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Spin Echo in Synchrotrons*

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Abstract

As a polarized beam is accelerated through a depolarization resonance, its polarization is reduced by a well-defined calculable reduction factor. When the beam subsequently crosses a second resonance, the final beam polarization is considered to be reduced by the product of the two reduction factors corresponding to the two crossings, each calculated independently of the other. This is a good approximation when the spread of spin precession frequency $\Delta\nu_{\rm spin}$ of the beam (particularly due to its energy spread) is sufficiently large that the spin precession phases of individual particles smear out completely during the time τ between the two crossings. This approximate picture, however, ignores two spin dynamics effects: an interference effect and a spin echo effect. This paper is to address these two effects.

The interference effect occurs when $\Delta\nu_{\rm spin}$ is too small, or when τ is too short, to complete the smearing process. In this case, the two resonance crossings interfere with each other, and the final polarization exhibits constructive or destructive patterns depending on the exact value of τ. Typically, the beam's energy spread is large and this interference effect does not occur. To study this effect, therefore, it is necessary to reduce the beam energy spread and to consider two resonance crossings very close to each other.

The other mechanism, also due to the interplay between two resonance crossings, is spin echo. It turns out that even when the precession phases appear to be completely smeared between the two crossings, there will still be a sudden and short-lived echo signal of beam polarization at a time τ after the second crossing; the magnitude of which can be as large as 57%. This echo signal exists even when the beam has a sizable energy spread and when τ is very large, and could be a sensitive (albeit challenging) way to experimentally test the intricate spin dynamics in a synchrotron.

After giving an analysis of the interference and the echo effects, two possible experiments to explore them are suggested.

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Spin Echo and Interference in Synchrotrons

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1 Introduction

In the study of nuclear magnetic resonance effects, spin echo is a well-known phenomenon [1]. A related phenomenon is expected to occur in accelerators. In a planar synchrotron, the spin of a particle precesses rapidly around the vertical y-axis with the spin tune $\nu_{\rm spin} = G\gamma$, where $G = (g-2)/2$ with g the gyromagnetic ratio of the particle under consideration, and γ is the Lorentz energy factor. As the particle is accelerated or decelerated, its γ changes and spin tune changes accordingly. As the spin tune varies, the spin motion of a particle will be strongly affected if the particle experiences perturbing electromagnetic fields as it executes orbital motion in the synchrotron, and if its spin tune comes close to, or crosses a depolarization resonance

$$G\gamma = \kappa \tag{1}$$

where κ specifies the resonance location (for example, κ = integer for imperfection resonances, κ = integer $\pm$ vertical betatron tune for intrinsic resonances, etc.). In this situation, it is well-known that the perturbation on spin motion can be characterized by a single complex quantity, the resonance strength ϵ, which can be expressed as an appropriate Fourier harmonic of the perturbing fields around the accelerator [2]-[8].

One such analysis was obtained by Froissart and Stora [8] when the spin tune crosses the resonance linearly in time starting and ending far from the resonance. Their result yielded the well-known Froissart-Stora formula that relates the final polarization after crossing to the initial polarization before crossing.

A matrix formalism [9] has recently been developed that allows the calculation of polarization near a resonance when the crossing pattern of the spin tune consists of a combination of constant in time, linear in time, and sudden discrete jumps. The condition of being far from the resonance before and after crossing is also removed. This matrix formalism can be used, for example, to study multiple crossings of a resonance. Using this formalism, we are able to explore constructive and destructive interference effects of these crossings. Condition for destructive interference between two crossings, for example, might be useful if one wishes to compensate a particularly strong depolarization resonance by another artificially induced resonance.

Experimentally, these interference effects are expected to be most readily observable when the polarized beam has a small spread $\Delta\nu_{\rm spin}$ in particles' spin tunes. To meet this requirement, the beam must have a small energy spread. Ways to produce a beam with small energy spread should help greatly the exploration of the interference effects. For a polarized beam of larger energy spreads, the multiple resonance crossings are far separated from each other and the interference effects are not readily observable. Fortunately, it turns out in this case that detection of interference can still be attempted using a more subtle effect, namely the echo effect, as suggested in [9]. In this echo experiment, two resonance crossings normally considered to be far separated can still interfere with each other to produce a spin echo signal at an unexpected long time after the second crossing.

The spin echo effect is not dissimilar to the orbital echo effects observed in storage rings [10]. By examining the orbital and the spin echoes, detailed and intricate orbital dynamics or spin dynamics can be studied. Since the spin echoes are expected to last for very long times, they are expected to be sensitive to weak and slow perturbations of spin diffusion, and their study can lead to quantitative examination of these spin diffusion mechanisms.

The first part of this paper gives an analysis of the interference and the echo effects. After this analysis, two possible experiments to study them will be suggested. It is hoped that some variation of these suggested experiments can be carried out in a synchrotron in a not too distant future, for example as part of the experimental efforts in [11, 12].

2 Equation of Motion

We assume the spin dynamics is determined by one and only one depolarization resonance of strength ϵ at $G\gamma = \kappa$. We assume $G\gamma$ is near the resonance and its distance to the resonance is a prescribed function of time.

In spinor notation, the spin state of a particle is described by a two-component complex spinor ψ, and the spin dynamics is described by [2]-[8]

$$\frac{d\psi}{d\theta} = -\frac{i}{2} \begin{bmatrix} -G\gamma & \epsilon e^{i\kappa\theta} \\ \epsilon^* e^{-i\kappa\theta} & G\gamma \end{bmatrix} \psi \tag{2}$$

where θ is the time variable (advancing by 2π per revolution of the particle around the synchrotron), $G\gamma$ depends on θ as

$$G\gamma(\theta) = \kappa + \alpha(\theta) \tag{3}$$

where $\alpha(\theta)$ is the way the resonance is approached or crossed. This spinor equation implicitly assumes that α and ϵ as functions of time vary slowly compared to spin precession around the vertical y-axis. This is a good assumption because spin precession is fast.

We define

$$\beta(\theta) = \int_{\theta_0}^{\theta} d\theta' \alpha(\theta') \tag{4}$$

where θ_0 is the initial time, and make the transformation

$$\psi = e^{\frac{i}{2}[\kappa\theta + \beta(\theta)]\sigma_y} \begin{bmatrix} f(\theta) \\ g(\theta)e^{i\beta(\theta)} \end{bmatrix} \tag{5}$$

with Pauli matrix $\sigma_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Equation (2) then becomes

$$\begin{aligned} \frac{df}{d\theta} &= -\frac{i\epsilon}{2} g \\ \frac{dg}{d\theta} &= -i\alpha g - \frac{i\epsilon^*}{2} f \end{aligned} \tag{6}$$

We still need to specify the initial condition of the spin at time $\theta = \theta_0$. Let us designate $\alpha_0 = \alpha(\theta_0)$ and $\epsilon_0 = \epsilon(\theta_0)$. We assume that these parameters have been held at these values, and that the spin has been in an eigenstate, from $\theta = -\infty$ to $\theta = \theta_0$, i.e.,

$$\begin{bmatrix} f \\ g \end{bmatrix}_{\theta_0} = \sqrt{\frac{\Omega + |\alpha_0|}{2\Omega}} \begin{bmatrix} 1 \\ -\frac{\text{sgn}(\alpha_0)}{\epsilon_0} (\Omega - |\alpha_0|) \end{bmatrix} \tag{7}$$

where

$$\Omega = \sqrt{\alpha_0^2 + |\epsilon_0|^2} \tag{8}$$

We will primarily be interested in P_y, the y-component of polarization, in this planar accelerator,

$$P_y(\theta) = \psi^\dagger \sigma_y \psi = |f(\theta)|^2 - |g(\theta)|^2 \tag{9}$$

Being in an eigenstate, the initial polarization is assumed to be 100% and is adiabatically brought to the initial position θ_0. At θ_0, the x- and z-components of the spin precess rapidly, and we shall concentrate on the y-component only. The initial y-component of polarization is given by

$$P_y(\theta_0) = |f(\theta_0)|^2 - |g(\theta_0)|^2 = \frac{|\alpha_0|}{\Omega} \tag{10}$$

The rapidly precessing phase of the x- and z-components of the polarization is contained in the phase of ϵ_0. Our results of P_y will not depend on the phase of ϵ_0, and will depend only on $|\epsilon_0|$.

3 Jump crossing a resonance

In crossing a resonance, the simplest case to treat is when the resonance is crossed by a sudden jump in the spin tune. In the present study, for simplicity, we shall consider jump crossings only. Slower resonance crossings with beam energy being varied linearly in time can also be treated using the matrix formalism, but are not studied below. Interference and echo effects are sufficiently illustrated by the case of sudden jump crossings. Consider first the case of a jumping pattern in $\alpha(\theta)$ as shown in Fig.1. A resonance of strength ϵ_0 is jump-crossed twice at times θ_1 and θ_2.

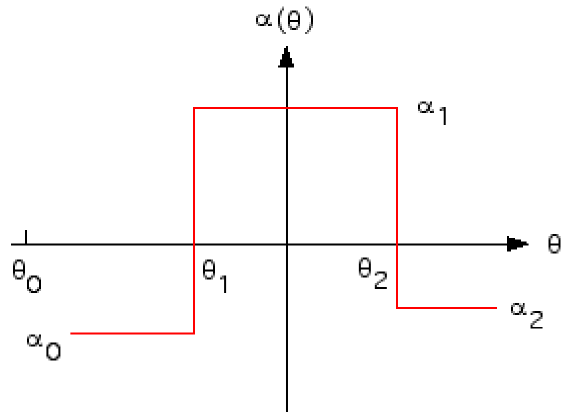

Figure 1: Two crossings of a resonance by sudden jumps of spin tune.

The initial spin state at $\theta = \theta_0$ is given by (7). Applying the matrix formalism [9], the spin state at time $\theta_1 > \theta > \theta_0$, i.e. before the first jump, is given by

$$\begin{bmatrix} f \\ g \end{bmatrix}_{\theta_1 > \theta > \theta_0} = T_{\alpha_0, \epsilon_0}(\theta, \theta_0) \begin{bmatrix} f \\ g \end{bmatrix}_{\theta_0} \tag{11}$$

where $T_{\alpha_0,\epsilon_0}(\theta,\theta_0)$ is the transfer matrix that brings the initial spin state at time θ_0 to its final state at time θ, and is defined by

$$\begin{aligned} T_{\alpha_0,\epsilon_0}(\theta,\theta_0) &\equiv e^{-\frac{i}{2}\alpha_0(\theta-\theta_0)} \begin{bmatrix} 1 & 0 \\ \frac{\alpha_0}{\epsilon_0} & \frac{i\Omega}{\epsilon_0} \end{bmatrix} \begin{bmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{i\alpha_0}{\Omega} & -\frac{i\epsilon_0}{\Omega} \end{bmatrix} \\ \Theta &= \frac{\Omega}{2}(\theta - \theta_0) \end{aligned} \tag{12}$$

Before the first resonance crossing, Eqs.(7) and (11) give

$$\begin{bmatrix} f \\ g \end{bmatrix}_{\theta_1 > \theta > \theta_0} = e^{\frac{i}{2}[\mathrm{sgn}(\alpha_0)\Omega_0 - \alpha_0](\theta-\theta_0)} \begin{bmatrix} f \\ g \end{bmatrix}_{\theta_0} \tag{13}$$

where $\Omega_0 = \sqrt{\alpha_0^2 + |\epsilon_0|^2}$. As one would expect, the spin stays in the initial eigenstate. The polarization in this time period is preserved, and is given by

Eq.(10), i.e.

$$P_y(\theta_1 > \theta > \theta_0) = \frac{|\alpha_0|}{\Omega_0} \tag{14}$$

After the first crossing and before the second crossing, the matrix formalism now gives

$$\begin{aligned}
\begin{bmatrix} f \\ \\ g \end{bmatrix}_{\theta_2>\theta>\theta_1} &= T_{\alpha_1,\epsilon_0}(\theta,\theta_1) \begin{bmatrix} f \\ \\ g \end{bmatrix}_{\theta_1} = e^{\frac{i}{2}(\theta_1-\theta_0)[\mathrm{sgn}(\alpha_0)\Omega_0-\alpha_0]}\, T_{\alpha_1,\epsilon_0}(\theta,\theta_1) \begin{bmatrix} f \\ \\ g \end{bmatrix}_{\theta_0} \\
&= e^{-\frac{i}{2}\alpha_1(\theta-\theta_1)+\frac{i}{2}[\mathrm{sgn}(\alpha_0)\Omega_0-\alpha_0](\theta_1-\theta_0)} \sqrt{\frac{\Omega_0+|\alpha_0|}{2\Omega_0}} \\
&\times \begin{bmatrix} e^{\frac{i}{2}\mathrm{sgn}(\alpha_0)\Omega_1(\theta-\theta_1)} + \frac{i}{\Omega_1}[(\Omega_0-\Omega_1)\mathrm{sgn}(\alpha_0) \\ \quad +\alpha_1-\alpha_0]\sin\frac{\Omega_1(\theta-\theta_1)}{2} \\ \\ \frac{1}{\epsilon_0\Omega_1}\Big\{[\alpha_0-\Omega_0\mathrm{sgn}(\alpha_0)]\Omega_0\cos\frac{\Omega_1(\theta-\theta_1)}{2} \\ \quad -i[|\epsilon_0|^2-\alpha_1\Omega_0\mathrm{sgn}(\alpha_0)+\alpha_1\alpha_0]\sin\frac{\Omega_1(\theta-\theta_1)}{2}\Big\} \end{bmatrix}
\end{aligned} \tag{15}$$

where $\Omega_1 = \sqrt{\alpha_1^2+|\epsilon_0|^2}$. Polarization during this time period is given by

$$P_y(\theta_2 > \theta > \theta_1) = \frac{\mathrm{sgn}(\alpha_0)}{\Omega_0\Omega_1^2}\Big[\alpha_1(\alpha_0\alpha_1+|\epsilon_0|^2)+(\alpha_0-\alpha_1)|\epsilon_0|^2\cos\Omega_1(\theta-\theta_1)\Big] \tag{16}$$

This polarization is sinusoidal in $\theta-\theta_1$ with frequency Ω_1. Its value oscillates between $\frac{|\alpha_0|}{\Omega_0}$ and $\frac{\mathrm{sgn}(\alpha_0)}{\Omega_0\Omega_1^2}[\alpha_0\alpha_1^2+|\epsilon_0|^2(2\alpha_1-\alpha_0)]$. The oscillation centers around the value $\frac{\mathrm{sgn}(\alpha_0)\alpha_1}{\Omega_0\Omega_1^2}(\alpha_0\alpha_1+|\epsilon_0|^2)$.

After the second crossing, we have

$$\begin{aligned}
\begin{bmatrix} f \\ \\ g \end{bmatrix}_{\theta>\theta_2} &= T_{\alpha_2,\epsilon_0}(\theta,\theta_2) \begin{bmatrix} f \\ \\ g \end{bmatrix}_{\theta_2} \\
&= e^{-\frac{i}{2}\{\alpha_2(\theta-\theta_2)+\alpha_1(\theta_2-\theta_1)+(\theta_1-\theta_0)[\alpha_0-\Omega_0\mathrm{sgn}(\alpha_0)]\}} \sqrt{\frac{\Omega_0+|\alpha_0|}{2\Omega_0}}
\end{aligned}$$

$$
\times \left[\begin{array}{l}
-\frac{i}{\Omega_1\Omega_2} \sin\Theta_2 \big\{ \Omega_0 \cos\Theta_1 (\alpha_0 - \Omega_0 \mathrm{sgn}(\alpha_0)) \\
\qquad -i(\alpha_0\alpha_1 + |\epsilon_0|^2 - \alpha_1 \Omega_0 \mathrm{sgn}(\alpha_0)) \sin\Theta_1 \big\} \\
+(\cos\Theta_2 + i\frac{\alpha_2}{\Omega_2} \sin\Theta_2) \big\{ \cos\Theta_1 \\
\qquad + \frac{i}{\Omega_1}(-\alpha_0 + \alpha_1 + \Omega_0 \mathrm{sgn}(\alpha_0)) \sin\Theta_1 \big\} \\
\\
\frac{\epsilon_0^*}{\Omega_1\Omega_2} \sin\Theta_2 \big\{ -ie^{\frac{i}{2}\Omega_1 \mathrm{sgn}(\alpha_0)(\theta_2-\theta_1)} \Omega_1 \\
\quad +(\alpha_1 - \alpha_0 + \Omega_0 \mathrm{sgn}(\alpha_0) - \Omega_1 \mathrm{sgn}(\alpha_0)) \sin\Theta_1 \big\} \\
+\frac{1}{\epsilon_0\Omega_1\Omega_2} (\Omega_2 \cos\Theta_2 - i\alpha_2 \sin\Theta_2) \\
\quad \big\{ \Omega_0 \cos\Theta_1 (\alpha_0 - \Omega_0 \mathrm{sgn}(\alpha_0)) \\
\qquad -i(\alpha_0\alpha_1 + |\epsilon_0|^2 - \alpha_1 \Omega_0 \mathrm{sgn}(\alpha_0)) \sin\Theta_1 \big\}
\end{array} \right] \tag{17}
$$

where $\Omega_2 = \sqrt{\alpha_2^2 + |\epsilon_0|^2}, \Theta_1 = \frac{\Omega_1}{2}(\theta_2 - \theta_1), \Theta_2 = \frac{\Omega_2}{2}(\theta - \theta_2)$.

Polarization P_y after the second crossing is given by $P_y(\theta > \theta_2) = |f|^2 - |g|^2$ with f and g given by Eq.(17). This final polarization oscillates in $\theta - \theta_2$ with frequency Ω_2.

Note that P_y oscillates with frequencies Ω_1 and Ω_2 after each of the two crossings. These are much slower frequencies than the spin precession frequency $\sim \kappa$ (by a few orders of magnitude).

A special case occurs when the two jumps have magnitudes such that

$$
\alpha_0 = -A, \quad \alpha_1 = A, \quad \alpha_2 = -A \tag{18}
$$

In this case, Eq.(17) gives a simpler expression for the polarization after the two jumps,

$$
P_y(\theta > \theta_2) \quad = \quad \frac{|A|}{\Omega^5} \Big[(A^2 - |\epsilon_0|^2)^2 + 2|\epsilon_0|^4 \cos\Omega(\theta - \theta_1) - 2A^2|\epsilon_0|^2 \cos\Omega(\theta + \theta_1 - 2\theta_2)
$$

$$+ \quad 2|\epsilon_0|^2(A^2 - |\epsilon_0|^2)\cos\Omega(\theta - \theta_2) + 4A^2|\epsilon_0|^2\cos\Omega(\theta_2 - \theta_1)\Big] \tag{19}$$

where $\Omega = \sqrt{A^2 + |\epsilon_0|^2}$.

4 Interference

From Eq.(19) for the special case, it can be observed that there is complete destructive interference between the two resonance jumps if, in addition to condition (18), we have

$$\Theta_1 = k\pi, \quad \text{or} \quad \Omega(\theta_2 - \theta_1) = 2k\pi \tag{20}$$

where k is an integer. When conditions (18) and (20) are satisfied, the final polarization is equal to the initial polarization $|\alpha_0|/\Omega_0$, and the two resonance jumps do not cause a loss of polarization. The two crossings have destructively annihilated each other.

There is also a constructive interference that occurs when

$$\Theta_1 = k\pi + \frac{\pi}{2}, \quad \text{or} \quad \Omega(\theta_2 - \theta_1) = (2k+1)\pi \tag{21}$$

In this case, the final polarization reads

$$\begin{aligned} P_y(\theta > \theta_2) &= \frac{|A|}{\Omega^5}\Big[(A^4 - 6A^2|\epsilon_0|^2 + |\epsilon_0|^4) \\ &+ 4|\epsilon_0|^2(|\epsilon_0|^2 - A^2)\cos\Omega(\theta - \theta_1)\Big] \end{aligned} \tag{22}$$

In this case of constructive interference, the highest (or lowest, as the case may be) value of the oscillating P_y after the second crossing is equal to the lowest value of P_y during the time between θ_1 and θ_2.

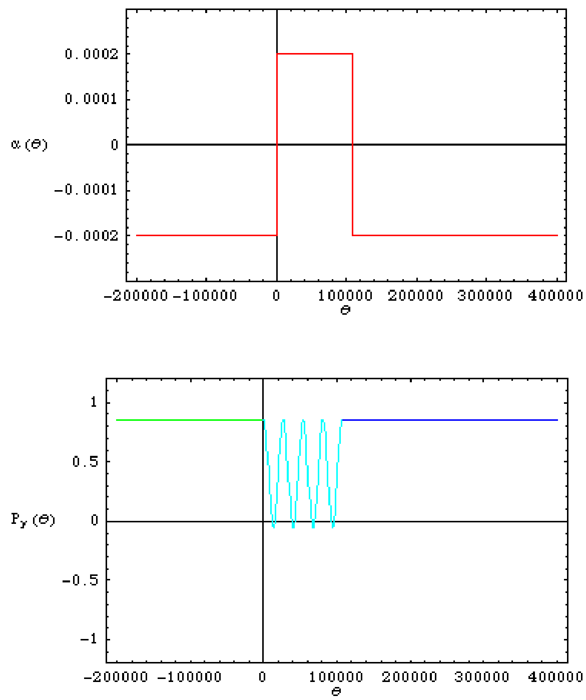

Figure 2: Upper figure shows the resonance crossing pattern $\alpha(\theta)$ of a particle as a function of θ. Lower figure shows the polarization $P_y(\theta)$ as the particle makes the resonance crossings. Parameters used are $A = 2 \times 10^{-4}, |\epsilon_0| = 1.2 \times 10^{-4}, \theta_0 = -2 \times 10^5, \theta_1 = 0, \Theta_1 = 4\pi$. The two jumps destructively interfere as the polarization makes 4 complete oscillations during the time between the two jumps. The three colors in the lower figure gives the polarization during the three time periods before first crossing, between the two crossings, and after the second crossing, respectively. The exact value of the launching time θ_0 does not matter.

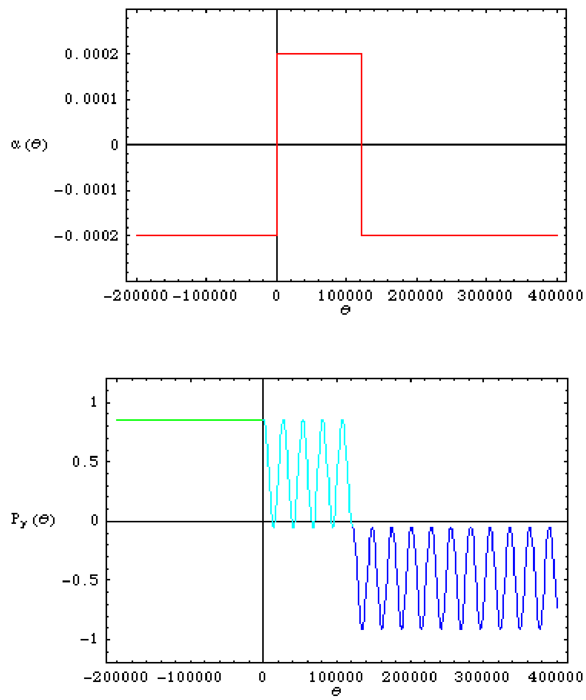

Figure 3: Same as Fig.2, except that $\Theta_1 = \frac{9\,\pi}{2}$. The two jumps constructively interfere as the polarization makes $4\frac{1}{2}$ complete oscillations during the time between the two jumps.

Examples of interferences are shown in Figs.2 (destructive interference) and 3 (constructive interference). The two figures differ in their values of Θ_1.

It is amusing to note that when the case of constructive interference has the additional condition that $|\epsilon_0| = |A|$, the polarization after the second jump will be simply a constant, as illustrated in Fig.4.

It should be emphasized that, at least in principle, after crossing a resonance, the potential of interfering strongly with a next resonance crossing lasts

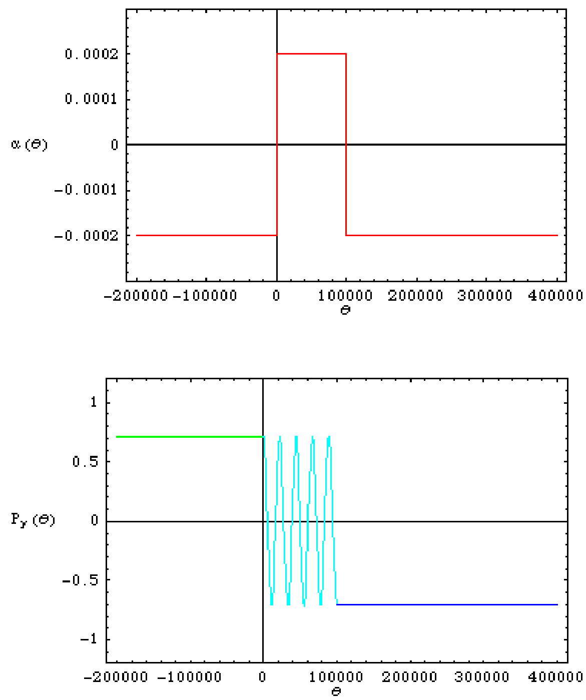

Figure 4: Same as Fig.3, except that $|\epsilon_0|$ has been changed to become equal to A so that $|\epsilon_0| = A = 2 \times 10^{-4}$. The two jumps constructively interfere as the polarization makes $4\frac{1}{2}$ complete oscillations during the time between the two jumps. The final polarization after the second crossing is constant in time.

indefinitely in time. The memory of crossing a resonance lasts indefinitely for each single particle of the beam. In this sense, resonance crossings should not be generally considered to be separate events. Having crossed a resonance will carry the memory indefinitely, and will necessarily interference with subsequent crossings of other resonances. However, this interference effect has conventionally not been taken seriously. In what follows, we will explore the conditions when ignoring the interference effects is justified.

5 Off-momentum particle

We consider a case when the on-momentum particle of the beam is made to double jump-cross the resonances according to the prescription (18). In other words, the on-momentum particle's energy as a function of time θ is such that

$$G\gamma_0(\theta) = \kappa + \begin{cases} -A, & \text{if } \theta < \theta_1 \\ A, & \text{if } \theta_1 < \theta < \theta_2 \\ -A, & \text{if } \theta_2 < \theta \end{cases} \tag{23}$$

For an off-momentum particle in the beam with energy deviation $\delta = \Delta\gamma/\gamma_0$, on the other hand, its spin tune will be given by

$$G\gamma(\theta) = \kappa + \begin{cases} -A + \kappa\delta, & \text{if } \theta < \theta_1 \\ A + \kappa\delta, & \text{if } \theta_1 < \theta < \theta_2 \\ -A + \kappa\delta, & \text{if } \theta_2 < \theta \end{cases} \tag{24}$$

We will assume that $|\delta| \ll 1, |\kappa\delta| \ll 1$ and $|\kappa\delta| \ll A$.

In the following, we assume that the first resonance crossing is a jump from below. This means $A > 0$. For the off-momentum particle, condition (18) is not fulfilled, we use Eqs.(14), (16) and (17) to obtain the polarization at various stages of the resonance jump process. In expressions (14), (16) and (17), we note that the momentum deviation makes important contributions only through the phases in the sinusoidal terms. Therefore we obtain, for an off-momentum particle,

$$P_y(\theta < \theta_1) \approx \frac{A}{\Omega}$$

$$
\begin{aligned}
P_y(\theta_2\!>\!\theta\!>\!\theta_1) &\approx \frac{A}{\Omega^3}\left\{A^2-|\epsilon_0|^2+2|\epsilon_0|^2\cos\left[(\Omega+\frac{A\kappa\delta}{\Omega})(\theta-\theta_1)\right]\right\} \\
P_y(\theta\!>\!\theta_2) &\approx \frac{A}{\Omega^5}\left\{(A^2\!-\!|\epsilon_0|^2)^2+2|\epsilon_0|^4\cos\left[\Omega(\theta\!-\!\theta_1)+\frac{A\kappa\delta}{\Omega}(2\theta_2\!-\!\theta\!-\!\theta_1)\right]\right. \\
&- 2A^2|\epsilon_0|^2\cos\left[\Omega(\theta\!+\!\theta_1\!-\!2\theta_2)\!-\!\frac{A\kappa\delta}{\Omega}(\theta\!-\!\theta_1)\right] \\
&+ 2|\epsilon_0|^2(A^2-|\epsilon_0|^2)\cos\left[(\Omega-\frac{A\kappa\delta}{\Omega})(\theta\!-\!\theta_2)\right] \\
&+ \left. 4A^2|\epsilon_0|^2\cos\left[(\Omega+\frac{A\kappa\delta}{\Omega})(\theta_2\!-\!\theta_1)\right]\right\} \qquad (25)
\end{aligned}
$$

where $\Omega = \sqrt{A^2+|\epsilon_0|^2}$, and we have used the fact that $\Omega_0 = \Omega_2 \approx \Omega - \frac{A\kappa}{\Omega}\delta$ and $\Omega_1 \approx \Omega + \frac{A\kappa}{\Omega}\delta$.

6 A beam of particles

The above result applies to the case of a single particle. For a beam of particles with a finite energy spread among the particles, an averaging on the result (25) over the beam's energy distribution will have to be performed. Assuming the energy distribution is Gaussian with rms σ_δ, the result is

$$
\begin{aligned}
P_y(\theta\!<\!\theta_1) &\approx \frac{A}{\Omega} \\
P_y(\theta_2\!>\!\theta\!>\!\theta_1) &\approx \frac{A}{\Omega^3}\left\{A^2-|\epsilon_0|^2+2|\epsilon_0|^2 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta-\theta_1)^2}\cos\Omega(\theta-\theta_1)\right\} \\
P_y(\theta\!>\!\theta_2) &\approx \frac{A}{\Omega^5}\left\{(A^2\!-\!|\epsilon_0|^2)^2+2|\epsilon_0|^4 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(2\theta_2-\theta-\theta_1)^2}\cos\Omega(\theta\!-\!\theta_1)\right. \\
&- 2A^2|\epsilon_0|^2 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta-\theta_1)^2}\cos\Omega(\theta\!+\!\theta_1\!-\!2\theta_2) \\
&+ 2|\epsilon_0|^2(A^2\!-\!|\epsilon_0|^2) e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta-\theta_2)^2}\cos\Omega(\theta\!-\!\theta_2) \\
&+ \left. 4A^2|\epsilon_0|^2 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta_2-\theta_1)^2}\cos\Omega(\theta_2\!-\!\theta_1)\right\}
\end{aligned}
$$

(26)

It may be useful at this point to discuss some features of the result (26):

- In $P_y(\theta_2 > \theta > \theta_1)$, there is a sinusoidal oscillating term with oscillation frequency Ω. This term is the shock response of the beam polarization to the first resonance jump, its phase depending on $\theta - \theta_1$.

- In $P_y(\theta > \theta_2)$, there are four oscillating terms, all with oscillation frequency Ω. Each term has its own physical meaning. The third oscillating term gives the shock response to the second resonance crossing, its phase depending on $\theta - \theta_2$. The fourth term describes an interference between the two crossings, its phase depending on $\theta_2 - \theta_1$. (This fourth term is independent of time θ, so strictly speaking, it is not an "oscillating" term.) The remaining two terms (the first and second terms) appear more mysterious. We will see later that they give rise to a spin echo effect, while the first term will dominate over the second term. These interference and the echo effects are the emphases of the present paper.

- Each of the oscillating terms in (26) contains an exponential factor corresponding to the effect of phase smearing due to the finite beam energy spread. The rate the phase information is lost is such that each of the oscillating terms is damped in N_{smear} turns, where

$$N_{\text{smear}} \approx \frac{\sqrt{2}\,\Omega}{2\pi |A| \kappa \sigma_\delta} \tag{27}$$

  Because of these exponential smearing factors, all the oscillating terms will be significant only within a time span of the order of $\Delta\theta \sim 2\pi N_{\text{smear}}$

centered around specific values of time θ. The shock terms will center around $\theta = \theta_{1,2}$, while the echo term will center around $\theta = 2\theta_2 - \theta_1$.

- To observe a significant interference effect, i.e., for the fourth oscillating term in $P_y(\theta > \theta_2)$ to be significant, it is necessary that

$$\theta_2 - \theta_1 \lesssim \frac{\sqrt{2}\,\Omega}{|A|\kappa\sigma_\delta} \tag{28}$$

The quantity on the right hand side, therefore, specifies how long the memory of crossing a resonance lasts. As discussed earlier, when $\sigma_\delta = 0$, such as for a single particle, the interference will be remembered indefinitely.

- Our analysis assumes the resonances are crossed by sudden jumps in the spin tune. In practice, spin tune is varied at a finite speed. A "sudden" jump means the crossing is made in a time short enough that the polarization has not made a significant change. This requires

$$N_{\text{jump}} \ll \frac{1}{\Omega} \tag{29}$$

where N_{jump} is the number of turns it takes to complete the jump.

- As mentioned earlier, the polarization oscillation frequency Ω is much slower than the precession frequency κ. In fact, in order for the spinor equation of motion (2) to hold, we require another condition, namely, one needs the jump not to be too fast,

$$N_{\text{jump}} \gg \frac{1}{\kappa} \tag{30}$$

and in any case the jump should be made in more than several turns. This condition, however, is easily fulfilled in practice.

- As mentioned, the interference effect is pronounced only when Eq.(28) holds, i.e. only when the two jumps are sufficiently close in time. More specifically, interference occurs most pronouncedly when

$$\frac{1}{\kappa} \ll N_{\text{jump}} \ll \left\{ \begin{array}{c} \frac{1}{\Omega} \\ \\ \\ \frac{1}{2\pi}(\theta_2 - \theta_1) \end{array} \right\} \ll \frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta} \tag{31}$$

- In comparison, the spin echo effect is pronounced only when Eq.(28) does not hold. In that case, the two shock responses and the echo signal are all clearly separated in time. To examine the echo effect, we are interested mainly in the parameters regime

$$\frac{1}{\kappa} \ll N_{\text{jump}} \ll \left\{ \begin{array}{c} \frac{1}{\Omega} \\ \\ \\ \frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta} \end{array} \right\} \ll \frac{1}{2\pi}(\theta_2 - \theta_1) \tag{32}$$

  In this regime, the interference term does not contribute, and can be dropped.

- In addition to (31) and (32), we should keep in mind that our analytic approximation (25) requires

$$\kappa\sigma_\delta \ll |A| \tag{33}$$

  although the absolute validity of this condition may not be too critical.

- In the above analysis, we have assumed that the spin tune spread comes from an energy spread of the beam particles. If there are other sources of spin tune spread, the same analysis applies as long as the spin tunes stay fixed throughout the crossing process and their changes come only from the acceleration [13]. In particular, if energy change has a contribution from synchrotron motion of the particles as in the case of a bunched-beam operation, the analysis will require modifications [14].

- The analysis assumes the same resonance strength ϵ_0 for all particles. This means it applies only to the cases of imperfection resonances or resonances driven by radio-frequency dipoles. Imperfection resonance, for example, will require more involved analysis [15].

7 Spin echo

We are now ready to calculate the echo effect for a beam with energy spread. One example is shown in Fig.5. Figure 5 (upper) reproduces the case of Fig.3 when $\sigma_\delta = 0$, as it should. Figures 5 (middle) and 5 (lower) are cases in the regime (32), and with increasing σ_δ. Each of these two figures contains three separated, peaked responses, centered around $\theta = \theta_1$ (shock response to first crossing), $\theta = \theta_1 + \tau$ (shock response to second crossing), and $\theta = \theta_1 + 2\tau$ (echo response), where $\tau = \theta_2 - \theta_1$ is the time separation between the two jumps. There is one and only one echo signal, i.e. there are no secondary echoes even if one waits for a longer time beyond the first echo.

In Eq.(32), the term $\frac{1}{\Omega}$ in the curly bracket represents the oscillatory motion of the polarization, while the term $\frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta}$ corresponds to the smear-caused damped behavior of the polarization. Depending on the relative values of $\frac{1}{\Omega}$ and

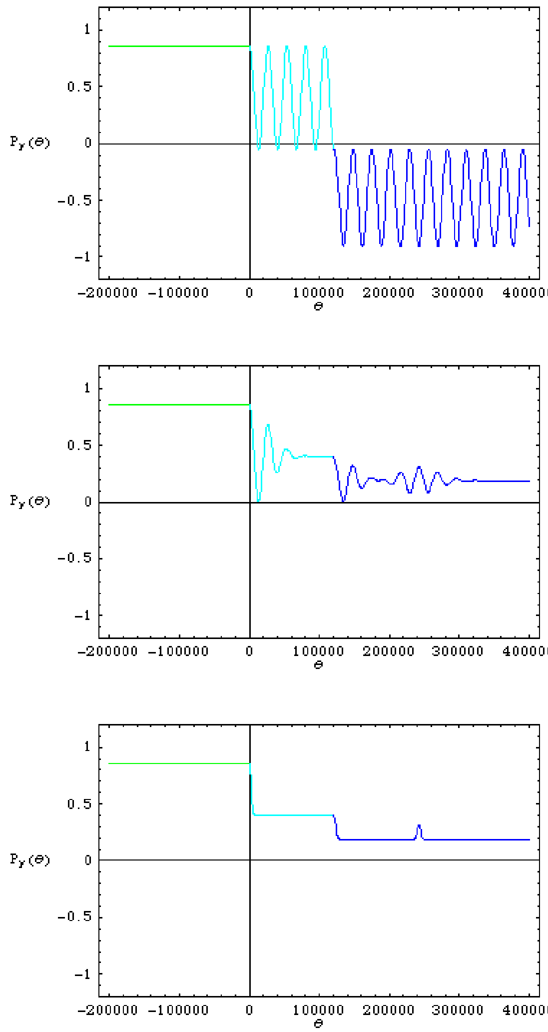

Figure 5: Conditions are the same as in Fig.3, except that this is for a beam with finite energy spread; $\sigma_\delta = 0$ (upper), $\sigma_\delta = 10^{-5}$ (middle), $\sigma_\delta = 10^{-4}$ (lower). As σ_δ increases, the interference effect is suppressed while an echo signal becomes more apparent. We have taken $\kappa = 4.4$.

$\frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta}$, the separated responses will appear damped-oscillatory or critically-damped. Figure 5 (middle) appears damped-oscillatory, while Fig.5 (lower) appears critically damped. When σ_δ is increased further from Fig.5 (lower), the polarization will look basically the same as Fig.5 (lower) except that the responses become increasingly sharply centered around $\theta_1, \theta_1 + \tau$, and $\theta_1 + 2\tau$.

After the oscillating terms are damped out, and ignoring the echo term, the level of polarization is given by

$$P_y = \begin{cases} \frac{A}{\Omega} & \text{if } \theta < \theta_1 \\ \frac{A}{\Omega}\left(\frac{A^2-|\epsilon_0|^2}{\Omega^2}\right) & \text{if } \theta_2 > \theta > \theta_1 \\ \frac{A}{\Omega}\left(\frac{A^2-|\epsilon_0|^2}{\Omega^2}\right)^2 & \text{if } \theta > \theta_2 \end{cases} \tag{34}$$

It is clear that the first jump gives a loss of polarization by a factor $\left(\frac{A^2-|\epsilon_0|^2}{\Omega^2}\right)$, while the second jump gives rise to the same loss factor. When condition (32) holds, i.e. when the beam has a sufficiently large energy spread and the two resonance crossings are sufficiently separated in time, therefore, it does seem justified to consider the two jumps as two separate independent events, and ignore any interference effects.

However, this is true only if one ignores the echo. Even when condition (32) is satisfied, one must remember that there is in addition an echo signal located at a distant time of τ beyond the second jump. The magnitude of the echo signal, relative to its background value, is

$$P_{y,\text{echo}} = \frac{2A|\epsilon_0|^4}{\Omega^5} \tag{35}$$

It follows that this echo signal is maximum when

$$|A|_{\text{max. echo}} = \frac{1}{2}|\epsilon_0| \tag{36}$$

When condition (36) is fulfilled, the echo signal is $(4/5)^{5/2} = 57\%$, a perhaps surprisingly large value. However, it should be pointed out also that condition (36) means the resonance jump is done in such a way that both the launching and the final spin tunes are within the width of the resonance.

Th echo effect demonstrated above applies when the one and only resonance is crossed twice. Whether and under what conditions two separate resonances crossed by an accelerated beam will produce an echo is yet to be studied.

The echo signal comes about because the shock response in the spin spinor produced by the first crossing contains a precessing term. When the second crossing at a time τ later produces a shock response that contains another precessing term of equal speed but opposite direction, the two terms cancel each other at a time τ after the second crossing. Although particles with different energy errors precess with different speeds, the time when cancellation occurs is exactly the same for all particles independent of their energy errors. An echo is then produced as a result.

8 Two experiments

We propose two possible experiments, one for detecting echo and the other for detecting interference, possibly using a 2.1 GeV/c proton beam of COSY [12]. The parameters chosen should be further optimized according to the experimental conditions, but the following proposals are meant to illustrate the possibilities.

In the experiments proposed below, resonances are introduced using a radio-frequency dipole [11, 12]. The strength of the resonance is controlled by the dipole strength. The resonance tune is determined by its radio frequency. The speed of resonance crossing is determined by the speed at which its radio frequency is varied.

We will suggest to cross the resonances rapidly to assimilate sudden jumps. If the speed turns out to be slower, analysis employing slower crossings will have to be carried out, which has not been done in the present paper. To complete the jump in $N_{\rm jump}$ turns, the resonance crossing speed needs to be such that

$$\frac{1}{f_c^2}\frac{df}{dt} = \frac{2A}{N_{\rm jump}} \tag{37}$$

where $\frac{df}{dt}$ is the rate at which dipole radio frequency is swept in time, and f_c is the revolution frequency of the beam in the synchrotron. For the COSY synchrotron, $f_c = 1.5$ MHz.

The beam energy spread is a key parameter for these experiments. In the proposed experiments below, a smallest experimentally achievable value of beam energy spread is assumed. This smallest value will require electron cooling to the polarized beam.

The beam is assumed to be 100% polarized initially away from the resonance. With the resonance strength turned on to the value ϵ_0, the beam is adiabatically brought to a launching position by bringing the spin tune of the beam's on-momentum particle to a distance $-A$ from the resonance $G\gamma = \kappa$. Starting from this launching position, a resonance jump is made (in $N_{\rm jump}$ turns). The on-momentum spin tune is made to be equal to $+A$ after the jump.

The beam is then parked there for a period of time τ (or $\tau/2\pi$ turns), while the resonance strength is kept at ϵ_0. At time τ after the first jump, a second resonance jump is performed, bringing the on-momentum spin tune from $+A$ back to $-A$.

The beam is then parked at this new position, while resonance strength is still kept at ϵ_0. Beam polarization P_y is then measured using a polarimeter. The measurement is gated at a short time window approximately only 0.5 ms wide. The timing of the gate is varied so that a range of polarization is mapped out as a function of time after the second jump. To take each data point, 120 spin-up and 120 spin-down cycles are accumulated to give sufficient statistics. A good statistics is expected to be a challenge in these experiments.

8.1 Echo experiment

For the echo experiment, we propose the following parameters, [16]

$$\begin{aligned}
\kappa &= 4.4 \\
\sigma_\delta &= 10^{-4} \\
|\epsilon_0| &= 10^{-3} \\
A &= 0.5 \times 10^{-3} \\
N_{\text{jump}} &< 100 \text{ turns}
\end{aligned} \tag{38}$$

These parameters give $\Omega = \sqrt{A^2 + |\epsilon_0|^2} = 1.12 \times 10^{-3}$, and they satisfy the conditions (32), (33), and (36). To make sure that resonance jumps are made in less than 100 turns, the resonance crossing speed needs to be $\frac{1}{f_c^2}\frac{df}{dt} > 10^{-5}$.

The time separation τ between the two resonance jumps should be very flexible and can be chosen by convenience. Having chosen τ, the echo signal is expected to occur at the time τ after the second resonance jump. Since the echo will last only for a time duration $\pm N_{\text{smear}} = \pm\frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta} = \pm 1150$ turns (± 0.8 ms), and will oscillate with period $\frac{1}{\Omega} = 900$ turns (0.6 ms), the detector time must be gated rather narrowly and rather accurately. Figure 6 shows the expected polarization behavior of this experiment when $\tau = \theta_2 - \theta_1 = 2\pi \times 8000$, or 8000 turns. To dramatize the echo effect, one may increase τ by a large factor, e.g. a factor of 1000.

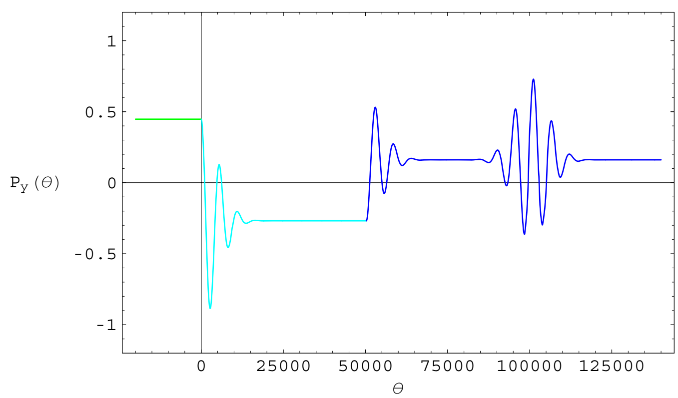

Figure 6: Expected polarization in an echo experiment.

Detecting the echo is further complicated by the counting rate statistics. In a COSY experiment, if we assume the polarization measurement accuracy of $\pm 1\%$ when gated at a 200 ms time window (assuming 30 spin-up and 30 spin-down cycles), the expected accuracy of 0.5 ms window would be $\pm 10\%$ assuming 120 spin-up and 120 spin-down cycles [16]. This $\pm 10\%$ statistics is to be compared

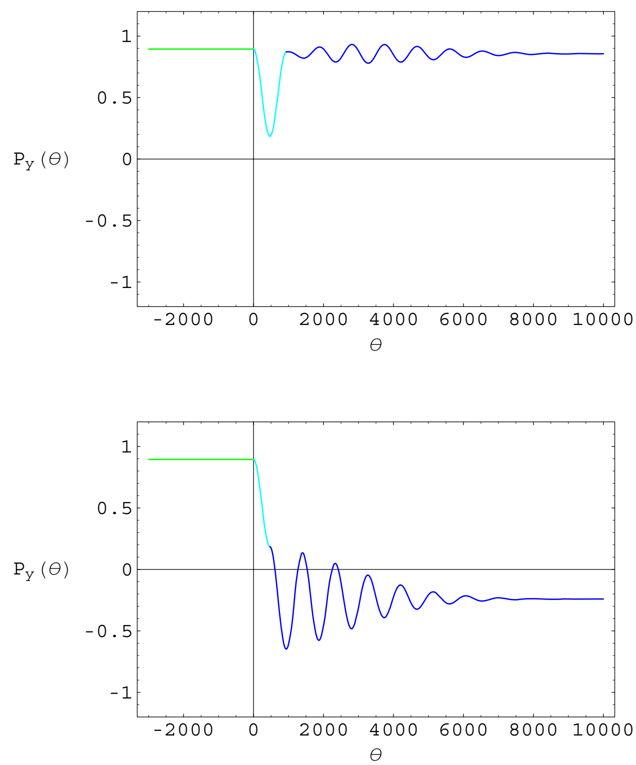

Figure 7: Expected polarization in an interference experiment. The upper figure is when the two resonance crossings interfere destructively. The lower figure is when they interfere constructively.

with the expected echo polarization signal of 57%.

Note that, as seen in Fig.6, the polarization at echo in this example is larger than the launching polarization. This is allowed because the beam is assumed to be initially 100% polarized and brought to its launching position adiabatically. If the beam turns out to be less than 100% to start with, the polarization level will need to be reduced throughout by the initial polarization.

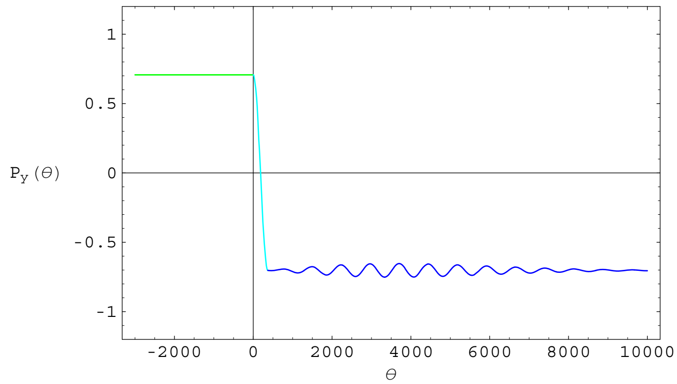

Figure 8: Expected polarization when the two resonance jumps constructively interfere, while $\epsilon_0| = |A|$.

8.2 Interference experiment

For an interference experiment, we assume the following parameters, [16]

$$
\begin{aligned}
\kappa &= 4.4 \\
\sigma_\delta &= 10^{-4} \\
|\epsilon_0| &= 3 \times 10^{-4} \\
A &= 6 \times 10^{-4} \\
\tau &= \begin{cases} \frac{2\pi}{\Omega} = 9.4 \times 10^3 = 1500 \text{ turns} \\ \qquad \text{for destructive interference} \\ \frac{\pi}{\Omega} = 4.7 \times 10^3 = 750 \text{ turns} \\ \qquad \text{for constructive interference} \end{cases} \\
N_{\text{jump}} &< 100 \text{ turns}
\end{aligned}
\tag{39}
$$

These parameters give $\Omega = \sqrt{A^2 + |\epsilon_0|^2} = 6.7 \times 10^{-4}$, and they satisfy the conditions (31) and (33).

Figure 7 shows the result expected in this experiment. The polarization after the second jump includes some contribution from the echo, but the interference effect is reflected by the fact that the final polarization long after the second jump depends sensitively on the choice of the time separation τ between the two jumps.

Figure 8 shows a special case of constructive interference when $|\epsilon_0| = |A|$. When the beam has no energy spread, we have shown the result in Fig.4. Here, we show the case when the parameters are the same as those of (39) except that $|\epsilon_0|$ has been changed to 6×10^{-4}. The shock response to the second jump has disappeared, while a small echo signal remains.

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