

Table 1: Parameters for LCLS BC1 and BC2.

	Q (nC)	ρ (m)	σ_z (μm)	λ (mm)	f (THz)
BC1	1	2.4	200	1.2	0.24
	0.2	2.2	60	0.38	0.8
BC2	1	14	20	0.12	2.4
	0.2	17	8	0.05	6.0

half opening angle $\theta_{\text{open}-1/2}(R, \lambda) = \theta_{\text{open}-1/2}(\infty, \lambda) = (\lambda/\rho)^{1/3}$ with ρ being the bending radius. Hence, the observe point is in the near field, when $R < R_f(\lambda) = \lambda/[\theta_{\text{open}-1/2}(\infty, \lambda)]^2 = \lambda^{1/3}\rho^{2/3}$. For BC1, we have typical $\lambda = 2\pi\sigma_z = 1.2$ mm and $\rho = 2.4$ m, hence, the formation length $R_f \approx 19$ cm. For BC2, $\lambda \approx 0.13$ mm and $\rho = 14.5$ m, hence, the formation length $R_f \approx 30$ cm. The second is the finite length of the bending magnet. For bending magnet with a finite length, the entrance edge and the exit edge have to be taken into consideration. This leads to the edge radiation in our following calculation.

Since in the BC1 current design, the mirror is only 20 cm downstream of the edge of the last bending magnet, the radiation is in the so-called near field regime. It is worthwhile to point out that, for the purpose of introducing the concept, the calculation in Ref. [3] did not consider the real geometry as shown in Fig. 1. The very purpose of this paper is to perform calculation for this particular setup shown in Fig. 1, even though further improvement on the calculation is needed along the development of the BL11.

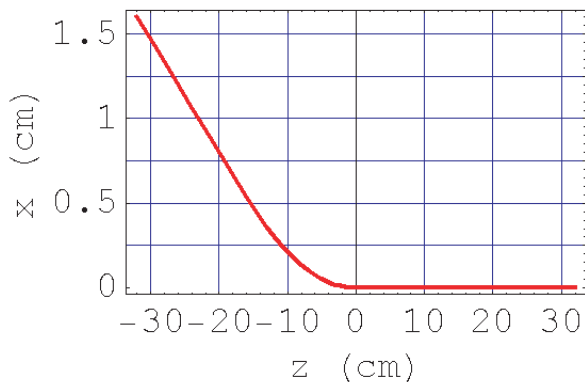


Figure 2: The simplified model of the trajectory.

Single Electron Radiation

The Fourier component of the electric field reads,

$$\mathbf{E}_\omega = \frac{e}{4\pi\epsilon_0 c} \mathbf{F}_\omega, \quad (1)$$

with

$$\mathbf{F}_\omega = i\omega \int_{-\infty}^{\infty} dt' \frac{e^{i\omega t(t')}}{R(t')} \left\{ \vec{\beta}(t') - \left[1 + \frac{ic}{\omega R(t')} \right] \mathbf{n}(t') \right\}. \quad (2)$$

In Eq. (1), e is the charge of the electron, ϵ_0 is the vacuum permittivity; c is the speed of light. In Eq. (2), $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ where $\mathbf{r}(\mathbf{r}')$ is the vector directing from the origin to the observer (electron); $\vec{\beta} = \mathbf{v}/c$ where \mathbf{v} is the velocity of the electron; and $\mathbf{n} = \mathbf{R}/R$ is the unit direction vector. We define the origin of the coordinates on the axis and at the exit-plane of the last bending magnet. Hence, the distance between the electron and the observer, R , is written as $R = [(X-x)^2 + (Y-y)^2 + (Z-z)^2]^{1/2}$ with $\mathbf{r} = (X, Y, Z)$ and $\mathbf{r}' = (x, y, z)$. Following [6], we use electron's longitudinal coordinate z as independent variable, so to write the retarded time t' as

$$t' = \int_0^z \frac{dz'}{c\beta_z} = \frac{1}{c} \int_0^z \left[1 + \frac{1}{2\gamma^2} + \frac{\beta_x^2}{2} + \frac{\beta_y^2}{2} \right] dz'. \quad (3)$$

Notice that we set the integral lower limit as $z' = 0$ for convenience. The retard condition determines the time $t = t' + R(t')/c$ in Eq. (2) as

$$t = \frac{1}{c} \left[z + \frac{z}{2\gamma^2} + \frac{1}{2} \int_0^z (\beta_x^2 + \beta_y^2) dz' + \sqrt{(X-x)^2 + (Y-y)^2 + (Z-z)^2} \right]. \quad (4)$$

Notice also that, according to Eq. (3), we have the Jacobian dt'/dz . The spectral flux density is calculated as

$$\frac{d^2 N_p}{dS d\omega/\omega} = \frac{d^2 \mathcal{E}}{\hbar dS d\omega} = \frac{\epsilon_0 c}{\hbar \pi} |\mathbf{E}_\omega|^2 = \frac{\alpha}{4\pi^2} |\mathbf{F}_\omega|^2, \quad (5)$$

where $\alpha = e^2/(\hbar c 4\pi\epsilon_0) \approx 1/137$ is the fine structure constant with $\hbar = h/(2\pi)$ and $h \approx 6.626 \times 10^{-34}$ is the Planck's constant.

Trajectory

For the setup in Fig. 1, the trajectory is simply modelled as what is in Fig. 2 and expression is given in Table 2. Notice that, $y = 0$ and $\beta_y = 0$ for the reference particle.

Table 2: Trajectory and velocity.

	$z < -z_{\text{dip}}$	$-z_{\text{dip}} < z < 0$	$z > 0$
$x(z)$	$\rho - \frac{zz_{\text{dip}} + \rho^2}{\sqrt{\rho^2 - z^2}}$	$\rho - \sqrt{\rho^2 - z^2}$	0
$\beta_x(z)$	$-\frac{z_{\text{dip}}\beta}{\rho}$	$\frac{z\beta}{\rho}$	0
$\beta_z(z)$	$\frac{\beta}{\rho} \sqrt{\rho^2 - z^2}$	$\frac{\beta}{\rho} \sqrt{\rho^2 - z^2}$	β

EXAMPLES

We work out an example for BL11 to illustrate some issues. We compute the spectral flux density for $\omega = c/\sigma_z$ in the $Y = 0$ plane.

Near Field Edge Radiation

When $R \ll \lambda\gamma^2$, with a “zero edge length” model, the edge radiation field is approximately equal to [4]

$$E(R, \theta) = \frac{e\gamma^2\theta}{\pi\epsilon_0 cR} \left[-i \exp\left(\frac{-i\pi R\theta^2}{2\lambda}\right) \right] \times \frac{\sin[\pi R\theta^2/(2\lambda)]}{1 + \gamma^2\theta^2}. \quad (6)$$

For a finite length magnet, the entrance edge and the exit edge both generate edge radiation, even though the generated field are directing in opposite direction.

Far Field Storage-Ring Synchrotron Radiation

The synchrotron radiation from a storage-ring in the far field regime reads [7]

$$E(R, \theta) = -\frac{\sqrt{3}e\gamma}{4\pi\epsilon_0 cR} \left(\frac{\omega}{\omega_c}\right) (1 + \gamma^2\theta^2) \times \left[\text{sign}(1/\rho)K_{2/3}(\xi)\mathbf{u}_\sigma - i\frac{\gamma\theta K_{1/3}(\xi)}{\sqrt{1 + \gamma^2\theta^2}}\mathbf{u}_\pi \right], \quad (7)$$

where $\omega_c = 3c\gamma^3/(2\rho)$; $r_e = e^2/(4\pi\epsilon_0 mc^2) \approx 2.82 \times 10^{-15}$ m is the classical radius of electron; and $\xi = \frac{1}{2}\frac{\omega}{\omega_c} (1 + \gamma^2\theta^2)^{3/2}$.

RESULTS AND DISCUSSION

In Fig. 3, the dots are for the numerical result for finite bending magnet; the solid curve is for the near field edge radiation including both entrance edge and the exit edge; and the dashed curve is for the storage-ring result. The conventional synchrotron far field result extends to about $X = -2$ cm, and peaks around $X = -1$ cm. It contains both the σ -mode and the π -mode contribution. The edge radiation shows the interference pattern originated from the entrance edge and the exit edge. The numerical result nicely show the contributions from the entrance edge, the body of the bending magnet, and the exit edge. Since the mirror has a hole with a diameter of 1.5 cm, part of the synchrotron radiation from the body part of the magnet leaks out.

Having the comparison in Fig. 3, we understand the contribution of what is normally called edge radiation and what is synchrotron radiation. We can then compute the coherent radiation. Assuming a detector responding to a central wavelength of $\lambda = 2.2$ mm, and a response bandwidth of $\Delta\lambda = 1$ mm, the coherent radiation within in this frequency region at the mirror is shown in Fig. 4. The dashed curve is obtained based on Eq. (6) and stands for the contribution from the edge radiation. The long-dashed curve is from Eq. (7) and stands for the storage-ring synchrotron radiation. The solid curve is the sum of them. As we expected, the edge radiation contribution dominates. The CDR from the mirror is negligible because it is small in the absolute value, and also it is not in the focal plane.

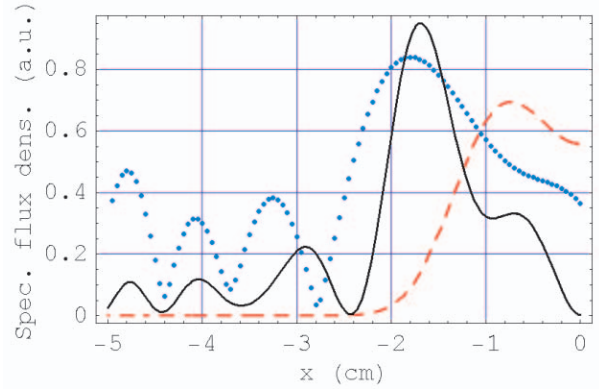


Figure 3: Comparison of the three contributions. Notations are explained in the text.

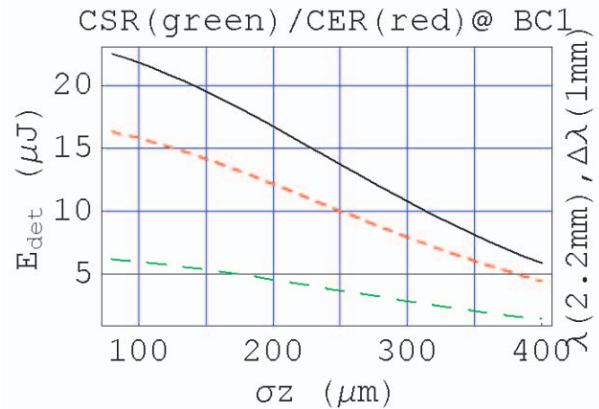


Figure 4: Comparison of CSR and CER. Notations are explained in the text.

Because of the fact that the mirror is close to the bending magnet and the bending magnet has a finite length, some details for calculating the coherent radiation signal is discussed in this paper. Further improvement of the calculation is always possible and needed with the development of the BLMs for LCLS.

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