# Measurement of the $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ Branching Fractions using $\Upsilon(4 S) \rightarrow B \bar{B}$ Events Tagged by a Fully Reconstructed $B$ Meson 

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#### Abstract

We report preliminary measurements of the exclusive charmless semileptonic branching fractions of the $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ decays. These measurements are based on $316 \mathrm{fb}^{-1}$ of data collected at the $\Upsilon(4 S)$ resonance by the BABAR detector. In events in which the decay of one $B$ meson to a hadronic final state is fully reconstructed, the semileptonic decay of the recoiling $B$ meson is identified by the detection of a charged lepton and an $\eta$ or $\eta^{\prime}$. We measure the branching fraction $\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)=(0.84 \pm 0.27 \pm 0.21) \times 10^{-4}$, where the first error is statistical and the second one systematic. We also set an upper limit on the branching fraction of $\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)<$ $1.4 \times 10^{-4}$ and $\mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)<1.3 \times 10^{-4}$ at the $90 \%$ confidence level.


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## 1 Introduction

Precise measurements of the Cabibbo-Kobayashi-Maskawa matrix [1] element $V_{u b}$ can be employed to test the consistency of the Standard Model description of $C P$ violation. $\left|V_{u b}\right|$ can be extracted from exclusive charmless semileptonic $B$ decays allowing for more stringent kinematical constraints and better background suppression than possible with inclusive measurements. However, the determination of $\left|V_{u b}\right|$ from exclusive decays is complicated by the presence of the strong interaction between the quarks in the initial and the final states. In the case of $B \rightarrow X_{u} \ell \nu$ decays, where $X_{u}$ is a pseudoscalar meson, and neglecting the mass of the lepton, the dynamics are described by a single form-factor $f\left(q^{2}\right)$ that depends on the square of the $B \rightarrow X_{u}$ momentum transfer $q$. The shape of the form-factors can in principle be measured, while we have to rely on theoretical predictions [2] for their normalization.

Exclusive charmless semileptonic $B$ decays have been previously measured by the CLEO [3], Belle [4] and BABAR [5-8] collaborations. An extensive study with independent measurements of various additional charmless semileptonic decay modes, such as those involving the $\omega, \eta, \eta^{\prime}, a_{0}^{0}, \ldots$, is important to further constrain the theoretical models and reduce the statistical and systematic uncertainties. In this paper, we present an update of our previous results [9] on the branching fractions for the $B^{+} \rightarrow \eta \ell^{+} \nu^{5}$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ decay modes.

The analysis is based on a sample of $B \bar{B}$ events produced at the $\Upsilon(4 S)$ resonance that are tagged by a fully reconstructed hadronic decay. The full reconstruction of the tagging $B$ provides a clean sample of $B \bar{B}$ events and allows us to determine the flavor of the reconstructed $B$ meson and to separate $B^{0}$ and $B^{+}$decays.

A semileptonic decay of the recoiling $B$ meson is identified by the presence of a charged lepton. The $\eta$ and $\eta^{\prime}$ mesons in the semileptonic decay are reconstructed, and the missing mass is calculated assuming that the $\eta\left(\eta^{\prime}\right)$ and the charged lepton are the only particles present in the recoil except for the undetected neutrino. Since the momentum of the tagging $B$ meson is measured, a transformation into the rest frame of the recoiling $B$ meson can be performed.

## 2 Data Sample

The preliminary results shown here are based on a data sample corresponding to an integrated luminosity of $316 \mathrm{fb}^{-1}$, containing about 347 million $B \bar{B}$ pairs, collected with the BABAR detector [10] at the SLAC PEP-II asymmetric-energy $e^{+} e^{-}$collider [11] operating at the $\Upsilon(4 S)$ resonance. A Monte Carlo (MC) simulation of the BABAR detector based on Geant4 [12] has been used to optimize the selection criteria and to determine the signal efficiencies and background distributions.

## 3 Event Reconstruction and Selection

The analysis proceeds in three steps: first, one of the two $B$ mesons is fully reconstructed in hadronic decays ( $B_{\text {reco }}$ ), second, for the recoiling $B$ meson ( $B_{\text {sig }}$ ) we only reconstruct a charged lepton, electron or muon, and then we select the exclusive decays $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$. In order to minimize the systematic uncertainties due to the $B_{\text {reco }}$ selection and lepton identification, the exclusive branching fractions are measured relative to the inclusive semileptonic branching fraction.

[^3]
### 3.1 Full Reconstruction of Hadronic $B$ Decays

To reconstruct a sample of $B$ mesons, the hadronic decays $B^{+} \rightarrow \bar{D}^{0} Y^{+}, \bar{D}^{* 0} Y^{+}$are selected. The system $Y^{+}$consists of hadrons with a total charge of +1 , composed of $n_{1} \pi^{ \pm} n_{2} K^{ \pm} n_{3} K_{S}^{0} n_{4} \pi^{0}$, where $n_{1}+n_{2} \leq 5, n_{3} \leq 2$, and $n_{4} \leq 2$. We reconstruct $\bar{D}^{* 0} \rightarrow \bar{D}^{0} \pi^{0}, \bar{D}^{0} \gamma$ and $\bar{D}^{0} \rightarrow K^{+} \pi^{-}$, $K^{+} \pi^{-} \pi^{0}, K^{+} \pi^{-} \pi^{-} \pi^{+}, K_{S}^{0} \pi^{+} \pi^{-}$and $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$. The kinematical consistency of $B_{\text {reco }}$ candidates is checked with two variables, the beam energy-substituted mass $m_{\mathrm{ES}}=\sqrt{s / 4-\vec{p}_{B}^{2}}$ and the energy difference $\Delta E=E_{B}-\sqrt{s} / 2$. Here $\sqrt{s}$ is the total energy in the $\Upsilon(4 S)$ center-of-mass (CM) frame, and $\vec{p}_{B}$ and $E_{B}$ denote the momentum and energy of the $B_{\text {reco }}$ candidate in the same frame. We require $|\Delta E|<3 \sigma_{\Delta E}$, where $\sigma_{\Delta E}=10$ to 35 MeV , depending on the decay mode, is the resolution on $\Delta E$ for signal $B_{\text {reco }}$ events. On average, we reconstruct one signal $B_{\text {reco }}$ candidate in $200 B^{+} B^{-}$ events.

The combinatorial background from $B \bar{B}$ events and $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ events is subtracted by performing an unbinned maximum likelihood fit to the $m_{\mathrm{ES}}$ distribution, using the following threshold function [13]:

$$
\begin{equation*}
\frac{d N}{d m_{\mathrm{ES}}} \propto m_{\mathrm{ES}} \sqrt{1-x^{2}} \exp \left(-\xi\left(1-x^{2}\right)\right) \tag{1}
\end{equation*}
$$

for the background (where $x=m_{\mathrm{ES}} / m_{\max }, m_{\max }$ is the endpoint of the curve and $\xi$ is a free parameter determined by the fit to the $m_{\mathrm{ES}}$ distribution). A Gaussian function corrected for radiation losses [14] peaked at the $B$ meson mass is used to describe the signal.

### 3.2 Selection of Semileptonic $B$ Decays

The semileptonic selection identifies a charged lepton with a momentum $p_{\ell}^{*}$ in the $B_{\text {sig }}$ rest frame greater than $0.5 \mathrm{GeV} / c$ for electrons and $0.8 \mathrm{GeV} / c$ for muons. Electron candidates are identified using a likelihood method whose efficiency is about $93 \%$ and the hadron misidentification rate is less than $0.1 \%$. Muons are identified with an efficiency of about $75 \%$ and the hadron misidentification


Figure 1: Fit to the $B_{\text {reco }} m_{\text {ES }}$ distribution for events with a fully reconstructed $B^{+}$decay, after semileptonic selection has been applied The fitted curve (solid line) to the data points is the sum of a radiation loss corrected Gaussian and a threshold function (dashed line) described by Eq. 1.
rate is about $3 \%$. We also require the lepton and the $B_{\text {reco }}$ candidate to have opposite charge and that the lepton track has not been used to reconstruct the $B_{\text {reco }}$ candidate. Tracks are assumed to be pions if they are not identified as either a muon or an electron. The number of events after the semileptonic selection is obtained with the $m_{\mathrm{ES}}$ fit described in Section 3.1. The fit result on data is shown in Fig. 1.

The distributions of the lepton momentum, $p_{\ell}^{*}$, computed in the recoiling $B$ rest frame, at this stage of the selection, are shown in Fig. 2.

### 3.3 Selection of $B \rightarrow \eta \ell \nu$ and $B \rightarrow \eta^{\prime} \ell \nu$ Decays

The $B^{+} \rightarrow \eta \ell^{+} \nu\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)$ decay of $B_{\text {sig }}$ is reconstructed by combining $\eta\left(\eta^{\prime}\right)$ candidates with the charged lepton. We reconstruct $\eta$ candidates in three decay modes: $\eta \rightarrow \gamma \gamma(B F=39.4 \%)$, $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}(B F=22.6 \%)$ and $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}(B F=32.5 \%)$. The $\pi^{0}$ candidates used to build the $\eta$ are defined as pairs of photons, each with an energy in the laboratory frame $E_{\gamma}>30 \mathrm{MeV}$, in the invariant mass window $110<m_{\gamma \gamma}<160 \mathrm{MeV} / c^{2}$. For the $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$ channel one of the three reconstructed $\pi^{0}$ mesons should satisfy additional requirements based on the shape of the neutral clusters of the electromagnetic calorimeter and a tighter cut on the invariant mass of the $\pi^{0}$ $\left(115<m_{\gamma \gamma}<150 \mathrm{MeV} / c^{2}\right)$. The aim of these additional cuts is the reduction of the combinatorial background.

We reconstruct $\eta^{\prime}$ candidates in two decay modes: $\eta^{\prime} \rightarrow \rho^{0} \gamma(\mathrm{BF}=29.5 \%)$ and $\eta^{\prime} \rightarrow \eta \pi^{+} \pi^{-}$ ( $\mathrm{BF}=44.3 \%$ ). The $\rho^{0}$ candidates used to build the $\eta^{\prime}$ are reconstructed as pairs of charged pions with opposite charge while the $\eta$ candidates are selected as described above. In the $\eta^{\prime} \rightarrow \rho^{0} \gamma$ channel we apply a cut on the momentum of the $\gamma$ at $p_{\gamma}^{*}>0.35 \mathrm{GeV} / c$ to remove the background from $B^{+} \rightarrow \rho^{0} \ell^{+} \nu$ and $b \rightarrow c \ell \nu$ decays.

After these selection criteria, the dominant background is due to $b \rightarrow c \ell \nu$ semileptonic decays with either a real or combinatorial $\eta^{\left({ }^{( }\right)}$. A good rejection variable against these events is the missing


Figure 2: Distributions of the electron and muon momenta, $p_{\ell}^{*}$, computed in the rest frame of the recoiling $B$, for data points and Monte Carlo (histogram) for $B^{+} \rightarrow \eta \ell^{+} \nu$ (left) and $B^{+} \rightarrow$ $\eta^{\prime} \ell^{+} \nu$ (right), after applying all cuts of the semileptonic selection except for the request on $p_{\ell}^{*}$, for events in which at least one $\eta\left(\eta^{\prime}\right)$ candidate has been found. The distributions are normalized to the same area. The ratio between data and MC simulation is shown below the histogram.
four momentum of the event

$$
\begin{equation*}
p_{\text {miss }}=p_{\Upsilon(4 S)}-p_{B_{\text {reco }}}-p_{\eta\left(\eta^{\prime}\right)}-p_{\ell}, \tag{2}
\end{equation*}
$$

where $p_{\Upsilon(4 S)}$ is the sum of the four-momenta of the colliding beams, $p_{B_{\text {reco }}}$ is the measured fourmomentum of the $B_{\text {reco }}, p_{\eta\left(\eta^{\prime}\right)}$ is the measured four-momentum of the $\eta$ or $\eta^{\prime}$ and $p_{\ell}$ is the measured four-momentum of the lepton. For signal events the only missing particle should be a single undetected neutrino, while for background events the missing momentum and energy in the event are due to other undetected or poorly measured particles. Thus, in signal events the resulting missing mass squared, defined as $m_{\text {miss }}^{2}=p_{\text {miss }}^{2}$, peaks at zero while for background events it tends to have larger values, and provides a discrimination of signal and background.

To select the decay modes of interest, the following additional selection criteria are applied:

- a cut on the invariant mass of the $\eta$ and $\eta^{\prime}$ candidates, different for each mode;
- event charge balance: $Q_{t o t}=Q_{B_{r e c o}}+Q_{B_{s i g}}=0$. This condition rejects preferentially $b \rightarrow c \ell \nu$ events, since their higher charge multiplicity implies a larger number of lost charged tracks;
- a cut on the squared missing mass, $\left|m_{m i s s}^{2}\right|<0.5 \mathrm{GeV}^{2} / c^{4}$;
- the only tracks allowed to be present in the recoil are the charged lepton and the tracks used to reconstruct the $\eta$ or $\eta^{\prime}$ candidate;
- for the $B^{+} \rightarrow \eta \ell^{+} \nu$ channel, the missing mass squared calculated assuming a $B^{+} \rightarrow \pi^{0} \ell^{+} \nu$ decay is required to be $\left|m_{\text {miss }}^{2}\right|_{\pi^{0}}>1.5 \mathrm{GeV}^{2} / c^{4}$. This condition rejects $B^{+} \rightarrow \pi^{0} \ell^{+} \nu$ events, which are the main $b \rightarrow u \ell \nu$ background source.

The selection criteria described above have been optimized by minimizing the expected statistical error of the measurement and are summarized in Table 1. After all cuts have been applied we have $10-15 \%$ signal events with more than one $\eta\left(\eta^{\prime}\right)$ candidates for event. When several candidates remain in an event after all the cuts, the one with $m_{\text {miss }}^{2}$ closest to zero is chosen. The selection efficiencies $\epsilon_{s e l}^{e x c l}$ as estimated from the Monte Carlo simulation are reported in Table 2. The number of events after all analysis cuts are obtained with the fit to the $m_{\mathrm{ES}}$ distribution. The fit results on data are shown in Fig. 3.

## 4 Measurement of Branching Fractions

In order to reduce systematic uncertainties, the exclusive branching fractions are measured relative to the inclusive semileptonic branching fraction.

After the combinatorial background has been subtracted using the $m_{\mathrm{ES}}$ fit, the number of inclusive $B \rightarrow X \ell \nu$ events, $N_{s l}^{\text {meas }}$, and the number of remaining background events, $N_{s l}^{B G}$, peaking at the $B$ mass in the $m_{\mathrm{ES}}$ distribution, are related to the true number of semileptonic decays $N_{s l}^{\text {true }}$ as:

$$
\begin{equation*}
N_{s l}^{\text {meas }}-N_{s l}^{B G}=\epsilon_{l}^{s l} \epsilon_{t}^{s l} N_{s l}^{\text {true }} . \tag{3}
\end{equation*}
$$

Here $\epsilon_{l}^{s l}$ refers to the efficiency for selecting a lepton from a semileptonic $B$ decay in an event with a hadronic $B$ decay, reconstructed with tag efficiency $\epsilon_{t}^{s l}$.

Table 1: Summary of event selection for $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$.

| Selection | $B^{+} \rightarrow \eta \ell^{+} \nu$ | $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ |
| :---: | :---: | :---: |
| $\begin{aligned} & \hline \hline \pi^{0} \text { mass } \\ & \eta \text { mass } \\ & (\eta \rightarrow \gamma \gamma) \\ & \left(\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \\ & \left(\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}\right) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \hline 110<m_{\gamma \gamma}<160 \mathrm{MeV} / c^{2} \\ & 115<m_{\gamma \gamma}<150 \mathrm{MeV} / c^{2}\left(\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}\right) \\ & 505<m_{\eta}<585 \mathrm{MeV} / c^{2} \\ & 530<m_{\eta}<560 \mathrm{MeV} / c^{2} \\ & 510<m_{\eta}<580 \mathrm{MeV} / c^{2} \\ & \hline \end{aligned}$ |  |
| $\begin{aligned} & \hline \eta^{\prime} \text { mass } \\ & \left(\eta^{\prime} \rightarrow \rho^{0} \gamma\right) \\ & \left(\eta^{\prime} \rightarrow \eta \pi^{+} \pi^{-}, \eta \rightarrow \gamma \gamma\right) \\ & \left(\eta^{\prime} \rightarrow \eta \pi^{+} \pi^{-}, \eta \rightarrow \pi^{+} \pi^{-} \pi^{0}\right) \\ & \left(\eta^{\prime} \rightarrow \eta \pi^{+} \pi^{-}, \eta \rightarrow \pi^{0} \pi^{0} \pi^{0}\right) \\ & \rho^{0} \text { mass } \\ & \gamma \text { momentum } \end{aligned}$ |  | $\begin{aligned} & 930<m_{\eta^{\prime}}<980 \mathrm{MeV} / c^{2} \\ & 940<m_{\eta^{\prime}}<970 \mathrm{MeV} / c^{2} \\ & 935<m_{\eta^{\prime}}<975 \mathrm{MeV} / c^{2} \\ & 910<m_{\eta^{\prime}}<1000 \mathrm{MeV} / c^{2} \\ & 595<m_{\pi^{+} \pi^{-}}<955 \mathrm{MeV} / c^{2} \\ & p_{\gamma}^{*}>0.35 \mathrm{GeV} / c \\ & \hline \end{aligned}$ |
| Lepton momentum Number of leptons Charge conservation Number of tracks Charge correlation Missing mass squared | $\begin{array}{r} p_{e l}^{*}>0.5 \mathrm{GeV} / \\ N_{l e_{1}} \\ Q_{t} \\ \text { no additiona } \\ Q_{b(r e} \\ \left\|m_{\text {misss }}^{2}\right\| \\ \hline \end{array}$ | $\begin{aligned} & p_{\mu}^{*}>0.8 \mathrm{GeV} / c \\ & o n=1 \\ & t=0 \end{aligned}$ <br> charged tracks $\begin{aligned} & Q_{\ell}<0 \\ & 0.5 \mathrm{GeV}^{2} / c^{4} \\ & \hline \end{aligned}$ |
| $B^{+} \rightarrow \pi^{0} \ell^{+} \nu$ rejection | $\left\|m_{\text {miss }}^{2}\left(\pi^{0}\right)\right\|>1.5 \mathrm{GeV}^{2} / c^{4}$ |  |



Figure 3: Fit to the $m_{\mathrm{ES}}$ distribution for $B \rightarrow \eta \ell \nu$ (left) and $B \rightarrow \eta^{\prime} \ell \nu$ (right) candidates, after all selection criteria have been applied. The fitted curve (solid line) to the data points is the sum of a radiation loss corrected Gaussian and a threshold function (dashed line) described by Eq. 1.

The number of $B^{+} \rightarrow \eta \ell^{+} \nu\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)$ events after the $m_{E S}$ combinatoric background subtraction, $N_{\text {excl }}^{\text {meas }}$, and the number of peaking background events, $N_{\text {excl }}^{B G}$, are related to the true number of $B^{+} \rightarrow \eta \ell^{+} \nu\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)$ decays $N_{\text {excl }}^{\text {true }}$ as

$$
\begin{equation*}
N_{\text {excl }}^{\text {meas }}-N_{\text {excl }}^{B G}=\epsilon_{\text {sel }}^{\text {excl }} \epsilon_{l}^{\text {excl }} \epsilon_{t}^{\text {excl }} N_{\text {excl }}^{\text {true }}, \tag{4}
\end{equation*}
$$

where the signal efficiency $\epsilon_{\text {sel }}^{\text {excl }}$ accounts for all selection criteria applied to the sample after the requirement of the presence of a charged lepton.

The background contributions $N_{s l}^{B G}$ and $N_{\text {excl }}^{B G}$ are estimated using the MC simulation and scaled to the luminosity of the data sample by the ratio of the number of semileptonic events in data and MC. For $N_{\text {excl }}^{B G}$ we further rescale to match the data in a sideband region $1<m_{\text {miss }}^{2}<4 \mathrm{GeV}^{2} / c^{4}$.

We measure the ratio of the branching fractions for $B^{+} \rightarrow \eta \ell^{+} \nu$ or $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ to the branching fraction of $B \rightarrow X \ell \nu$ decays as

$$
\begin{equation*}
R_{e x c l / s l}=\frac{\mathcal{B}\left(B \rightarrow \eta\left(\eta^{\prime}\right) \ell \nu\right)}{\mathcal{B}(B \rightarrow X \ell \nu)}=\frac{N_{e x c l}^{\text {true }}}{N_{s l}^{t r u e}}=\frac{\left(N_{e x c l}^{\text {meas }}-N_{e x c l}^{B G}\right) /\left(\epsilon_{s e l}^{\text {excl }}\right)}{\left(N_{s l}^{\text {meas }}-N_{s l}^{B G}\right)} \times \frac{\epsilon_{l}^{s l} \epsilon_{t}^{s l}}{\epsilon_{l}^{\text {excl }} \epsilon_{t}^{\text {excl }}} . \tag{5}
\end{equation*}
$$

The ratio of efficiencies for $B \rightarrow X \ell \nu$ and signal events in Eq. 5 is expected to be close to, but not equal to unity. Due to the difference in multiplicity and the different lepton momentum spectra, we expect the tag efficiencies $\epsilon_{t}^{s l, \text { excl }}$ and the charged lepton efficiencies $\epsilon_{l}^{s l, \text { excl }}$ to be slightly different for the two classes of events. Using the semileptonic branching ratio $\mathcal{B}(B \rightarrow X \ell \nu)=(10.73 \pm 0.28) \%$ [15] and the ratio of the $B^{0}$ and $B^{+}$lifetimes $\tau_{B^{+}} / \tau_{B^{0}}=1.086 \pm 0.017$ [15], the branching ratios $\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)$ and $\mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)$ are derived.

The results for $R_{\text {excl/sl }}$ and all the corresponding input measurements are shown in Table 2. Fig. 4 shows the resulting data $m_{\text {miss }}^{2}$ distributions. The signal and background components from the Monte Carlo has been scaled to the number of events passing the semileptonic selection and further rescaled by a factor $1.29 \pm 0.06$ for $B^{+} \rightarrow \eta \ell^{+} \nu$ and a factor $0.97 \pm 0.09$ for $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$. We also report in Table 3 the contribution to the peaking background from the different components.

Table 2: Measurement of $R_{e x c l / s l}$ for $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ and corresponding inputs. The reported errors are statistical only.

| mode | $N_{e x c l}^{\text {meas }}$ | $N_{\text {excl }}^{\text {BG }}$ | $\epsilon_{\text {sel }}^{\text {excl }}$ | $N_{s l}^{\text {meas }}-N_{s l}^{B G}$ |  | $R_{\text {excl/sl }}\left[\times 10^{-3}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B^{+} \rightarrow \eta \ell^{+} \nu$ | $45.9 \pm 7.1$ | $23.8 \pm 4.9$ | $0.24 \pm 0.02$ | $109000 \pm 450$ | $0.88 \pm 0.06$ | $0.75 \pm 0.24$ |
| $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ | $14.0 \pm 5.3$ | $11.0 \pm 3.3$ | $0.10 \pm 0.01$ | $109000 \pm 450$ | $1.05 \pm 0.08$ | $0.30 \pm 0.53$ |

Table 3: Breakdown of background events for $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$. For each studied channel (columns) the background contributions from the different components (rows) are shown.

|  | $B^{+} \rightarrow \eta \ell^{+} \nu$ | $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ |
| :--- | :---: | :---: |
| $B^{0} \rightarrow \pi^{-} \ell^{+} \nu$ |  |  |
| $B^{+} \rightarrow \pi^{0} \ell^{+} \nu$ | $0.77 \pm 0.57$ |  |
| $B^{0} \rightarrow \rho^{+} \ell \nu$ | $0.19 \pm 0.14$ | $0.15 \pm 0.19$ |
| $B^{+} \rightarrow \rho^{0} \ell^{+} \nu$ |  |  |
| $B^{ \pm} \rightarrow \omega \ell \nu$ | $0.39 \pm 0.29$ |  |
| $B^{+} \rightarrow \eta \ell^{+} \nu$ |  |  |
| $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ | $0.77 \pm 0.57$ |  |
| Other $b \rightarrow u \ell \nu$ | $0.19 \pm 0.14$ | $0.45 \pm 0.58$ |
| $B \rightarrow D \ell \nu$ | $2.44 \pm 0.61$ | $1.48 \pm 0.51$ |
| $B \rightarrow D^{*} \ell \nu$ | $17.08 \pm 4.32$ | $5.94 \pm 2.06$ |
| Other $b \rightarrow c \ell \nu$ | $0.82 \pm 0.21$ | $0.89 \pm 0.31$ |
| Others | $1.21 \pm 1.25$ | $2.33 \pm 1.53$ |

## 5 Systematic Uncertainties

The different sources of systematic uncertainties and their impact on the final results are reported in Table 4 and briefly described in the following, using the same order as in the table.

The track-finding efficiency is different between data and MC simulation, therefore we apply a flat $0.36 \%$ correction to the simulation in order to match the efficiency in data. The systematic uncertainty related to the reconstruction of charged tracks is determined by removing randomly a fraction of tracks corresponding to the uncertainty in the track finding efficiency. The systematic uncertainty due to the reconstruction of neutral particles in the EMC is studied by varying the resolution and efficiency to match those found in control samples in data. Moreover we assign a systematic uncertainty of $1.8 \%$ per photon and a $3.0 \%$ uncertainty due to $\pi^{0}$ reconstruction. We estimate the systematic uncertainties due to particle identification by varying the electron and muon identification efficiency by $\pm 2 \%$ and $\pm 3 \%$, respectively, and by applying a $\pm 15 \%$ uncertainty on mis-identification efficiency.

The uncertainty of the $B_{\text {reco }}$ background subtraction is estimated by using an alternative approach to evaluate $N_{s l}$, based on a binned $\chi^{2}$ fit, which is compared to the one obtained from the $m_{\mathrm{ES}}$ fit to estimate the systematic error. After applying the semileptonic selection, we consider the $m_{\mathrm{ES}}$ distribution obtained from the data and from background components modeled with distri-


Figure 4: $m_{\text {miss }}^{2}$ distribution from data (dots) and signal and background (open and shaded histograms) contributions from Monte Carlo for $B^{+} \rightarrow \eta \ell^{+} \nu$ (left) and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ (right).
butions taken from Monte Carlo simulation: $B^{0} \bar{B}^{0}, B^{+} B^{-}$and $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$. We fit the background normalization on data in the $m_{\mathrm{ES}}$ sideband region, defined by $m_{\mathrm{ES}}<5.26 \mathrm{GeV} / c^{2}$. The relative normalization of each component is determined by a binned $\chi^{2}$ fit. The $\chi^{2}$ function is defined as

$$
\begin{equation*}
\chi^{2}\left(C_{b k g}\right)=-\sum_{i}\left(\frac{N_{i}^{\text {meas }}-C_{b k g} N_{i}^{b k g, M C}}{\sqrt{\delta N_{i}^{\text {meas } 2}+\delta N_{i}^{M C^{2}}}}\right)^{2} \tag{6}
\end{equation*}
$$

where $N_{i}^{\text {meas }}$ is the number of observed events in the $i$-th bin, $N_{i}^{b k g, M C}$ is the total background component, $C_{b k g}$ is the normalization of the background component and $\delta N_{i}^{\text {meas }}$ and $\delta N_{i}^{M C}$ are the statistical uncertainties for data and Monte Carlo respectively. The normalization for $e^{+} e^{-} \rightarrow q \bar{q}$ ( $q=u, d, s, c$ ) is fixed and the scaling factor is obtained from a comparison with off-peak data. Instead, the $B^{+} B^{-}$and $B^{0} \bar{B}^{0}$ components and the normalization of the background component are left to vary in the fit. The total background contribution is then subtracted from the events in the $m_{\mathrm{ES}}$ signal region ( $m_{\mathrm{ES}}>5.27 \mathrm{GeV} / c^{2}$ ) in order to extract the number of semileptonic events, separately for $B^{0}$ and $B^{+}$and the difference is taken as a systematic error.

We evaluate the effect of cross-feed between $B^{0}$ and $B^{+}$decays by repeating the analysis with only the $B^{+} B^{-}$Monte Carlo sample. The impact of the charm semileptonic branching fractions has been estimated by varying each of the exclusive branching fractions within one standard deviation of the current world average [15]. The effects due to exclusive $B \rightarrow X_{u} \ell \nu$ decays have been evaluated by varying their branching fractions by $15 \%$ for $B \rightarrow \pi \ell \nu, 30 \%$ for $B \rightarrow \rho \ell \nu$ and by $100 \%$ for the remaining decay modes.

The use of different theoretical form-factor calculations changes the shape of the lepton momentum spectrum for the signal and, as a consequence, affects the efficiencies $\epsilon_{l}^{e x c l}, \epsilon_{l}^{s l}$ and $\epsilon_{s e l}^{e x c l}$. The Monte Carlo samples used in this analysis were generated using the ISGW2 model [16]. We reweight the event distributions according to the recent calculations by Ball and Zwicky [2] based on light-cone sum rules since, among the calculations currently available, these calculations result in distributions that differ most from those predicted by ISGW2. We assign the variations with
respect to ISGW2 as systematic uncertainties. This contribution is small because the selection efficiencies for $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ are largely flat over the phase space.

We take into account the possible effects of the excess around $1.5 \mathrm{GeV} / c^{2}$ in the $B^{+} \rightarrow \eta \ell^{+} \nu$ case (left plot of Fig. 4) on the yield extraction. We varied the sideband region definition used to normalize the background from $1<m_{\text {miss }}^{2}<4 \mathrm{GeV}^{2} / c^{4}$ to $1<m_{\text {miss }}^{2}<2.5 \mathrm{GeV}^{2} / c^{4}$, that corresponds to a variation on the number of background events $N_{\text {excl }}^{B G}$ of $11 \%$. The difference in the branching fraction has been taken as systematic uncertainty.

The statistical uncertainty on the ratio of efficiencies for $B \rightarrow X \ell \nu$ and signal events in Eq. 5, due to limited Monte Carlo statistics, has been taken as a systematic uncertainty.

The total systematic uncertainties on the $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ branching ratios are given by the sum in quadrature of all the individual contributions to the systematic uncertainties (Table 4).

Table 4: Systematic uncertainties in the measurement of $R_{\text {excl/sl }}$.

|  | Uncertainty on $R_{\text {excl/sl }}$ |  |
| :--- | :---: | :---: |
|  | $B^{+} \rightarrow \eta \ell^{+} \nu$ | $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$ |
| Statistical error | $\pm 0.24$ | $\pm 0.53$ |
| Track reconstruction efficiency | $\pm 0.04$ | $\pm 0.02$ |
| Photon resolution, $\pi^{0}$ reconstruction | $\pm 0.03$ | $\pm 0.03$ |
| Electron identification | $\pm 0.03$ | $\pm 0.01$ |
| Muon identification | $\pm 0.03$ | $\pm 0.02$ |
| $m_{\text {ES fit }}$ | $\pm 0.09$ | $\pm 0.04$ |
| Cross-feed $B^{0} \leftrightarrow B^{+}$ | $\pm 0.01$ | $\pm 0.09$ |
| $B \rightarrow D l \nu X$ and $D$ branching fractions | $\pm 0.04$ | $\pm 0.12$ |
| $B \rightarrow X_{u} \ell$ branching fractions | $\pm 0.02$ | $\pm 0.05$ |
| Form-factor model | $\pm 0.03$ | $\pm 0.02$ |
| Background normalization | $\pm 0.08$ | $\pm 0.07$ |
| MC statistics | $\pm 0.12$ | $\pm 0.20$ |
| Total systematic error | $\pm 0.19$ | $\pm 0.27$ |

## 6 Conclusions

We measured the branching fractions relative to the inclusive charmless semileptonic branching fraction for $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$. We obtain:

$$
\begin{aligned}
& \frac{\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)}{\mathcal{B}(B \rightarrow X \ell \nu)}=\left(0.75 \pm 0.24_{\text {stat }} \pm 0.19_{\text {syst }}\right) \times 10^{-3}, \\
& \frac{\mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)}{\mathcal{B}(B \rightarrow X \ell \nu)}=\left(0.30 \pm 0.53_{\text {stat }} \pm 0.27_{\text {syst }}\right) \times 10^{-3} .
\end{aligned}
$$

Using the inclusive semileptonic branching fraction $\mathcal{B}(B \rightarrow X \ell \nu)=(10.73 \pm 0.28) \%$ and the ratio of the $B^{0}$ and $B^{+}$lifetimes $\tau_{B^{+}} / \tau_{B^{0}}=1.086 \pm 0.017$ [15], we derive the branching fractions for $B^{+} \rightarrow \eta \ell^{+} \nu$ and $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$. We obtain:

$$
\begin{aligned}
& \mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)=\left(0.84 \pm 0.27_{\text {stat }} \pm 0.21_{\text {syst }}\right) \times 10^{-4}, \\
& \mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)=\left(0.33 \pm 0.60_{\text {stat }} \pm 0.30_{\text {syst }}\right) \times 10^{-4} .
\end{aligned}
$$

The significance and the upper limit has been calculated including all the systematic and statistical uncertainties on the background.
The resulting significance is $2.55 \sigma$ for $B^{+} \rightarrow \eta \ell^{+} \nu$ and $0.95 \sigma$ for $B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu$.
For these branching fractions we get the following $90 \%$ confidence level (C.L.) upper limits:

$$
\begin{aligned}
& \mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)<1.4 \times 10^{-4}(90 \% \text { C.L. }) \\
& \mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)<1.3 \times 10^{-4}(90 \% \text { C.L. })
\end{aligned}
$$

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