SLAC-PUB-12042
EFI 06-16
hep-ph/0608040
August 2006

RATE AND CP-ASYMMETRY SUM RULES IN $B \to K\pi$ [1]

Michael Gronau[a,b] and Jonathan L. Rosner[c]

[a] *Physics Department, Technion – Israel Institute of Technology*
32000 Haifa, Israel

[b] *Stanford Linear Accelerator Center, Stanford University*
Stanford, CA 94309, USA

[c] *Enrico Fermi Institute and Department of Physics, University of Chicago*
Chicago, Illinois 60637, USA

ABSTRACT

The observed violation of $A_{CP}(B^0 \to K^+\pi^-) = A_{CP}(B^+ \to K^+\pi^0)$ has been recently mentioned as a puzzle for the standard model. We point out that while this violation may be accounted for by a large color-suppressed tree amplitude, a sum rule involving three or four $B \to K\pi$ CP asymmetries should hold. The current experimental status of these sum rules and of a sum rule for $B \to K\pi$ decay rates is presented.

Recently [1, 2] the fact that $A_{CP}(B^0 \to K^+\pi^-) \neq A_{CP}(B^+ \to K^+\pi^0)$ was mentioned as a puzzle for the Standard Model. The equality of these two CP asymmetries was proposed eight years ago [3] in the limit that only penguin (P) and color-favored tree (T) amplitudes contributed to these decays. Since then it has been recognized for some time (e.g., through detailed flavor-SU(3) fits of B decays to two charmless pseudoscalar mesons [4]) that the color-suppressed (C) tree amplitude also plays an important role in $B^+ \to K^+\pi^0$ decays. When this amplitude is included in the discussion, a more exact sum rule was proposed [5]:

$$A_{CP}(K^+\pi^-) = A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0) \quad , \tag{1}$$

or, taking account of a small annihilation amplitude (A) as well [6],

$$A_{CP}(K^+\pi^-) + A_{CP}(K^0\pi^+) = A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0) \quad . \tag{2}$$

These relations also hold approximately in the presence of an electroweak contribution P_{EW}, and the second can be derived using isospin [6, 7]. Rather than expressing a discrepancy with the Standard Model, they serve as an important *test* of it once the CP asymmetry in $B^0 \to K^0\pi^0$ is measured with sufficient accuracy [8, 9].

Eqs. (1) and (2) are derived in the limit of the leading-order (P) contributions to decay rates. More accurate versions are expressed in terms of *rate differences*

$$\Delta_{ij} \equiv \Gamma(B \to K^i\pi^j) - \Gamma(\bar{B} \to K^{\bar{i}}\pi^{\bar{j}}) \quad . \tag{3}$$

[1] Research supported in part by the US Department of Energy, contract DE-AC02-76SF00515

Submitted to Physical Review D

Table I: Branching ratios for $B \to K\pi$ presented at ICHEP06 and their averages, in units of 10^{-6}.

Mode	BaBar [10]	Belle [2]	Average
$K^+\pi^-$	$19.7 \pm 0.6 \pm 0.6$	$20.0 \pm 0.4^{+0.9}_{-0.8}$	19.83 ± 0.63
$K^+\pi^0$	$13.3 \pm 0.56 \pm 0.64$	$12.4 \pm 0.5^{+0.7}_{-0.6}$	12.83 ± 0.59
$K^0\pi^+$	$23.9 \pm 1.1 \pm 1.0$	$22.9^{+0.8}_{-0.7} \pm 1.3$	23.40 ± 1.06
$K^0\pi^0$	$10.5 \pm 0.7 \pm 0.5$	$9.2^{+0.7+0.6}_{-0.6-0.7}$	9.89 ± 0.63

Neglecting the annihilation amplitude A one finds [5]

$$\Delta_{+-} \simeq 2(\Delta_{+0} + \Delta_{00}) \tag{4}$$

while including A one has [6]

$$\Delta_{+-} + \Delta_{0+} \simeq 2(\Delta_{+0} + \Delta_{00}) \tag{5}$$

At the moment the CP asymmetry $A_{CP}(K^0\pi^0)$ agrees well with the nearly-identical predictions of Eqs. (4) and (5). A corresponding sum rule relating the *rates* for the four $B \to K\pi$ processes is now seen to be satisfied at the 1σ level.

We use the latest measured branching ratios and asymmetries from BaBar [1, 10] and Belle [2] summarized in Tables I and II, respectively. CP asymmetries, by convention, are defined in terms of the rate differences Δ_{ij} by

$$A_{CP}(K^i\pi^j) \equiv -\Delta_{ij}/[\Gamma(B \to K^i\pi^j) + \Gamma(\bar{B} \to K^{\bar{i}}\pi^{\bar{j}})] \quad . \tag{6}$$

We first write the sum rule (5) for rate asymmetries, which, when expressed in terms of CP asymmetries, reads [6]

$$A_{CP}(K^+\pi^-) + A_{CP}(K^0\pi^+)\frac{\mathcal{B}(K^0\pi^+)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_0}{\tau_+} =$$

$$A_{CP}(K^+\pi^0)\frac{2\mathcal{B}(K^+\pi^0)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_0}{\tau_+} + A_{CP}(K^0\pi^0)\frac{2\mathcal{B}(K^0\pi^0)}{\mathcal{B}(K^+\pi^-)} \quad . \tag{7}$$

Here we have converted ratios of branching ratios to ratios of rates where necessary using the ratio $\tau_+/\tau_0 = 1.076 \pm 0.008$ of B^+ and B^0 lifetimes [11]. The sum rule (4) is evaluated by omitting the term containing the very small CP asymmetry $A_{CP}(K^0\pi^+)$. Using the averaged branching ratios and CP asymmetries in Tables I and II, we predict

$$\text{Eq. (5)} \Rightarrow A_{CP}(K^0\pi^0) = -0.151 \pm 0.043 \ , \tag{8}$$

$$\text{Eq. (4)} \Rightarrow A_{CP}(K^0\pi^0) = -0.159 \pm 0.036 \ , \tag{9}$$

Table II: CP asymmetries for $B \to K\pi$ presented at ICHEP06 and their averages.

Mode	BaBar [1, 12]	Belle [2]	Average
$K^+\pi^-$	$-0.108 \pm 0.024 \pm 0.007$	$-0.093 \pm 0.018 \pm 0.008$	-0.099 ± 0.016
$K^+\pi^0$	$0.016 \pm 0.041 \pm 0.010$	$0.07 \pm 0.03 \pm 0.01$	0.050 ± 0.025
$K^0\pi^+$	$-0.029 \pm 0.039 \pm 0.010$	$0.03 \pm 0.03 \pm 0.01$	0.007 ± 0.025
$K^0\pi^0$	$-0.20 \pm 0.16 \pm 0.03$	$-0.05 \pm 0.14 \pm 0.05$	-0.12 ± 0.11

to be compared with the observed value

$$A_{CP}(K^0\pi^0) = -0.12 \pm 0.11 \quad . \tag{10}$$

Either prediction is consistent with the observed value. Nearly identical predictions of $A_{CP}(K^0\pi^0) = (-0.142 \pm 0.039, -0.149 \pm 0.030)$ are obtained using Eqs. (2) and (1), respectively.

The rate sum rule [3, 13]

$$\Gamma(K^+\pi^-) + \Gamma(K^0\pi^+) = 2[\Gamma(K^+\pi^0) + \Gamma(K^0\pi^0)] \quad , \tag{11}$$

where isospin-breaking corrections are suppressed by a ratio of tree and penguin amplitudes [14], may be expressed in terms of branching ratios by correcting for the lifetime ratio:

$$\mathcal{B}(K^+\pi^-) + \mathcal{B}(K^0\pi^+)\frac{\tau_0}{\tau_+} = 2[\mathcal{B}(K^+\pi^0)\frac{\tau_0}{\tau_+} + \mathcal{B}(K^0\pi^0)] \quad . \tag{12}$$

In units of 10^{-6}, the left-hand side is 41.58 ± 1.18, while the right-hand side is 43.63 ± 1.68. The difference is 2.05 ± 2.05, or 1σ. Both this sum rule and the rate difference sum rule (5) are useful tests for new physics in the $b \to s$ penguin diagram, which has shown hints of exhibiting new contributions elsewhere [9, 15].

Note added: The rate sum rules are dominated by a common penguin contribution for which they are trivially satisfied. They may be rearranged so that each side is an interference term between the dominant penguin and subdominant color-favored or color-suppressed tree contributions [16]. They are, of course, still satisfied in this form, but present experimental errors are still too large to tell whether each side of the sum rule is nonzero with sufficient significance.

We thank H. J. Lipkin for helpful comments. M.G. is grateful to the SLAC Theory Group for its kind hospitality. This work was supported in part by the Israel Science Foundation under Grant No. 1052/04, by the German-Israeli Foundation under Grant No. I-781-55.14/2003, and by the U. S. Department of Energy under Grant No. DE-FG02-90ER40560.

References

[1] E. DiMarco, presented at XXXIII International Conference on High Energy Physics (ICHEP06), Moscow, USSR, July 26 – August 2, 2006.

[2] Y. Unno, presented at ICHEP06, Moscow, USSR, July 26 – August 2, 2006.

[3] M. Gronau and J. L. Rosner, Phys. Rev. D **59**, 113002 (1999) [arXiv:hep-ph/9809384].

[4] C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, Phys. Rev. D **70**, 034020 (2004) [arXiv:hep-ph/0404073].

[5] M. Gronau and J. L. Rosner, Phys. Rev. D **71**, 074019 (2005) [arXiv:hep-ph/0503131].

[6] M. Gronau, Phys. Lett. B **627**, 82 (2005) [arXiv:hep-ph/0508047].

[7] D. Atwood and A. Soni, Phys. Rev. D **58**, 036005 (1998) [arXiv:hep-ph/9712287].

[8] K. Hara, presented at ICHEP06, Moscow, USSR, July 26 – August 2, 2006.

[9] M. Hazumi, presented at ICHEP06, Moscow, USSR, July 26 – August 2, 2006.

[10] M. Bona, presented at ICHEP06, Moscow, USSR, July 26 – August 2, 2006.

[11] We use the averages of the Heavy Flavor Averaging Group, `http://www.slac.stanford.edu/xorg/hfag/`, hep-ex/0603003.

[12] B. Aubert *al.* [BABAR Collaboration], arXiv:hep-ex/0608036.

[13] H. J. Lipkin, Phys. Lett. B **445**, 403 (1999).

[14] M. Gronau, Y. Grossman, G. Raz and J. L. Rosner, Phys. Lett. B **635**, 207 (2006) [arXiv:hep-ph/0601129].

[15] M. Gronau, Invited talk presented at the First Workshop on Theory, Phenomenology and Experiments in Heavy Flavor Physics, Anacapri, Italy, 29–31 May 2006, hep-ph/0607282.

[16] H. J. Lipkin, Phys. Lett. B **633**, 540 (2006) [arXiv:hep-ph/0507225].