB^0 - q\bar{B}^0$) neutral mass eigenstate, and $S = \frac{2 \cdot \Im \lambda}{1+|\lambda|^2}$ and $C = -A = \frac{1-|\lambda|^2}{1+|\lambda|^2}$ where the parameter C is used by the BABAR collaboration and A by the Belle collaboration. The parameter λ, given by $\eta_{CP}\frac{q}{p}\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$, η_{CP} being the CP eigenvalue of the final state, is a phase-convention-independent quantity which contains possible sources of CP violation in mixing ($|\frac{q}{p}| \neq 1$), direct CP violation ($|\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}| \neq 1$), and CP violation in the interference between decay with and without mixing ($\Im \lambda \neq 0$). In the $B^0 - \bar{B}^0$ sys-

tem, to a very good approximation $\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}$ and CP-violation in mixing is expected to be small, which is confirmed by the experimental constraints: $|\frac{q}{p}| - 1 = 0.0015 \pm 0.0039$ [2]. For the decay $B^0/\bar{B}^0 \to (c\bar{c})K^0/\bar{K}^0$, the dominant amplitude is a tree-mediated $b \to c\bar{c}s$ quark-transition. There are also contributions from penguin amplitudes. In the case of the c- and t-penguin, the CKM phases are equal or almost equal, respectively, to the tree-decay phase ($\propto V_{cs}V_{cb}^*$). Only in the case of the u-quark penguin the CKM phase ($\propto V_{us}V_{ub}^*$) differs significantly from the tree amplitude. However, it is doubly-Cabibbo suppressed with respect to the tree amplitude. An additional effect comes from CP-violation in the $K^0 - \bar{K}^0$ system, however which changes the relative phase by only a small amount. Hence, for the CP asymmetry, one expects to a very good approximation $A_{CP}(t) = \eta_{CP}\sin 2\beta \sin(\Delta m_d \cdot t)$.

2. MEASUREMENTS OF $\sin 2\beta$

The most precise determinations of $\sin 2\beta$ are coming currently from the B-factory experiments BABAR and Belle thanks to the excellent performance of their accelerators PEP II and KEKB. The new peak luminosity records achieved in 2006 are 1.207×10^{34} cm^{-2}s^{-1} for PEP II and 1.6517×10^{34} cm^{-2}s^{-1} for KEKB. By summer 2006, PEP II/BABAR had collected 390 fb^{-1} of data while KEKB/Belle had collected a data sam-

ple of 630 fb^{-1}. This results in a sum of 10^9 $B\bar{B}$ events written to tape. Both experiments have updated their $\sin 2\beta$ analyses recently [3,4].

At the B-factories, the $\Upsilon(4S)$ resonance is produced in e^+e^- collisions with asymmetric electron and positron beam energies resulting in a boost factor of $\beta\gamma = 0.56$ at PEP II and $\beta\gamma = 0.425$ at KEKB, respectively. The $\Upsilon(4S)$ decays in about 50% of the cases to neutral B-meson pairs, produced in a coherent quantum state. The time-dependent CP asymmetry is measured by determining the decay-time difference Δt between the decay of one B to a CP eigenstate (B_{CP}) and the decay of the other B meson (B_{tag}) to flavour-specific final states. These final states are not reconstructed exclusively and the flavor of B_{tag} is determined ("tagged") only on a statistical basis by determining the sign of the charge mainly from high-energy leptons, and kaons and low-energy pions mutually stemming from D^* decays. The figure of merit for the tagging performance is given by the quality factor $Q = \sum_i e_i(1 - 2 \times w_i)^2$, where e_i is the tagging efficiency and w_i the mistag fraction in tagging category i. Q enlarges the statistical uncertainty of the decay-rate asymmetry measurements and has been measured by BABAR (Belle) to be $Q = (30.4 \pm 0.3)\%$ ($Q = (29.0 \pm 1.0)\%$). The decay-time difference Δt is estimated from the distance between the two B-decay vertices Δz in the beam-direction z. Due to the $\Upsilon(4S)$ boost, Δz can be measured with sufficient precision by means of silicon vertex detectors. The average Δz is of order 200 μm at Belle and about 250 μm at BABAR. The Δz resolution is dominated by the B_{tag} vertex resolution and is of order 190 μm.

The most recent BABAR analysis [3] was performed on a sample of 348×10^6 $B\bar{B}$ events. The CP-odd final states ($\eta_{CP} = -1$) used are $B^0/\bar{B}^0 \rightarrow (c\bar{c})K_S^0$ where $c\bar{c} = J/\psi \rightarrow e^+e^-, \mu^+\mu^-, \psi(2S) \rightarrow e^+e^-, \mu^+\mu^-, J/\psi\pi^+\pi^-, \chi_{c1} \rightarrow J/\psi\gamma$, and $\eta_c \rightarrow K_S^0 K^+\pi^-$. The CP-even final state $B^0/\bar{B}^0 \rightarrow J/\psi K_L^0$ is also reconstructed. Compared to the high-purity sample $B^0/\bar{B}^0 \rightarrow (c\bar{c})K_S^0$, this final state suffers from a significantly higher background level.

In addition, the vector-vector final state $B^0/\bar{B}^0 \rightarrow J/\psi K^{*0}$ with $K^{*0} \rightarrow K_S^0\pi^0$ is used. A recent angular analysis performed by BABAR finds an effective CP eigenvalue of $\eta_{CP} = 0.504 \pm 0.033$ for this final state [5]. For the CP-odd final states the result is $\sin 2\beta = +0.713 \pm 0.038_{stat}$, for the final state $B^0/\bar{B}^0 \rightarrow J/\psi K_L^0$ it is $\sin 2\beta = +0.716 \pm 0.080_{stat}$ (see Fig. 1). The average of all modes including $B^0/\bar{B}^0 \rightarrow J/\psi K^{*0}$ results in $\sin 2\beta = +0.710 \pm 0.034_{stat} \pm 0.019_{sys}$. The main systematic uncertainties are due to the
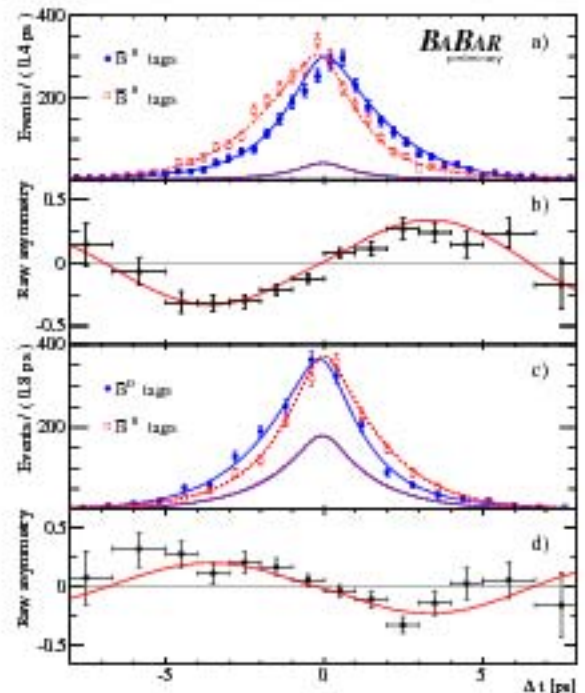

Figure 1. Measured decay-time distributions and corresponding decay-rate asymmetries for the BABAR $\sin 2\beta$ analysis. a) and b) CP odd finals states, c) and d) CP even final state. The shaded regions represent the estimated background contribution. The solid (dashed) curves represent the fit projections in Δt for B ($\bar{B}$) tags.

CP content of the background, the Δt resolution

functions, the background in $B^0/\overline{B^0} \to J/\psi K_L^0$, the differences in the mistag fractions between B^0 and $\overline{B^0}$, and the knowledge of the event-by-event beam spot position. For the final state $B^0/\overline{B^0} \to (c\bar{c})K_S^0$, which has the highest purity (92%), BABAR fits $\sin 2\beta$ and $|\lambda|$ simultaneously and finds $C = +0.070 \pm 0.028_{stat} \pm 0.018_{sys}$, translating into a 2 σ deviation from zero. The dominant systematic uncertainty in this case (0.014) is due to the interference between the suppressed $\bar{b} \to \bar{u}c\bar{d}$ amplitude and the $\bar{b} \to \bar{c}u\bar{d}$ amplitude for certain hadronic B_{tag} decays.

The Belle analysis [4] uses a sample of 532 million $B\bar{B}$ events where only the CP-odd final state $B^0/\overline{B^0} \to J/\psi K_S^0$ and the CP-even final state $B^0/\overline{B^0} \to J/\psi K_L^0$ are considered. Both results for the S and C coefficients are consistent: $\sin 2\beta(J/\psi K_S^0) = +0.643 \pm 0.038_{stat}$ and $\sin 2\beta(J/\psi K_L^0) = +0.641 \pm 0.057_{stat}$, respectively, $C(J/\psi K_S^0) = -0.018 \pm 0.021_{stat}$ and $C(J/\psi K_L^0) = -0.045 \pm 0.033_{stat}$. The averages between both results are: $\sin 2\beta = +0.642 \pm 0.031_{stat} \pm 0.017_{sys}$ and $C = -0.018 \pm 0.021_{stat} \pm 0.014_{sys}$. The dominant systematic uncertainties on $\sin 2\beta$ come from the vertex reconstruction, the resolution function, the background fraction, flavor tagging, and the effect of the tag-side interference. The systematic uncertainty on C, as in the case of BABAR, is dominated by the tag-side interference (0.009).

The world average computed by the Heavy Flavor Averaging Group (HFAG) [2] is $\sin 2\beta = +0.675 \pm 0.023_{stat} \pm 0.012_{sys}$, where most of the systematic uncertainties have been treated as uncorrelated. Correlated error sources like the B-lifetime, the mixing frequency Δm_d or the effect of the tag-side interference play only a minor role at this stage. This suggests that on the time scale of 2008, when an integrated luminosity of order 2 ab^{-1} is expected from BABAR and Belle together, the total uncertainty on $\sin 2\beta$ will fall below 0.020. Theoretical estimates for the difference between the measured S coefficient and the true $\sin 2\beta$ value are of order 0.01 or below [6]. The average value of BABAR and Belle for the cosine coefficient is $C = 0.012 \pm 0.017_{stat} \pm 0.014_{sys}$ and hence consistent with zero as expected in the Standard Model at this level of precision. The

systematic uncertainty is dominated by the tag-side interference effect. Under the assumption that both measurements have estimated their uncertainties correctly and that the uncertainties are Gaussian-like, the p-value (denoted CL by HFAG [2]) to find a deviation between the two experiments as large as or larger than the one observed is 0.02.

3. MEASUREMENTS OF $\cos 2\beta$

Since only the parameter $\sin 2\beta$ is measured, there are four different solutions for β. Two solutions correspond to $\cos 2\beta > 0$, $\beta = 21.2°$ or $\beta = 21.2° + 180°$, while the ones corresponding to $\cos 2\beta < 0$ are $\beta = 68.8°$ or $\beta = 68.8° + 180°$. The constraints on the unitarity triangle excluding $\sin 2\beta$ as an input result is $\beta = (27.7^{+0.8}_{-3.9})°$ [7], suggesting that $\beta = 21.2°$ is the correct solution. However, this argument relies on the validity of the Standard Model. If one allows for New Physics contributions in $B^0 - \overline{B^0}$ mixing a new phase, $2\theta_d$, is introduced. As a result, the measured parameter is $\sin(2\beta + 2\theta_d)$ and the new degree of freedom invalidates the argument given above. Only the measurement of a positive sign of $\cos(2\beta + 2\theta_d)$ allows a restriction of the possible solutions to $\beta + \theta_d = 21.2°$ or $\beta + \theta_d = 21.2° + 180°$. Even then the allowed space for β and θ_d is quite large and can only be reduced by adding other experimental inputs (see e.g. Ref. [7]).

3.1. $B^0/\overline{B^0} \to J/\psi K^{*0}$

The first constraints on $\cos 2\beta$ have been obtained from a time-dependent angular analysis of the final state $B^0/\overline{B^0} \to J/\psi K^{*0}$. Since this is a vector-vector mode there are three amplitudes contributing to this final state: two CP-even amplitudes, $|A_0|e^{i\delta_0}$ and $|A_{||}|e^{i\delta_{||}}$, and a CP-odd amplitude $|A_\perp|e^{i\delta_\perp}$, where the δ_i represent strong phases. The sizes and relative strong phases of these amplitudes can be measured from the angular distributions (usually described in the transversity basis) of the decay products of the J/ψ and K^{*0}. The measured time-dependent decay-rate asymmetry in the three-dimensional phase space of the transversity angles is sensitive to $\cos 2\beta$ due to the interference between the

CP-even amplitudes A_0 and $A_{||}$, and the CP-odd amplitude $A_\perp$. However, there is an ambiguity in the solution of the strong phase differences $(\delta_{||} - \delta_0, \delta_\perp - \delta_0) \to (\delta_0 - \delta_{||}, \pi + \delta_0 - \delta_\perp)$ resulting in a sign ambiguity $\cos 2\beta \to -\cos 2\beta$ which seems to spoil the $\cos 2\beta$ extraction.

BABAR has performed such a time-dependent angular analysis on a sample of 88×10^6 $B\bar{B}$ pairs [8] and finds $\sin 2\beta = +0.10 \pm 0.57_{stat} \pm 0.14_{sys}$ and $\cos 2\beta = +3.32 \pm^{+0.76}_{-0.96 stat} \pm 0.27_{sys}$. When fixing $\sin 2\beta = 0.731$ as their best measured value, BABAR finds $\cos 2\beta = +2.72 \pm^{+0.50}_{-0.79 stat} \pm 0.27_{sys}$. The sign ambiguity in $\cos 2\beta$ has been resolved in BABAR's analysis by taking advantage of the interference in the $K\pi$ system between the P-wave coming from the K^{*0} decay and the underlying S-wave.

Belle has also performed a time-dependent angular analysis on a sample of 275×10^6 $B\bar{B}$ pairs [9] and finds $\sin 2\beta = +0.24 \pm 0.31_{stat} \pm 0.05_{sys}$ and $\cos 2\beta = +0.56 \pm 0.79_{stat} \pm 0.11_{sys}$. When fixing $\sin 2\beta = 0.726$, $\cos 2\beta = +0.87 \pm 0.74_{stat} \pm 0.12_{sys}$ is found. The sign ambiguity in $\cos 2\beta$ has not been resolved. Instead, Belle uses the theoretical argument of s-quark helicity conservation [10] to select the solution for the strong phase difference. This solution is consistent with BABAR's experimental result on the strong phase difference and leads to the positive sign for $\cos 2\beta$.

3.2. $B^0/\bar{B}^0 \to D^{(*)0}/\bar{D}^{(*)0}h^0$

It has been proposed recently [11] that the sign of $\cos 2\beta$ can also be extracted from a time-dependent Dalitz plot analysis of the decay $B^0/\bar{B}^0 \to D^{(*)0}/\bar{D}^{(*)0}h^0$ where h^0 denotes a light neutral meson. These color-suppressed decay topologies are mediated by tree diagrams. If one neglects doubly-Cabibbo suppressed diagrams, the leading relevant quark transitions are $\bar{b} \to \bar{c}u\bar{d}$ for $B^0 \to \bar{D}^{(*)0}h^0$, and $b \to c\bar{u}d$ for $\bar{B}^0 \to D^{(*)0}h^0$, respectively. Interference between these amplitudes is obtained by reconstructing the neutral D mesons in the common final state $D^{(*)0}/\bar{D}^{(*)0} \to K_S^0\pi^+\pi^-(\pi)$. We assume no CP violation in the $D^{(*)0}/\bar{D}^{(*)0}$ system and denote $f_{+-} = f(m^2_{K_S^0\pi^+}, m^2_{K_S^0\pi^-})$ for the $\bar{D}^{(*)0}$ and $f_{-+} = f(m^2_{K_S^0\pi^-}, m^2_{K_S^0\pi^+})$ for the $D^{(*)0}$

decay amplitudes, respectively. Then, the time-dependent B decay amplitudes are given by:

$$
\begin{aligned}
M_{\bar{B}^0}(t) &= f_{-+}\cos\left(\Delta m_d \cdot t/2\right) \\
&\quad - ie^{-2i\beta}\eta_{h^0}(-1)^\ell f_{+-}\sin\left(\Delta m_d \cdot t/2\right) \\
M_{B^0}(t) &= f_{+-}\cos\left(\Delta m_d \cdot t/2\right) \\
&\quad - ie^{+2i\beta}\eta_{h^0}(-1)^\ell f_{-+}\sin\left(\Delta m_d \cdot t/2\right),
\end{aligned}
$$

where also direct CP violation as well as CP violation in mixing in the neutral B system has been neglected. Here, η_{h^0} is the CP eigenvalue of h^0 and ℓ is the relative orbital angular momentum in the $D^{(*)}h^0$ system. Due to the interference between f_{+-} and f_{-+} over the Dalitz plane, the time-dependent decay-rate asymmetry allows us to extract simultaneously $\sin 2\beta$ and $\cos 2\beta$ once the model for the Dalitz plot amplitudes is fixed.

A time-dependent Dalitz plot analysis on a sample of 386 million $B\bar{B}$ pairs has been first performed by Belle [12] by reconstructing $B^0/\bar{B}^0 \to D^0/\bar{D}^0h^0$ with $h^0 = \pi^0, \eta, \omega$ and $B^0/\bar{B}^0 \to D^{*0}/\bar{D}^{*0}h^0$ with $D^{*0}/\bar{D}^{*0} \to D^0/\bar{D}^0\pi^0$ and $h^0 = \pi^0, \eta$. The measured Dalitz plot is shown in Fig. 2. A simultaneous fit results in $\sin 2\beta = +0.78 \pm 0.44_{stat} \pm 0.22_{sys+Dalitz}$ and $\cos 2\beta = +1.87^{+0.44}_{-0.53}(stat)^{+0.22}_{-0.32}(sys + Dalitz)$.

In a similar BABAR analysis on a sample of 311 million $B\bar{B}$ pairs, the decays $B^0/\bar{B}^0 \to D^0/\bar{D}^0h^0$ with $h^0 = \pi^0, \eta, \eta', \omega$ and $B^0/\bar{B}^0 \to D^{*0}/\bar{D}^{*0}h^0$ with $D^{*0}/\bar{D}^{*0} \to D^0/\bar{D}^0\pi^0$ and $h^0 = \pi^0, \eta$ have been reconstructed [13]. BABAR finds $\sin 2\beta = +0.45 \pm 0.36_{stat} \pm 0.05_{sys} \pm 0.07_{Dalitz}$ and $\cos 2\beta = +0.54 \pm 0.54_{stat} \pm 0.08_{sys} \pm 0.18_{Dalitz}$. In this fit also $|\lambda|$ has been determined and found to be consistent with 1 (0.98 ± 0.09). This is as expected due to the smallness of the neglected doubly-Cabibbo suppressed tree-amplitudes carrying a different CKM phase with respect to the dominant tree amplitudes. In principle, the neglected amplitude could be taken into account in the analysis but this is likely to be impractical since the ratio of amplitudes $\frac{|A(\bar{B}^0 \to \bar{D}^0h^0)|}{|A(\bar{B}^0 \to D^0h^0)|} \propto |\frac{V_{ub}V_{cd}^*}{V_{cb}V_{ud}^*}|$ is expected to be of order 0.02. With higher statistics the precision on $\cos 2\beta$ might be dominated by the systematic uncertainty on the Dalitz plot model which is already the largest systematic error.
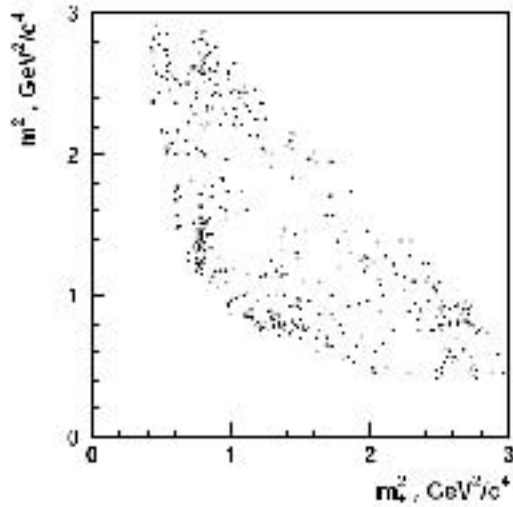
Figure 2. Dalitz plot distribution $m_-^2 = m_{K_S^0 \pi^-}^2$ versus $m_+^2 = m_{K_S^0 \pi^+}^2$ as measured in the Belle $B^0/\overline{B}^0 \to D^0/\bar{D}^0 h^0$ analysis.

3.3. $B^0/\overline{B}^0 \to D^{*+}D^{*-}K_S^0$

Another possible way to extract $\sin 2\beta$ and $\cos 2\beta$ simultaneously has been proposed in Ref. [14] and has then been studied in more detail in Ref. [15]. The method uses the final state $B^0/\overline{B}^0 \to D^{*+}D^{*-}K_S^0$. The time-dependent CP asymmetry is given by:

$$A_{CP}(t; \sin 2\beta, \cos 2\beta) = \eta_y \frac{J_c}{J_0} \cos(\Delta m \cdot t)$$

$$-(\frac{2J_{s1}}{J_0} \sin 2\beta + \eta_y \frac{2J_{s2}}{J_0} \cos 2\beta) \sin(\Delta m \cdot t).$$

The J_i are integrals of functions of the amplitudes $A = A(B^0 \to D^{*+}D^{*-}K_S^0)$ and $\bar{A} = A(\overline{B}^0 \to D^{*+}D^{*-}K_S^0)$ over the half Dalitz space in the variables $s^+ = m_{D^{*+}K_S^0}^2$ and $s^- = m_{D^{*-}K_S^0}^2$:

$$J_{0(c)} = \int_{s^+ < s^-} (|A|^2 + (-)|\bar{A}|^2) ds,$$

$$J_{s1} = \int_{s^+ < s^-} \Re(\bar{A}A^*) ds, \quad J_{s2} = \int_{s^+ < s^-} \Im(\bar{A}A^*) ds.$$

The parameter $\cos 2\beta$ can be measured if $J_{s2} \neq 0$. This could be realized if a broad intermediate resonance contributed. In this case, one expects that J_c becomes large.

The BABAR collaboration has performed such an analysis [16]. In the invariant $m_{D^{*\pm}K_S^0}$ spectrum there is a 4.6 σ evidence for the $D_{s1}^*(2536)$ narrow width resonance. In addition, there is evidence for a broad structure below 2.9 GeV/c^2 (Fig. 3). The results of the time-dependent anal-
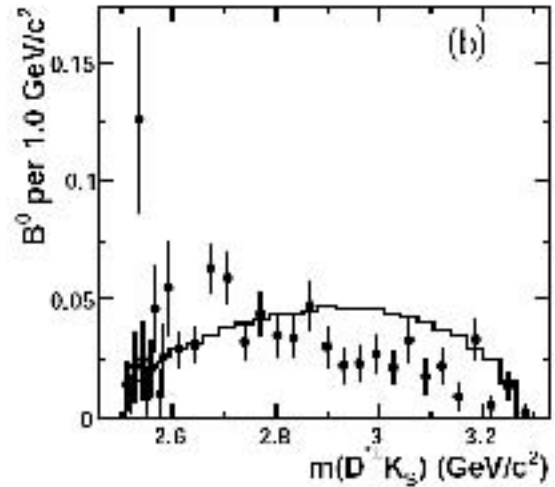

Figure 3. The $m_{D^{*\pm}K_S^0}$ distribution as measured in the BABAR $B^0/\overline{B}^0 \to D^{*+}D^{*-}K_S^0$ analysis.

ysis are $J_c/J_0 = 0.76 \pm 0.18_{stat} \pm 0.07_{sys}$, being consistent with the presence of a broad resonance in the $D^{*\pm}K_S^0$ system, $2J_{s1}/J_0 \sin 2\beta = +0.10 \pm 0.24_{stat} \pm 0.06_{sys}$, and $2J_{s2}/J_0 \cos 2\beta = 0.38 \pm 0.24_{stat} \pm 0.05_{sys}$. In Ref. [15], it has been predicted that the sign of J_{s2} is positive which would lead to a positive sign for $\cos 2\beta$.

An interesting question is the nature of the broad structure in the $m_{D^{*\pm}K_S^0}$ spectrum. Besides the $D_{s1}^*(2536)$ resonance a second $J^P = 1^+$ state is expected. A possible candidate for this state is the $D_{SJ}(2460)$ which however lies below

Table 1
Confidence level to exclude $\cos 2\beta_0 < 0$.

Mode	BABAR	Belle
$B \to J/\psi K^*$	86%	Not quantified
$B \to D^{(*)0}/\bar{D}^{(*)0}h^0$	87%	98.3%
$B \to D^{*+}D^{*-}K_S^0$	94%	Not measured

the $D^{*\pm}K_S^0$ threshold. In this case, the nature of this possible broad resonance is unclear.

3.4. Summary of $\cos 2\beta$ determinations

The analyses reviewed here have not extracted a confidence level as a function of β. Instead, the $B \to J/\psi K^*$ BABAR analysis [8] has calculated a p-value for $\cos 2\beta_0 < 0$, where β_0 corresponds to the value obtained from the the precise $\sin 2\beta$ measurement. For $\cos 2\beta_0 < 0$ the p-value is small (0.6%). However, since the BABAR-measured $\cos 2\beta$ value lies outside the physical region, also the p-value for $\cos 2\beta_0 > 0$ is small (5.7%). For this reason, also the likelihood ratio $h_-/(h_+ + h_-)$ has been considered where $h_\pm$ is the probability density function value for the measured $\cos 2\beta$ value if the true value is $\pm \cos 2\beta_0$. This likelihood ratio has then been interpreted in the framework of Bayesian statistics by assuming equal a-priori probabilities for the two hypotheses, $\cos 2\beta_0 < 0$ and $\cos 2\beta_0 > 0$, and denoted a confidence level. The other analyses discussed in this review followed this procedure and the confidence levels obtained are summarized in Table 1.

An average of all $\cos 2\beta$ measurements is beyond the scope of this review. In some cases the results have central values outside the physical region and large asymmetric uncertainties which makes an average cumbersome. The interpretation of the $B^0/\bar{B}^0 \to D^{*+}D^{*-}K_S^0$ result relies on a theoretical assumption which requires further investigation, including the understanding of the Dalitz plot structure. The systematic uncertainty due to the Dalitz plot model in $B^0/\bar{B}^0 \to D^{(*)0}/\bar{D}^{(*)0}h^0$ should be reduced since more statistics will come in the future. This channel provides a good opportunity to check whether $\sin 2\beta$ is measured to be the same to a good approximation in $b \to c\bar{c}s$ and $b \to c\bar{u}d$ transitions.

Besides the above quantitative difficulties, the qualitative results suggest that the prefered solution is $\beta + \theta_d = 21.2°$ or $\beta + \theta_d = 21.2° + 180°$, in agreement with the Standard Model. This finding removes half of the possible range in the New Physics parameter $2\theta_d$.

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