Supercritical Stability, Transitions and (Pseudo)tachyons

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Highly supercritical strings ($c \gg 15$) with a time-like linear dilaton provide a large class of solutions to string theory, in which closed string tachyon condensation is under control (and follows the worldsheet renormalization group flow). In this note we analyze the late-time stability of such backgrounds, including transitions between them. The large friction introduced by the rolling dilaton and the rapid decrease of the string coupling suppress the back-reaction of naive instabilities. In particular, although the graviton, dilaton, and other light fields have negative effective mass squared in the linear dilaton background, the decaying string coupling ensures that their condensation does not cause large back-reaction. Similarly, the copious particles produced in transitions between highly supercritical theories do not back-react significantly on the solution. We discuss these features also in a somewhat more general class of time-dependent backgrounds with stable late-time asymptotics.

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1. Introduction

It is of interest to understand cosmological solutions of string theory. Most solutions of general relativity coupled to quantum field theory evolve non-trivially with time, leading to a weakly coupled description only (at best) at asymptotically late times (or only at early times). This general class of backgrounds includes Calabi-Yau manifolds with flux[1], curved target spaces, and many others.

For a variety of applications, one might like to understand the physics of the perturbation spectrum about such backgrounds in the easily controlled weakly coupled regime. This is important for a stability analysis, as well as for calculating the particle content and density perturbations in cosmological solutions. Moreover, in perturbative string theory, the infrared perturbation spectrum is related by modular invariance to microphysical information, i.e. to the high-energy density of single-string states. More generally, one would like to understand the transitions connecting different tractable limits, which in some cases may arise as closed string tachyon condensation processes.

Perhaps the simplest background with a weakly coupled future asymptotic region is the time-like linear dilaton solution of supercritical superstring theory $[1\text{-}11]^2$, formulated in $d = 10 + 2Q^2$ dimensions ($Q > 0$). In the string frame this background is flat. In the Einstein frame its metric is

$$ds_E^2 = -dt^2 + \frac{4Q^2}{(d-2)^2}t^2 d\vec{x}^2, \tag{1.1}$$

with the string coupling

$$g_s(t) = \left(\frac{d-2}{2Qt}\right)^{(d-2)/2}. \tag{1.2}$$

We will refer to this as the supercritical linear dilaton (SCLD) phase. As shown in early works, it constitutes an exact classical solution to string theory, including all α' corrections. These backgrounds are strongly coupled at early times, so in order to completely define them we need to "cap off" the strong coupling region. One approach to this is to imagine tunneling into the SCLD phase from some meta-stable background (perhaps during eternal inflation) such as the background considered in [14] (and generalizations), though this does not address the physics arbitrarily far back in time. The details of this capping will not be important for most of our considerations.

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[1] Except when these lead to an anti-de Sitter solution.

[2] For more recent discussions, see [12,13] and references therein.

In this note, we analyze the stability and late-time perturbation spectrum of these theories and of closely related solutions describing transitions between them. Some of our considerations apply more generally as we will indicate as we go along. We begin in §2 by reviewing the properties of SCLD backgrounds and the fluctuations around them, and the fact that in the large Q limit, the tachyon condensation process in SCLD backgrounds follows the renormalization group flow on the worldsheet.

The analysis of the SCLD spectrum leads to what we call *pseudotachyons*. These are mode solutions which do not oscillate in time, and which naively cause an IR divergence in the perturbative string partition function at one-loop. In §3 we discuss these modes, and we argue that they do not actually cause an instability of the SCLD backgrounds, since their condensation does not cause a large back-reaction (a condition we will quantify).

This phenomenon appears much more generally in weakly coupled asymptotic regimes of general relativity and string theory; a prototypical example is the perturbations produced during inflationary expansion periods. Modular invariance relates this IR divergence to the supercritical effective central charge arising for $d > 10$. Since there are many similar backgrounds with pseudotachyonic perturbations, an interesting corollary is that any consistent perturbative string background with pseudotachyons is effectively supercritical, even if extra dimensions are not specified explicitly. An interesting class of examples of this is compact negatively curved target spaces, where the requisite effective central charge arises from winding modes [15-17].

In §4 we discuss the particle production during tachyon condensation processes in SCLD backgrounds. We argue that even though there is large particle production (whose details depend on the initial conditions for the SCLD phase), the produced particles do not back-react strongly on the SCLD background, so they do not change the classical fact that the tachyon condensation process follows the worldsheet RG flow. We end in §5 with a summary of our results and some future directions.

While this paper was nearing completion, we received the interesting work [18], which has some overlap with the present note, particularly in regards to the mostly harmless effects of pseudotachyons, and which shows that a tachyon condensation process in SCLDs for which the tachyon depends on a light-like coordinate follows the RG flow even for small Q.

2. Supercritical linear dilaton theories and the relation of tachyon condensation to renormalization group flow

The simplest non-trivial time-dependent background of string theory is a linear dilaton background in which the dilaton is linear in the time coordinate,

$$\Phi = -QX^0, \qquad g_s = e^{-QX^0}, \tag{2.1}$$

with $Q > 0$ and with a flat string frame metric. We arbitrarily choose the string coupling to decrease rather than to increase, leading to a weakly coupled future asymptotic region in the space-time.

As discussed above, systems such as this with weak coupling only in the future or past asymptotia are generic, even in situations with a lower scale of supersymmetry breaking (the background (2.1) breaks supersymmetry at the string scale). For example, although type II string theory on a Calabi-Yau manifold admits an exactly static solution, and adding 3-form flux breaks the supersymmetry well below the KK scale of the geometry at large radius, the flux changes the behavior of the system in the far past to be strongly coupled and infinitely far from this static solution. With sufficiently generic sources included in the system, the system may sit in a metastable minimum of the effective potential immediately before tunneling into the weakly coupled future asymptotic phase. This is a reasonably natural way to "cap off" the strong coupling regime, but still does not fully account for the behavior arbitrarily far back into the past. Another possibility is a transition from nothing, by tunneling or tachyon dynamics.

The linear dilaton CFT has a central charge $c_{LD} = 1 - 3Q^2$ (we use conventions in which $\alpha' = 1/2$). Together with any (decoupled) unitary matter CFT with central charge $c_{mat} = c_{crit} - c_{LD}$ (where for superstrings $c_{crit} = 15$) it gives an exact time-dependent background of classical string theory (of course, in the past the coupling constant becomes strong so we need to embed this into some complete quantum theory). We will call these theories supercritical linear dilaton theories (SCLDs)[3].

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[3] One can think about these theories as theories with a large central charge coupled to worldsheet gravity, where the worldsheet gravity gives rise to a time-like Liouville field which we denote by X^0. However, in this interpretation [19,20] one generally obtains also a worldsheet Liouville potential for X^0, while we will assume that no such potential is present, so our theory is a fine-tuned deformation of the usual coupling of a supercritical theory to gravity.

The linear dilaton significantly changes the behavior of deformations of the worldsheet theory. The dimension of the operator $e^{\kappa X^0}$ is given by

$$\dim(e^{\kappa X^0}) = \frac{1}{4}\kappa(\kappa + 2Q), \tag{2.2}$$

so a deformation of the worldsheet action by

$$\int d^2z \sum_i \mu_i \mathcal{O}_i e^{\kappa_i X^0}, \tag{2.3}$$

where the operator $\mathcal{O}_i$ has dimension Δ_i, is marginal when

$$\Delta_i + \frac{1}{4}\kappa_i(\kappa_i + 2Q) = 2, \tag{2.4}$$

or

$$\kappa_i = -Q \pm \sqrt{Q^2 - 4(\Delta_i - 2)}. \tag{2.5}$$

The behavior of the deformation as a function of time depends on Δ_i. For relevant deformations of the "matter CFT", with $\Delta_i < 2$ (or $m^2 + \vec{k}^2 < 0$ where m is the space-time mass of the corresponding field and $\vec{k}$ is its spatial momentum in any flat non-compact directions), one of the solutions for κ is negative and one is positive, leading to a mode which grows with time. This is similar to the usual case of tachyons in string theory. For $2 < \Delta_i < 2 + Q^2/4$ (or $0 < m^2 + \vec{k}^2 < Q^2$), both solutions for κ in (2.5) are real and negative. In this case the deformation decreases with time, so it is not a tachyon; nevertheless such modes are unstable in some senses that we will discuss below, so we will call them "pseudotachyons". Finally, for $\Delta_i > 2 + Q^2/4$ the solutions for κ are complex, and have a damped oscillatory behavior at late times.

If we start from a matter theory with no relevant operators, the background is stable, as no perturbations grow with time. In this background, modes are either oscillatory for all time, or non-oscillatory for all time, and are not created or destroyed by the time evolution (1.1), (1.2). The population of these modes depends on the initial conditions coming from the strong coupling regime in the far past. Tunneling from a metastable de Sitter minimum, for example, would imbue the system with a scale invariant spectrum of fluctuations.

Various examples of such stable SCLD backgrounds can be constructed. For instance, if we are in a superstring theory in which the matter theory is a free theory of $9 + 16n$ superfields (in addition to the fermionic partner of X^0), corresponding to superstring

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theory in $d = 10 + 16n$ dimensions, one can take a type II GSO projection and obtain a modular-invariant partition function with no tachyons. Obviously, the matter theory in such constructions does not have to be a free theory (it could be, for example, a sigma model on a Calabi-Yau manifold), so there is an infinite number of examples of such stable theories (with various values of n). Presumably, there are also many other possibilities for tachyon-free GSO projections (such as heterotic theories).

If our matter theory has a relevant operator $\mathcal{O}_i$, corresponding to a tachyonic field in space-time, the theory is unstable toward condensing this field, and it is interesting to ask what is the end-point of this closed string tachyon condensation process, which is described (at leading order in the deformation) by adding (2.3) (with the positive solution for κ_i) to the worldsheet action. In general this question is very complicated. However, it turns out that in the large Q (or large c_{mat}) limit the answer is simple – the tachyon condensation process precisely follows the renormalization group (RG) flow in the matter CFT[4] corresponding to the deformation by the operator $\mathcal{O}_i$, with the RG scale proportional to $e^{-2X^0/Q}$. In the context of strings propagating on flat space (or nearly flat space) this can be seen in two (equivalent) ways [4-7] (see also [8,9]). One way is by analyzing the beta function equations of the deformed worldsheet action and noting that they are equivalent to the matter RG equations to leading order in $1/Q$. Alternatively, one can analyze the space-time equations of motion, and note that the equation of motion of the tachyon T (as of all other NS-NS fields) contains a damping term coming from the linear dilaton

$$\frac{d^2T}{(dX^0)^2} + 2Q\frac{dT}{dX^0} = -\frac{\partial V}{\partial T}. \tag{2.6}$$

In the large Q limit there is a solution to this equation in which the first term on the left-hand side is negligible. This solution has the tachyon slowly rolling down its potential, and this is identified with the RG flow of the matter theory. A more general argument (valid also when the matter CFT is far from free) for this relation between the tachyon condensation process and the RG flow is that in the large Q limit, the interactions induced by (2.3) between the matter CFT and the X^0 CFT are of order $\kappa_i \sim 2(2 - \Delta_i)/Q$. Thus,

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[4] Note that both the tachyon condensation process and the renormalization group flow are reversible in principle; however, in general, reversing these processes requires significant fine-tuning in order to go back exactly to the maximum of the tachyon potential or to the UV fixed point of the RG flow.

at leading order in $1/Q$ there are no interactions between the matter CFT and the X^0 CFT, and the corrections to this picture scale as $1/Q$ [5].

At first sight this picture does not make sense, since the RG flow in the matter CFT leads to a smaller final central charge $c_{mat,f} < c_{mat,i}$, while the full string theory must remain critical also at late times after the tachyon has condensed (we are assuming for simplicity that there is only a single tachyon, whose condensation leads to a stable string theory). However, this change in the central charge is compensated by a change in the linear dilaton slope from Q_i to Q_f, whose magnitude in the large Q limit is

$$3Q_i^2 - 3Q_f^2 = c_{mat,i} - c_{mat,f} \qquad \rightarrow \qquad Q_f = Q_i - \frac{c_{mat,i} - c_{mat,f}}{6Q_i} + \cdots, \qquad (2.7)$$

which is of order $1/Q$ (compared to the difference between the initial and final matter theories, which is of order one in the large Q limit)[6]. Thus, it can be generated by the interactions between the two sectors, as discussed in a set of heterotic examples in [23].

In the large Q limit, the tachyon grows slowly: $T \sim \mu e^{X^0(2-\Delta_T)/Q}$. This means that in this limit, the string coupling decreases dramatically during a tachyon condensation transition. When the tachyon increases from some initial value T_i to T_f, the string coupling changes by :

$$\frac{g_{s,f}}{g_{s,i}} \sim \left(\frac{T_i}{T_f}\right)^{Q^2/(2-\Delta_T)}. \qquad (2.8)$$

Note that this dramatic decrease in the string coupling during the SCLD phase makes it difficult to obtain a transition to a realistic FRW cosmology from a SCLD phase (even if the tachyon condensation eventually leads to a critical background).

The relation described above between tachyon condensation and RG flow is very general, but there is one important caveat; one must make sure that the tachyon associated with the identity operator in the "matter CFT" is not turned on, so this operator must be projected out by some GSO-type projection both in the early-time and in the late-time

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[5] Note that this argument only applies to flows by relevant operators, not by marginally relevant operators. Note also that another way to obtain small values of κ_i is by choosing Δ_i to be very close to two. This also leads to a tachyon condensation process which begins by following the RG flow, even when Q is not large (see the recent discussions in [21,22]), but generally during the flow Δ_i would deviate from two and the tachyon condensation would deviate from the RG flow.

[6] From the space-time point of view, the change in the linear dilaton slope ensures that there is still a flat space solution in the string frame even after the tachyon potential V decreases from a maximum to a minimum.

theory. When this "ultimate tachyon" is turned on, it seems to lead to the end of space-time [24], instead of the flow we discussed which ends with the IR matter CFT coupled to a time-like linear dilaton. Except for this caveat, the relation is completely general[7], and it seems that many RG flows (between unitary theories with arbitrarily large central charges) can be embedded in string theory in this way; the only requirement is that one should be able to add to the matter theory we are interested in some additional matter fields in a way which allows for appropriate GSO projections that remove the "ultimate tachyon". For example, one can discuss flows generated by mass terms for scalars (or by sine-Liouville type interactions), which reduce the number of dimensions of space (as do RG flows in linear sigma models). In some cases (such as $\mathcal{N} = (4, 4)$ linear sigma models) an RG flow can lead to a sum of decoupled CFTs [25,26]; in such cases the string theory at late times lives on a sum of disconnected spaces (as in [27]) which could have different dimensions.

The discussion above provides a large class of examples of closed string tachyon condensation processes over which we have complete control in classical string theory (assuming that we understand the RG flow in the "matter CFT", that the corresponding operator is in the NS-NS sector, and that the string coupling is already weak when the tachyon starts condensing). So, one can use these theories to ask various questions about closed string tachyon condensation, such as how is the reduction in the high-energy density of states implemented, which state does the vacuum of the initial theory evolve into, how many particles are produced, etc. Of course, the answers to these questions are not necessarily the same in the large Q limit as in the $Q = 0$ case (where the tachyon condensation is definitely not the same as RG flow), but still it is nice to have controllable theories in which these questions can be answered. When one tries to ask questions about quantum effects in the SCLD theories, one immediately encounters a problem – the one-loop (torus) amplitude in these theories diverges. In the next section we will discuss the interpretation of this divergence, and how we believe it can be resolved. In section 4 we will discuss the particle production during the large Q closed string tachyon condensation process, and verify that it does not change the classical picture of the time evolution described above.

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[7]   At least for supersymmetric worldsheet theories, in which one does not need to worry about the energy difference between the vacuum energies of the initial and final matter theories.

3. Pseudotachyons

As mentioned above, the supercritical linear dilaton theory has a spectrum of small perturbations including modes with $m^2 + \vec{k}^2 < Q^2$ which do not oscillate. The deformation of the worldsheet corresponding to such a mode η decays exponentially, but in space-time, the canonically normalized field $\tilde{\eta}$ behaves as

$$\tilde{\eta} \sim \frac{\eta}{g_s} \sim e^{QX^0} \eta \sim e^{\pm QX^0 \sqrt{1 - (m^2 + \vec{k}^2)/Q^2}} \tag{3.1}$$

which can grow exponentially. From this point of view, these modes might seem like tachyonic instabilities. However, even though these modes grow exponentially, their back-reaction on the geometry, and their self-interactions, are negligible because of the decreasing string coupling $g_s \sim e^{-QX^0}$. From the worldsheet point of view, as discussed in §2, these modes involve operators in the matter CFT which are not relevant and do not decrease the effective central charge.

In this section, we will characterize more generally this phenomenon of *pseudotachyons* and its relation to the partition function and the effective central charge. We begin in §3.1 with a review of the one-loop divergence of the SCLD partition function, and a general discussion of tachyonic instabilities in time-dependent backgrounds. In §3.2 we discuss why in some cases (like SCLD backgrounds) these instabilities are not deadly, and how to make sense of such backgrounds.

3.1. 1-loop IR divergences in SCLDs and other backgrounds

We begin by computing the one-loop partition functions of SCLDs and noting that the modes with $m^2 + \vec{k}^2 < Q^2$ lead to an IR divergence, as follows. In order to discuss the one-loop partition function we first have to address the divergence caused by the negative kinetic term for the time-like field X^0. Staying in Lorentzian space-time signature, this may perhaps be accomplished (at least in the field theory limit) by inserting a convergence factor $e^{-\epsilon \int (X^0)^2}$; it would be interesting to analyze this in detail. Another method which is available in some backgrounds (including the SCLD background) is to perform a smooth Euclidean continuation which renders the path integral manifestly convergent (up to IR effects that are accessible in quantum field theory). Since our worldsheet Lagrangian includes a coupling of the form $QX^0 R^{(2)}$, where $R^{(2)}$ is the worldsheet curvature, it may be natural to Wick rotate also Q when we Wick rotate X^0; however, this does not make

sense since it would change the value of the central charge (the total central charge would no longer vanish), so we will leave Q real.

In a critical limit of string theory in d non-compact space-time directions, the one-loop partition function per unit volume can be expanded in the limit of a large imaginary part τ_2 for the modular parameter τ of the torus, and it takes the form

$$i \int^\infty \frac{d\tau_2}{2\tau_2} (2\pi^2 \tau_2)^{-d/2} \sum_i e^{-\pi m_i^2 \tau_2/2} \tag{3.2}$$

in terms of the masses m_i of the string states. This diverges exponentially when there are tachyons with $m_i^2 < 0$, but not otherwise.

In the SCLD theory the exponent is shifted by $\pi Q^2 \tau_2/2$. There are several ways to see this. From the point of view of computing the partition function on the worldsheet, the presence of the linear dilaton term has no effect on the torus, because we can choose the worldsheet curvature to vanish everywhere. Thus, the result is similar to that of the critical string theory, but with the matter theory central charge bigger by $c_{mat} - c_{crit} + 1$. Since the matter contribution to the integrand of (3.2) (from a specific state with a given worldsheet dimension) depends on the central charge as $e^{\pi c \tau_2/6}$, the integrand is multiplied compared to the critical case by $e^{\pi(c_{mat} - c_{crit} + 1)\tau_2/6} = e^{\pi Q^2 \tau_2/2}$.

Another way to see the same result is from the fact that the worldsheet dimension of an operator with a specific frequency is changed (2.2) by $\kappa Q/2$ from its original dimension. Since the worldsheet path integral is a sum over $e^{-4\pi\tau_2 \Delta_i}$, the usual integral giving the contribution of a particle with mass m,

$$\int \frac{d^d k}{(2\pi)^d} e^{-\pi\tau_2(k^2 + m^2)/2}, \tag{3.3}$$

which gives rise to the integrand of (3.2), is replaced, using (2.2), by

$$\int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} \frac{d\omega}{2\pi} e^{-\pi\tau_2(\vec{k}^2 + m^2 + \omega(\omega + 2Q))/2} = \int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} \frac{d\tilde{\omega}}{2\pi} e^{-\pi\tau_2(\vec{k}^2 + m^2 + \tilde{\omega}^2 - Q^2)/2} \tag{3.4}$$

in which all the masses are shifted by $m^2 \to m^2 - Q^2$, giving the same result as before.

This means that any string state whose mass squared is smaller than Q^2 (recall we are using units of $\alpha' = 1/2$) behaves like a tachyon in the sense of giving a divergence in the one-loop partition function. Note that the states in the range $0 \le m^2 < Q^2$ are precisely the states which we called *pseudotachyons* in the previous section; their zero momentum modes evolve as real exponentials of the time variable, rather than having an oscillatory

behavior, and when canonically normalized they can grow exponentially in time. The IR divergence they produce is related by modular invariance to the supercritical density of states, as discussed in general in [28], but as we will discuss further below they do not produce a significant instability.

Before moving to this interpretation, let us examine this effect in greater generality. One-loop divergences of this type will occur whenever we have fields whose effective mass squared (when they are canonically normalized) becomes negative at late times, since this leads to an exponential growth in these fields at late times. Since this is an infrared issue, we can describe the phenomenon in terms of low energy effective quantum field theory and general relativity. For simplicity we discuss the case of massless scalar fields (adding mass and/or spin is straightforward). The one-loop amplitude in general is of the form

$$\int d^d x \sqrt{-g} \Lambda(x) = \text{Tr } \log(\mathcal{H}) = \int \frac{d\tau_2}{\tau_2} \text{Tr } e^{-\pi\tau_2 \mathcal{H}/2} \tag{3.5}$$

where $\mathcal{H} = \nabla^2$ is the worldline Hamiltonian (equivalently the space-time Laplacian). If in a complete basis of normalizable functions, the trace in (3.5) diverges in the infrared, one has a tachyon or a pseudotachyon.

The Feynman propagator contributing to perturbative amplitudes satisfies

$$\nabla^2 D = \delta(x - x')/\sqrt{-G}. \tag{3.6}$$

Given a complete set of eigenfunctions of the Laplacian $\{\Psi_n(x)\}$, satisfying

$$\nabla^2 \Psi_n = \Delta_n \Psi_n, \quad \sum_n \Psi_n^*(x)\Psi_n(x') = \delta(x - x')/\sqrt{-G}, \tag{3.7}$$

we can construct the Feynman propagator as

$$D(x, x') = \sum_n \frac{\Psi_n^*(x)\Psi_n(x')}{\Delta_n + i\epsilon}. \tag{3.8}$$

If Δ_n is negative for any n, then this produces an IR divergence in (3.5) indicating the presence of a pseudotachyon or tachyon.

Let us examine this effect in a family of flat $n + 1$ dimensional FRW cosmologies

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \tag{3.9}$$

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with $H \equiv \dot{a}/a$ as usual. Each Fourier mode with spatial momentum $\vec{k}$ of a massless scalar field η in this background has an action

$$S_\eta \sim \int dt a(t)^n ((\dot{\eta})^2 - \frac{\vec{k}^2}{a(t)^2}\eta^2) = \int dt (\dot{\tilde{\eta}}^2 - m_{eff}^2(t)\tilde{\eta}^2), \tag{3.10}$$

with $\tilde{\eta}$ the canonically normalized field and

$$m_{eff}^2(t) = -\frac{n}{2}(\dot{H} + \frac{n}{2}H^2) + \frac{\vec{k}^2}{a(t)^2}. \tag{3.11}$$

In the inflationary limit of nearly constant H, there is a manifest pseudotachyonic insta-bility with the nearly constant negative mass squared from the Hubble friction dominating over the gradient energy at late times, leading to pseudotachyonic density perturbations. Generically, this effective mass squared is time-dependent, but whenever it becomes neg-ative at late times we will get an exponential growth of the canonically normalized field and IR divergences.

To investigate this further, let us consider a single-component homogeneous source, leading to a power law scale factor $a(t) \sim t^\beta$,

$$ds^2 = -dt^2 + t^{2\beta}d\vec{x}^2. \tag{3.12}$$

In this case,

$$m_{eff}^2 = \frac{w}{t^2(1+w)^2} + \frac{\vec{k}^2}{t^{2\beta}}, \tag{3.13}$$

where $w \equiv (2 - n\beta)/(n\beta)$ is the ratio of pressure to energy density in the source.

For $\beta < 1$, the gradient energy beats the Hubble friction contribution at late times, and one might suspect from this that no IR divergence results for these modes. This is indeed correct, as can be seen as follows. The two-point function satisfies (3.6). This is proportional to

$$D(x, x') = \int d\vec{k} d\omega \omega \frac{U_\omega^*(t)U_\omega(t')e^{i\vec{x}\cdot\vec{k}}}{\omega^2 + \vec{k}^2}, \tag{3.14}$$

where

$$U_\omega(t) \sim t^{-\beta n/2}t^{1/2}J_{(1-n\beta)/(2(1-\beta))}(\omega t^{1-\beta}/(1-\beta)). \tag{3.15}$$

This expansion is in a complete set of modes since

$$\int_0^\infty d\omega \omega J_\nu(\omega t^{1-\beta})J_\nu(\omega t'^{1-\beta}) = \delta(t^{1-\beta} - t'^{1-\beta})/t^{1-\beta}. \tag{3.16}$$

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Thus, for $\beta < 1$, there is a complete set of modes for which the denominator in the expression (3.8) is never negative, so we do not see the kind of divergence present in (pseudo-)tachyonic theories. The case $\beta = 1$ arises in the SCLD phase analyzed above, giving pseudotachyonic modes as discussed there. Note that within this family of solutions (3.12), we find no pseudotachyons for a decelerating scale factor, but for $\ddot{a} \geq 0$ the Hubble friction competes with the gradient energy and pseudotachyons appear.

Having discussed the issue in some generality, let us make one final remark. In cases where it occurs in a weakly coupled string theory, an IR divergence in the partition function is related by a modular transformation to a supercritical effective central charge, as explained for example in [28][8]. In the SCLD case of interest here, $c_{eff} = 3Q^2 + c_{crit}$ is part of the construction. However, pseudotachyons also appear in the infrared in many other late-time cosmological solutions. This leads to a large class of interesting backgrounds where the supercritical density of states arises in novel ways, as in compact negatively curved spaces where it arises from winding strings supported by the fundamental group [15,16].

3.2. Pseudotachyons and their resolution

At first sight the divergence we found above in the one-loop partition function implies that the theory is ill-defined, just like any other theory containing tachyons, and that we are (in some sense) expanding around an unstable vacuum of the theory. However, we find that this is not correct as a general statement: many backgrounds, such as the SCLD theory, are not significantly affected by the condensation of the pseudotachyonic modes. The real problem with tachyons is not the fact that they grow exponentially with time, but that their back-reaction on the original configuration grows exponentially with time, so that including this back-reaction moves us far from our original starting point (which is why it does not make sense to expand around this original starting point). This is not the case for the pseudotachyons in the SCLD background. Since all the interactions of the pseudotachyons with other fields come with factors of $g_s = e^{-QX^0}$ (when they are canonically normalized), their back-reaction on other fields (which is proportional to g_s times the pseudotachyon field) actually decays exponentially with time, unlike that of standard tachyons (with $m^2 < 0$). Thus, we claim that even though the one-loop

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[8] As we will discuss in the next subsection, this IR divergence does not necessarily imply a catastrophic instability on equal footing with that in the bosonic string theory.

partition function of SCLDs diverges because there is an instability toward producing exponentially-growing (canonically normalized) pseudotachyon fields, this instability does not take us away from our original SCLD configuration, so this configuration is actually stable. A similar claim can be made about other cosmological backgrounds of the type described above, as long as all the coupling constants decay fast enough. As is the case for perturbations of light fields in the inflationary universe, we expect that the expectation values $\langle \tilde{\eta}^2 \rangle$ of the squares of the canonically normalized pseudotachyon fields will grow with time, as in a random walk process, and these fields will be in a highly squeezed state. In the standard inflationary case the perturbations eventually decohere, but it seems that in the SCLD background this will not happen because of the exponentially decaying interactions; however, this difference between these two examples of pseudotachyons will not be important for our discussion here.

Note that zero momentum massless string modes, or moduli, do affect the background at late times so it is important to keep track of them (if they exist) in SCLD backgrounds (as in standard string backgrounds). These modes would shift by a finite amount during the closed string tachyon condensation process discussed in the previous section. Even when a potential for these modes is generated at string loops, the friction and the exponential decay in the potential generally prevent them from going to the minimum of this potential, so these moduli really are parameters of the SCLD background even though it is not supersymmetric. To see this, consider the equation of motion for such a rolling scalar field ϕ in the presence of a 1-loop tadpole V_1 (taken to be a constant for simplicity), which is of the form

$$\frac{d^2\phi}{(dX^0)^2} + 2Q\frac{d\phi}{dX^0} = -e^{-2QX^0}V_1. \tag{3.17}$$

The solution takes the form

$$\phi = \phi_0 + \phi_1 e^{-2QX^0} + e^{-2QX^0}\frac{V_1 X^0}{2Q}. \tag{3.18}$$

This shows that the field at late times reaches a constant, which can take different values ϕ_0.

Although it is interesting to monitor the evolution of the moduli during the pseudotachyon condensation process, it is also sometimes useful to consider a weaker notion of stability, namely to ask whether the pseudotachyon mode changes the effective central charge of the background. This measure of stability does not depend on where the system ends up on its effective moduli space. It simply measures whether the process lifts enough

worldsheet degrees of freedom to modify the leading Hagedorn density of states. Tachyons built from relevant operators in the worldsheet matter CFT do change this quantity; c_{eff} decreases in their condensation process. In space-time this corresponds to a process in which masses are increased by the tachyon condensation [29,30,24]. As we have seen here, modes built from marginal or irrelevant operators in the matter CFT on the worldsheet do not change c_{eff}, at least in the SCLD case.

Therefore, we claim that SCLDs are consistent stable string backgrounds, even when the loop corrections are taken into account. The divergence in the one-loop partition function arises because there is an instability toward creating the exponentially growing pseudotachyon fields, and in our naive partition function above we got the divergence by summing over all possible values of these fields (since larger values lead to a lower action). However, in the actual physical situation that we are interested in, these fields will be in some initial state which depends on the initial conditions (in any case, in order to really be able to sensibly discuss loop corrections, we need to assume that the strong coupling region at early times has been capped and replaced by some other background). This initial state could be a classical state which would sit at some distance away from the origin of field space and then exponentially grow from there, or a quantum state which could be a superposition of such states. However, for a given initial state we should not sum over all possible values of the pseudotachyon fields, so we would not encounter the IR divergence. Since the back-reaction of the pseudotachyons is small, the details of the state they are in do not affect the fact that the future will be well-described by the SCLD background (2.1). They do, however, affect various measurements performed in this background, like the one-point or two-point functions of the pseudotachyon fields. Thus, we cannot predict the results of such measurements without knowing the initial state. Nevertheless, this does not affect the stability of the SCLD background.

One specific initial state which is particularly simple is the one defined by analytically continuing $Q \to iQ$ (and then continuing back to obtain the final answer). As discussed above, it is not clear how to implement this directly in string theory, but (since the pseudo-tachyon divergence comes from the IR) we can implement it at the level of the low-energy effective action. For modes which are not pseudotachyonic, with $\vec{k}^2 + m^2 > Q^2$, the one-loop partition function may be written as a sum of the zero-point energies $\sqrt{\vec{k}^2 + m^2 - Q^2}$. For the pseudotachyonic modes, we can use the same expression, defined by the analytic continuation to be purely imaginary and equal to $i\sqrt{Q^2 - m^2 - \vec{k}^2}$. Essentially, we are taking the contribution of every mode corresponding to a negative harmonic oscillator to be the

(imaginary) frequency of this oscillator; this is related to the instability of this oscillator towards generating an exponential growth of the field at this rate. Using this prescription, we find that the contribution of a field of mass m to the one-loop partition function takes the form $|m^2 - Q^2|^{d/2} \log(m^2 - Q^2)$, which agrees with the standard Coleman-Weinberg answer when $m^2 > Q^2$ but has an imaginary part (described above) otherwise. Note that this formula is for each string mode separately, so it should be multiplied by a Hagedorn density of states. As we mentioned above, it is not clear how to perform this analytic continuation directly in the worldsheet action; we believe that a similar continuation can make all higher-loop corrections to SCLD computations well-defined, but we do not know how to do this explicitly, and it would be interesting to verify this. In any case, we stress that this continuation corresponds to a specific choice of the initial state, and that most initial states will lead to different results. Presumably, the description of generic initial states on the worldsheet would be non-local [31] since they would look like squeezed states.

4. Particle production

In section 2 we described how in the large Q limit, tachyon condensation in SCLDs followed the RG flow on the worldsheet. The arguments for this were based on a classical analysis, which ignored the particles which are produced quantum-mechanically in time-dependent backgrounds. It is interesting to ask how many particles are produced during the tachyon condensation process, and whether these particles change the claim that the end-point of the process is another SCLD theory with smaller Q or not.

As described in section 2, the SCLD background and the tachyon condensation have a simple description on the worldsheet. However, it seems difficult to analyze the particle production directly on the worldsheet, which requires computing 2-point (and higher) correlation functions in the time-dependent CFT (including the tachyon operator (2.3)) of section 2. Instead, we will estimate the particle production by computations done in the space-time effective field theory. A priori one would not expect to be able to trust such an effective field theory in a background in which the dilaton changes at a very fast pace (compared to the string scale). However, the linear dilaton is an exact classical solution. It introduces friction on other modes such as the tachyon, and the light modes change slowly compared to the string scale. We consider the physics at sufficiently late times that the coupling is very weak, and self-consistently work to linearized order in all other fields.

As we discussed above, only particle modes with $m^2 + \vec{k}^2 > Q^2$ oscillate in the SCLD background, so they can be given the standard particle interpretation and we can ask about their production[9]. Let us begin with a simple toy model – the particle production of such a mode Φ (with mass m and momentum $\vec{k}$) when it is coupled to the tachyon T by a $\frac{1}{2g_s^2}\lambda T^2\Phi^2$ potential in the string-frame space-time effective action. In the presence of the condensing tachyon, going as $T = \mu e^{\kappa X^0}$, we obtain an equation of motion

$$\frac{d^2\Phi}{(dX^0)^2} + 2Q\frac{d\Phi}{dX^0} + (m^2 + \vec{k}^2 + \lambda\mu^2 e^{2\kappa X^0})\Phi = 0. \tag{4.1}$$

It is easy to find the general solution to this equation, which is a generalization of the solution for $Q = 0$ discussed in [32,29,24]. It may be written as a sum of two Bessel functions in the form

$$\Phi = \alpha e^{-QX^0} J_{\frac{\sqrt{Q^2-m^2-\vec{k}^2}}{\kappa}}\left(\frac{\sqrt{\lambda}\mu}{\kappa}e^{\kappa X^0}\right) + \beta e^{-QX^0} J_{-\frac{\sqrt{Q^2-m^2-\vec{k}^2}}{\kappa}}\left(\frac{\sqrt{\lambda}\mu}{\kappa}e^{\kappa X^0}\right). \tag{4.2}$$

When $m^2 + \vec{k}^2 > Q^2$, the phase of these solutions oscillates both at very early and at very late times, and a standard computation of the Bogolubov coefficients gives the average number of created particles, which is $\exp(-\pi\sqrt{m^2 + \vec{k}^2 - Q^2}/\kappa)$. For large Q, when κ is small (of order $1/Q$), and when the momentum or the mass of Φ are large enough, this particle production is very small. Note that in the large Q SCLD vacua (at weak coupling), most of these produced particles would not decay (despite the exponentially growing phase space factor which is available to them due to their exponentially growing mass) because of the rapid exponential decrease in the coupling constant.

If we consider the case of several particles with more complicated couplings to the tachyon which mix them together, we can no longer exactly solve the equation of motion as above. However, in the limit in which all particles have $m^2 + \vec{k}^2 - Q^2 \gg \kappa^2$, the time evolution is very slow (assuming polynomial couplings to the tachyon), and we can use the adiabatic argument (as presented in [32]) to prove that the particle production rate is smaller than any power of $\kappa^2/(m^2 + \vec{k}^2 - Q^2)$, suggesting that it is still exponentially small as in the explicit computation above. This argument can be used both for the production of particles whose mass eventually goes to infinity, as in (4.1) (this is the case for most

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[9] In this section we ignore the fact that Q changes with time, since this is small in the large Q limit as discussed in §2.

particles when the central charge of the matter theory decreases), and for the production of particles whose mass remains finite.

For large Q, as discussed in the previous section, many of the string modes (with low momentum) are actually pseudotachyons which do not propagate (their wavefunctions do not oscillate). During the tachyon condensation, such pseudotachyons (or linear combinations of pseudotachyons and regular particles) can develop exponentially growing masses as above, and start propagating; this is somewhat similar to what happens to modes which start outside the horizon in an expanding decelerating FRW universe[10]. In this case there is no exponential suppression of the particle production of the type discussed above.

The precise details of the particle production in this case depend strongly on the initial state of the pseudotachyons; if we compute it by analytical continuation in Q we find a particle production of order one. More generally, we can estimate the energy density contained in these modes as follows. During the SCLD phase, while their effective mass is still negative, these fields roll down their effective potential $-m_{eff}^2 \tilde{\eta}^2$ (3.11) by a distance in field space $\tilde{\eta}_0$ depending on their initial value at the beginning of the SCLD phase, and on the time spent in the SCLD phase before the tachyon turns over their mass squared. Once the tachyon dominates, these modes roll back toward the origin and oscillate about the minimum, with an energy density of order $\lambda \mu^2 e^{2\kappa X^0} \tilde{\eta}_0^2$ for each mode. This energy density grows exponentially with time, and on top of it there is a Hagedorn density of such modes, that is a density of states of mass m of order $e^{\pi m Q \sqrt{2}}$, for $m_s \ll m \leq Q$.

Despite this large density of states, the energy density is finite and we can begin with a sufficiently small string coupling such that the process is under control throughout. The back-reaction of these modes is down by a factor $g_s^2 \sim g_{s,0}^2 e^{-2QX^0}$, which guarantees that their effect is negligible (except when we directly ask questions about these particles).

To summarize, in the large Q limit of the tachyon condensation process we expect to have a large production of particles which start out as pseudotachyons with $m^2 + \vec{k}^2 < Q^2$ but become propagating degrees of freedom, both for the particles which become infinitely massive and for the particles which remain of finite mass. Particles which begin with a larger mass or momentum are produced only in small numbers. This large particle production creates an exponentially large density of particles, and the mass of most of

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[10]   Modes could also become propagating even without interacting with the tachyon, just because of the decrease in the value of Q. However, this effect can be neglected in the large Q limit.

these particles grows as a power of $e^{\kappa X^0}$ [11]. However, since the string coupling decays as e^{-QX^0}, the back-reaction of this large energy density decays very fast with time (note that the total energy density diverges in the large Q limit, but it is finite for a given large value of Q). Thus, we believe that the large quantum creation of particles during the tachyon condensation process does not change the end-point of this process which we described in section 2.

5. Summary and future directions

In this note we discussed SCLD backgrounds of string theories and related backgrounds. We showed that despite the naive instability of these backgrounds, related to the divergence of the one-loop partition function, these backgrounds are actually well-defined (assuming that they start from an appropriate initial state). We reviewed the fact that in the large Q limit of SCLD backgrounds, the closed string tachyon condensation process on the worldsheet follows the RG flow of the worldsheet CFT, and we argued that this is not changed by quantum effects despite the fact that these effects lead to a very large particle production in space-time.

It would be interesting to analyze further the possible initial "capping" states for SCLD backgrounds. One could imagine smooth cappings involving either some strong coupling completion of string theory (if this exists for supercritical strings), or a smooth evolution from "nothing" as in [24]. Alternatively, the capping could involve a tunneling from some meta-stable background of string theory (or perhaps from nothing). Given a specific capping, one can compute the initial state of the SCLD phase, and use this to predict the details of the final state of this phase.

Our stability analysis in this paper was purely perturbative, and it would be interesting to analyze whether SCLD backgrounds are stable also with respect to tunneling into other backgrounds or not. Naively it seems that they should be, since the rapidly decreasing coupling constant should suppress all tunneling amplitudes. If these backgrounds are completely stable, then they provide an infinite class of possible non-supersymmetric end-points for evolution in the landscape (which in some cases are labeled by moduli). It would be interesting to try to estimate the probability of an eternal inflation process

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[11] Note that this growth is in units of the string scale; the mass decreases very fast in Planck units.

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ending up at such a background, as compared to the probability of a more conventional string background.

The SCLD backgrounds provide controlled examples of time-dependent backgrounds, and it is interesting to ask if they could have any cosmological applications. One interesting point that was noted already in [3] is that the SCLD backgrounds can solve the horizon and flatness problems, so they might provide an alternative to inflation. However, as mentioned above, the rapid decrease in the string coupling in the SCLD backgrounds seems to prevent a smooth transition from such backgrounds to our current universe.

In this paper we focused on discussing the tachyon condensation processes in the limit of large Q, in which one can rigorously argue that the tachyon condensation process follows the RG flow on the worldsheet. One may conjecture that also for smaller values of Q, when the tachyon condensation process deviates from the RG flow, it could still end up at the same end-point as the RG flow, as long as Q_f in (2.7) is positive (or maybe even vanishing). However, for small Q our arguments suggest that the quantum effects are very important, so it does not seem possible to check this conjecture just by using classical string theory. It would be interesting to investigate what can be said about tachyon condensation processes when Q is not very large, and in particular when the final Q vanishes. This is necessary in order to check the conjectures made in [33,23].

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