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An Analysis of Shot Noise Propagation and Amplification in Harmonic Cascade FELs *

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Abstract

The harmonic generation process in a harmonic cascade (HC) FEL is subject to noise degradation which is proportional to the square of the total harmonic order [1]. In this paper, we study the shot noise evolution in the first-stage modulator and radiator of a HC FEL that produces the dominant noise contributions. We derive the effective input noise for a modulator operating in the low-gain regime, and analyze the radiator noise for a density-modulated beam. The significance of these noise sources in different harmonic cascade designs is also discussed.

INTRODUCTION

Harmonic cascade (HC) FELs are envisioned to generate fully coherent x-ray pulses [2] and are currently under active development for several VUV and soft x-ray projects (see, e.g., Refs. [3, 4]). It was pointed out in Ref. [1] that electron shot noise can be amplified by at least the square of the total harmonic order in this process, much like a frequency multiplication chain in radar communications [5]. Thus, it is important to understand the shot noise contributions in the harmonic generation process which may be the limiting factors in determining the temporal coherence or the final wavelength reach of these seeded FELs.

In a self-amplified spontaneous emission (SASE) FEL, the one-dimensional (1D) shot noise power spectrum is $\rho\gamma mc^2/(2\pi)$ [6], where ρ is the FEL Pierce parameter [7] and γmc^2 is the electron energy. The shot noise power spectrum can be identified to be about the forward-direction spontaneous undulator radiation in the first two power gain lengths [8]. The three-dimensional (3D) correction to this simple 1D result including effects of energy spread and emittance is given in Refs. [9, 10]. If the first undulator of a HC FEL operates in the high-gain regime (i.e., much longer than the gain length), the SASE noise power (integrated over the gain bandwidth) may be used to estimate its noise contribution to a HC FEL. However, due to the availability of high-power seed laser, the typical design of the first undulator of a HC FEL is a short (energy) modulator that operates in the low-gain (or even no-gain) regime [3, 4, 11]. Thus, the shot noise content of this modulator can be different from a high-gain undulator. After the dispersion section, the density-modulated electron beam entering the radiator generates additional shot noise. In this paper, we analyze the shot noise evolution in the first-stage modulator and radiator of a HC FEL that produces the dominant noise contributions. We also discuss the significance of these noise sources in different harmonic cascade designs.

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ANALYSIS

To illustrate this noise degradation process, we consider a seed signal at the fundamental wavelength $\lambda_1 = 2\pi/k_1 = 2\pi c/\omega_1$:

$$E_1 = (E_0+\Delta E)e^{i\theta+i\Delta\theta} \approx (E_0+\Delta E)e^{i\theta}(1+i\Delta\theta)\,. \quad (1)$$

Here E_0 and $\theta = -\omega_1 t$ are the amplitude and the phase of the signal, ΔE and $\Delta\theta$ represent any small amplitude and phase noises (such as caused by the electron shot noise and/or any noise carried by the seed laser). After a total of $N_h = h_1 h_2 ...$ frequency multiplication, the electric field at the output harmonic is

$$\begin{aligned} E_{N_h} =& G(E_0 + \Delta E)\exp\left(iN_h\theta + iN_h\Delta\theta\right) \\ \approx& G(E_0 + \Delta E)e^{iN_h\theta}\left(1 + iN_h\Delta\theta\right)\,, \end{aligned} \quad (2)$$

where we have assumed that $N_h\Delta\theta \ll 1$ (otherwise the effect is rather large), and G is an arbitrary function of the field amplitude (such as the Bessel function bunching factor). Thus, the noise-to-signal ratio at the final harmonic radiation is [1, 5]

$$\left(\frac{P_n}{P_s}\right)_{N_h} = {N_h}^2\left(\frac{P_n}{P_s}\right)_1\,. \quad (3)$$

N_h can be a very large number (a few hundred to a few thousand when harmonic cascading a UV laser to an x-ray FEL). $(P_n)_1$ is the initial noise power which includes both the intrinsic laser noise and the electron shot noise. Suppose that the seed laser noise is controlled to a tolerable level, the shot noise fluctuations of the electron beam provide the essential contributions, which will be studied here.

We focus our analysis on the first-stage of a harmonic cascade that includes a modulator, a dispersion section and a radiator (tuned to the h^{th} harmonic of the seed wavelength) as shown in Fig. 1. This first stage has the largest total harmonic conversion factor and hence produces the dominate noise sources. The initial longitudinal phase space distribution function is

$$F(\theta_0,\eta_0) = \frac{k_1}{\chi}\sum_{j=1}^{N_e}\delta(\theta_0-\theta_j)\delta(\eta_0-\eta_j)\,, \quad (4)$$

where $\theta_0 = -ck_1t_0$ describes the electron phase relative to the EM wave (i.e., input laser field), η_0 describe the initial relative energy deviation, and $\chi = N_e/l_b$ is the line density of the electron bunch with N_e electrons and l_b bunch

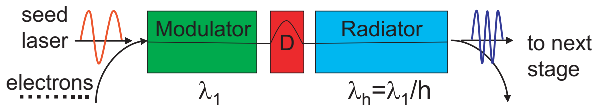

Figure 1: Schematic of the first-stage HC FEL.

length. The normalization in Eq. (4) is chosen so that the ensemble average

$$\left\langle \int F(\theta_0,\eta_0)d\eta_0 \right\rangle = 1 \tag{5}$$

for a constant current profile.

In typical HC FEL designs [3, 4, 11], the first undulator is relatively short and is mainly an energy modulator, then we have

$$\eta_1 = \eta_0 + \eta_s \sin\theta_0 + \eta_n(\theta_0)\,, \tag{6}$$

where η_s is the energy modulation amplitude induced by the seed laser field, and η_n is the energy modulation induced by the noisy spontaneous undulator radiation.

A dispersion section immediately after the modulator can convert the beam energy modulation into a density modulation. This is accompanied by a magnetic chicane that changes the phase of the electron according to its energy deviation:

$$\theta_1 = \theta_0 + k_1 R_{56}\eta_1 = \theta_0 + D\eta_1\,. \tag{7}$$

Here R_{56} is the net momentum compaction of the chicane together with the first undulator (modulator), and $D = k_1 R_{56}$. The harmonic bunching near the h^{th} harmonic (when $\nu \sim h$) can be found as

$$\begin{aligned} b_\nu &= \int \frac{d\theta_1}{k_1 l_b} d\eta_1 e^{-i\nu\theta_1} F(\theta_1,\eta_1) \\ &= \int \frac{d\theta_0}{k_1 l_b} d\eta_0 e^{-i\nu[\theta_0 + D(\eta_0 + \eta_s \sin\theta_0 + \eta_n)]} F(\theta_0,\eta_0)\,. \end{aligned} \tag{8}$$

Here we have assumed the laser pulse length is at least as long as the electron bunch length. If the laser pulse only overlaps a fraction of the electron bunch, l_b should be taken to be the laser pulse length instead of the electron pulse length, then N_e represents number of electrons within l_b. Let us also assume that the modulated part of the electron bunch is long compared to the laser wavelength (i.e., $k_1 l_b \gg 1$), and that the electron energy distribution is Gaussian with a slice rms energy spread σ_η, we expand Eq. (8) in Bessel series of η_s and to the first-order in η_n (as $|hD\eta_n| \ll 1$) to obtain

$$\begin{aligned} b_\nu &= \int \frac{d\theta_0}{k_1 l_b} \int d\eta_0 \sum_{p=-\infty}^{\infty} J_p(-hD\eta_s) e^{i(p-\nu)\theta_0} e^{-i\nu D\eta_0} \\ &\quad \times (1 - ihD\eta_n) F(\theta_0,\eta_0) \\ &= \sum_{p=-\infty}^{\infty} J_p(-hD\eta_s) \Bigg[\frac{1}{N_e} \sum_{j=1}^{N_e} e^{i(p-\nu)\theta_j} e^{-i\nu D\eta_j} \\ &\quad - ihD \exp\left(-\frac{h^2 D^2 \sigma_\eta^2}{2}\right) \int \frac{d\theta_0}{k_1 l_b} e^{i(p-\nu)\theta_0} \eta_n(\theta_0) \Bigg]\,, \end{aligned} \tag{9}$$

where we have applied the smooth distribution function in the second term of the bracket as η_n is treated as a small perturbation.

When $\nu = p = h$, the first term in the bracket produces the desired harmonic bunching signal as

$$\begin{aligned} b_h &= \frac{J_h(-hD\eta_s)}{N_e} \sum_{j=1}^{N_e} e^{-ihD\eta_j} \\ &= J_h(-hD\eta_s) \exp\left(-\frac{h^2 D^2 \sigma_\eta^2}{2}\right)\,. \end{aligned} \tag{10}$$

When $\nu \neq h$, the first term produces the noise bunching in the radiator as the ensemble average

$$\langle |b_\nu^r|^2 \rangle = \frac{1}{N_e} \sum_{p=-\infty}^{\infty} J_p^2(-hD\eta_s) = \frac{1}{N_e}\,. \tag{11}$$

Thus, the modulated bunch generates the same amount of the shot noise bunching in the radiator as a fresh electron bunch.

The second term in the bracket of Eq. (9) is the noise bunching originated from the modulator. As η_n is nearly a sinusoidal function of θ_0, it can be written as

$$\begin{aligned} b_\nu^m &= \exp\left(-\frac{h^2 D^2 \sigma_\eta^2}{2}\right) \Big[- ihD J_{h+1}(-hD\eta_s) \eta_n^+(\Delta\nu) \\ &\quad - ihD J_{h-1}(-hD\eta_s) \eta_n^-(\Delta\nu) \Big]\,, \end{aligned} \tag{12}$$

where $\Delta\nu = \nu - h$ and

$$\eta_n^\pm(\Delta\nu) = \int \frac{d\theta_0}{k_1 l_b} e^{i(\pm 1 - \Delta\nu)\theta_0} \eta_n(\theta_0) \tag{13}$$

is the Fourier component of η_n near the first undulator (modulator) resonant frequency ck_1.

If $x = hD\eta_s \ll 1$, we can expand $J_h(x) \sim (x/2)^h/h!$; If the dispersion strength is optimized to yield the maximum $|b_h|$ at $hD\eta_s \sim h$ or $D \sim 1/\eta_s$, then $J_h \sim J_{h\pm1} \sim 0.3$. In either case, we can approximate the bunching ratio as

$$\left| \frac{b_\nu^m}{b_h} \right|^2 \approx 4h^2 \frac{|\eta_n^\pm(\Delta\nu)|^2}{\eta_s^2}\,. \tag{14}$$

The power spectrum $dP/d\omega$ in the radiator is proportional to $|b|^2$. Integrating over their respective bandwidths in the radiator, we obtain the modulator noise-to-signal power ratio as

$$\left(\frac{P_n^m}{P_s}\right)_h = 4h^2\frac{\Delta\omega_n^m\langle|\eta_n^\pm(\Delta\nu)|^2\rangle}{\Delta\omega_s\eta_s^2}\,, \tag{15}$$

where $\Delta\omega_s = 2\pi c/l_b$ is the Fourier transform limited bandwidth for the signal, and $\Delta\omega_n^m$ is the bandwidth of the modulator noise. For simplicity, we assume that the full modulator bandwidth is much smaller than the full radiator bandwidth centered around a much higher frequency (i.e., $\Delta\omega_m = \Delta\nu_m\omega_1 < \Delta\omega_r = \Delta\nu_r\omega_h$), then we have $\Delta\omega_n^m = \Delta\omega_m = \Delta\nu_m\omega_1$ without convoluting the bandwidths of the modulator and the radiator.

The laser-induced energy modulation amplitude can be estimated as

$$\eta_s^2 = K_1^2[\mathrm{JJ}]^2\frac{L_{u1}^2}{\gamma^4\sigma_L^2}\frac{P_L}{P_0}\,, \tag{16}$$

where $P_0 = I_Amc^2/e \approx 8.7$ GW. Using the one-dimensional FEL theory, we find that the Fourier component of the shot-noise-induced energy modulation is

$$\begin{aligned}\eta_n^\pm(\Delta\nu) =&\frac{1}{8\gamma^3}\frac{I}{I_A}\left(\frac{\lambda_{u1}K_1[\mathrm{JJ}]}{\sigma_x}\right)^2 N_{u1}^2 f(\bar\nu)\\ &\times\frac{1}{N_e}\sum_{j=1}^{N_e}e^{i(\pm1+\Delta\nu)\theta_j}\,,\end{aligned} \tag{17}$$

where $\bar\nu = \pi\Delta\nu N_{u1}$ is the scaled detune in the first undulator with N_{u1} period, and

$$f(\bar\nu) = \left[\frac{e^{-i\bar\nu}\sin(\bar\nu)/\bar\nu - 1}{i\bar\nu}\right] \tag{18}$$

describes the energy modulation bandwidth due to the shot noise with a relative bandwidth given by $\Delta\nu_m = 1/N_{u1}$ for $N_{u1} \gg 1$. Inserting Eq. (16) and (17) into Eq. (15), we obtain finally

$$\begin{aligned}\left(\frac{P_n^m}{P_s}\right)_h =&h^2\frac{\lambda_{u1}N_{u1}r_e\sigma_L^2}{8\sigma_x^4}\frac{K_1^2[\mathrm{JJ}]^2}{1+K_1^2/2}\frac{(mc^2/e)\,I}{P_L}\\ =&h^2\left(\frac{P_n^m}{P_L}\right)_1\,.\end{aligned} \tag{19}$$

Thus, the effective modulator noise is

$$P_n^m = \frac{\lambda_u N_{u1}r_e\sigma_L^2}{8\sigma_x^4}\frac{K_1^2[\mathrm{JJ}]^2}{1+K_1^2/2}\frac{mc^2}{e}I\,. \tag{20}$$

For efficient laser-beam interaction in the modulator, the laser spot size is usually chosen to be $\sigma_L = \sqrt{\lambda_1\lambda_{u1}N_{u1}/8\pi}$ (i.e., the Rayleigh length is one half the undulator length with the laser waist located at the middle of the undulator). When the electron beam matches the laser spot (i.e., $\sigma_x \approx \sigma_L$), we have

$$P_n^m \approx\frac{\lambda_{u1}N_{u1}r_e}{8\sigma_x^2}\frac{K_1^2[\mathrm{JJ}]^2}{1+K_1^2/2}\frac{mc^2}{e}I \tag{21}$$

$$\approx\frac{\pi r_e}{\lambda_1}\frac{mc^2}{e}I\,. \tag{22}$$

Equation (21) can be shown to be the spontaneous undulator radiation in the forward direction (within a solid angle $\lambda_1^2/(2\pi\sigma_x^2)$ and a full bandwidth ω_1/N_{u1}). Equation (22) holds for $K_1^2 \gg 1$ and can be used for a quick estimation of modulator noise power. If the modulator length is much shorter than two power gain lengths, then the modulator noise power is much smaller than the usual SASE noise power as discussed in the introduction.

As shown in Eq. (11), the density-modulated beam generates the same shot noise bunching. The additional radiator noise-to-signal ratio can be estimated as

$$\left(\frac{P_n^r}{P_s}\right)_h = \frac{\Delta\omega_n^r\langle|b_\nu^r|^2\rangle}{\Delta\omega_s b_h^2} = \frac{\Delta\omega_n^r/N_e}{\Delta\omega_s J_h^2(hD\eta_s)e^{-h^2D^2\sigma_\eta^2}}\,, \tag{23}$$

where $\Delta\omega_n^r = \Delta\omega_r = \Delta\nu_r\omega_h$ is the noise bandwidth in the radiator. If the radiator is also a low-gain device, then $\Delta\nu_r \approx 1/N_{u2}$. If the radiator is a high-gain device, then $\Delta\nu_r \approx 2\rho$. In either case we can write

$$\left(\frac{P_n^r}{P_s}\right)_h = \frac{1/N_{lc}}{b_h^2} = \frac{1/N_{lc}}{J_h^2(hD\eta_s)e^{-h^2D^2\sigma_\eta^2}}\,, \tag{24}$$

where $N_{lc} = N_e l_c/l_b$ is the number of electrons within the radiator coherence length $l_c = \lambda_h/(\Delta\nu_r)$. Note that the radiator noise-to-signal ratio is independent of the signal laser power P_L, but depends strongly on the harmonic bunching strength. This has implications on different designs of HC FELs to be discussed below.

NUMERICAL EXAMPLES AND DISCUSSIONS

Equations (20), (22), and (24) are the main results of this paper and may be used to estimate the shot-noise-to-signal ratio of a HC FEL using a high-power seed laser in a short modulator. Let us take some numerical examples to illustrate the significance of various noise contributions. Consider the BESSY HC FEL design at the final radiation wavelength $\lambda_f = 1.24$ nm [3]. For $I = 1.75$ kA and $\lambda_1 = 297.50$ nm, the modulator noise power according to Eq. (22) is 26 W. If the laser power is 100 MW, then the modulator noise-to-signal ratio at the final wavelength after the total harmonic number $N_h = 297.50/1.24 = 240$ is

$$\left(\frac{P_n^m}{P_s}\right)_{N_h} = N_h^2 \times \frac{26}{100\times10^6} \approx 1.5\%\,. \tag{25}$$

Thus, the modulator noise contribution is noticeable but still small. In passing, we note the bandwidth of the modulator noise is about ω_1/N_{u1}, hence the final relative bandwidth of the first-stage modulator noise is

$$\frac{\Delta\omega_n^m}{\omega_f} = \frac{1}{N_h N_{u1}}\,. \tag{26}$$

This may still be a small relative bandwidth than a SASE FEL at 1 nm. For example, , $N_h = 240$, $N_{u1} = 18$ in the BESSY FEL, and $\Delta\omega_n^m/\omega_f \sim 2\times10^{-4}$. Thus, even when the final noise level due to the first modulator is comparable to the signal strength, the temporal coherence of the HC FEL is still improved as any noisy structure within the slippage length $\lambda_1 N_{u1}$ of the first modulator is naturally smoothed. This noise filtering effect was observed in the LUX HC FEL simulations [11].

Let us now consider the radiator noise. First, we take $I = 1.75$ kA, $N_{u2} = 40$, and $\lambda_h = 297.50/5 = 59.50$ nm, then we have $N_{lc} = N_e N_{u2}\lambda_h/l_b \sim 10^8$ in Eq. (24). The BESSY FEL employs a fresh bunch approach that shifts the output radiation to a fresh part of the bunch for the next-stage interaction and hence allows for a large energy modulation to be induced in the part of the electron bunch that overlaps with a very short laser signal [12]. In this approach, the harmonic bunching is usually maximized by choosing $D \sim 1/\eta_s$. If $hD\sigma_\eta \ll 1$, we have $b_h^2 \sim 0.1$. The increase of the energy spread due to the large energy modulation is not an issue as the next stage interaction occurs at a fresh part of the bunch with the same initial energy spread. In this case, the radiator noise-to-signal ratio given by Eq. (24) is extremely small, at the 10^{-7} level. Even after another harmonic conversion factor of 48 (from the radiator wavelength $\lambda_h = 59.50$ nm to the final wavelength $\lambda_f = 1.24$ nm), the contribution from the radiator noise is still small.

Nevertheless, the fresh bunch technique requires a tight timing control between the short laser pulse and the electron bunch. In addition, experiments demanding most photons in a narrow bandwidth may benefit from using a laser pulse longer than the electron bunch length to seed the whole bunch. In this case, the induced energy modulation must be controlled to a small level in order not to degrade the beam energy spread (i.e., $\eta_s < \sigma_\eta$). In view of Eq. (10), the harmonic bunching cannot be maximized as $hD\eta_s < hD\sigma_\eta < 1$, then the radiator noise-to-signal ratio can increase dramatically. For example, in the whole-bunch seeding example described in Ref. [1], the second-harmonic bunching at the radiation wavelength 130 nm is only $b_2^2 \sim 0.25\times10^{-4}$ in order to avoid a significant increase in the energy spread. Thus, the radiator noise-to-signal ratio given by Eq. (24) can be much larger ($\sim 10^{-3}$). An additional harmonic conversion factor of 16 (to the final wavelength at 8 nm) will amplify this radiator noise-to-signal ratio to 25%. Therefore, the shot noise contribution, especially in the radiator section, may limit the temporal coherence of such a harmonic cascade.

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