# Measurement of Time-dependent $C P$ Asymmetries in $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ Decays 

The BABAR Collaboration

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#### Abstract

We present an updated measurement of the time-dependent $C P$-violating asymmetry in $B^{0} \rightarrow$ $K_{S}^{0} K_{S}^{0} K_{S}^{0}$ decays based on 347 million $\Upsilon(4 S) \rightarrow B \bar{B}$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $B$ factory at SLAC. We obtain the $C P$ asymmetries $S_{f}=-0.66 \pm$ $0.26 \pm 0.08$ and $C_{f}=-0.14 \pm 0.22 \pm 0.05$, where the first uncertainties are statistical and the second systematic.


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## 1 Introduction

In the Standard Model (SM) of particle physics, the decays $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ are dominated by $b \rightarrow s \bar{s} s$ gluonic penguin amplitudes. Let $2 \beta_{\text {eff }}$ be the $C P$-violating phase difference between $B^{0} \rightarrow$ $K_{S}^{0} K_{S}^{0} K_{S}^{0}$ decays with and without mixing, and $\beta=\arg \left(-V_{c d} V_{c b}^{*} / V_{t d} V_{t b}^{*}\right)$ where $V_{i j}$ are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1]. The difference $\mid \sin 2 \beta-$ $\sin 2 \beta_{\text {eff }} \mid$ is expected to be nearly zero using calculations in SM, with theoretical uncertainties at the level of $\mathcal{O}(0.01)$ [2], thanks to the factor $\left|V_{u b} V_{u s} / V_{t b} V_{t s}\right|$ which suppresses the other contributions in the SM.

On the other hand, $b \rightarrow s \bar{s} s$ decays involve one-loop transitions, so contributions from heavy new particles entering such loops can introduce new $C P$-violating phases, and these may contribute to $\beta_{\text {eff }}$ [3]. The value of $\sin 2 \beta$ has been measured at the $B$ factories in recent years with high precision $[4,5]$, with a world average of $0.685 \pm 0.032$ [6].

The Belle and BABAR collaborations have already reported measurements of $C P$ asymmetries for $B^{0} \rightarrow \phi K_{s}^{0}[7,8,9]$ ( $C P$-odd) and $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ (CP-even) [10, 11].

The time-dependent $C P$ asymmetry is obtained by measuring the proper-time difference $\Delta t \equiv$ $t_{C P}-t_{\text {tag }}$ between a fully reconstructed decay $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ and the decay of a partially reconstructed tagging $B$ meson ( $B_{\mathrm{tag}}$ ). The expected asymmetry in the decay rate $f_{+}\left(f_{-}\right)$when the tagging meson is a $B^{0}\left(\bar{B}^{0}\right)$ is given as

$$
f_{ \pm}(\Delta t)=\frac{e^{-|\Delta t| \mid \tau_{B^{0}}}}{4 \tau_{B^{0}}} \times\left[1 \pm S_{f} \sin \left(\Delta m_{d} \Delta t\right) \mp C_{f} \cos \left(\Delta m_{d} \Delta t\right)\right]
$$

where $\tau_{B^{0}}$ is the $B^{0}$ lifetime and $\Delta m_{d}$ is the $B^{0}-\bar{B}^{0}$ mixing frequency. The parameter $S_{f}$ is nonzero if there is $C P$ violation in the interference between decays with and without mixing, while a non-zero value for $C_{f}$ would correspond to direct $C P$ violation. In the limit of one dominant decay amplitude in $b \rightarrow s \bar{s} s$ transition, the SM predicts no direct $C P$ violation, and that $S_{f}=-\eta_{f} \sin 2 \beta$, within the theoretical hadronic uncertainties already mentioned. The $C P$ eigenvalue $\eta_{f}=+1$ for $C P$-even $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0} \quad$ decays.

In this paper, we update our measurement of the time-dependent $C P$-violating asymmetries in the decay $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ previously presented in [11], using a larger data set and reconstructing the submode with one $K_{S}^{0}$ decaying into $\pi^{0} \pi^{0}$. The absence of charged decay tracks originating at the $B^{0}$ decay vertex requires special techniques to deal with its reconstruction [12]. In addition, the final state has a definite $C P$ content [13], so that an angular analysis is not needed.

## 2 The BABAR detector and dataset

The results presented here are based on $347.5 \pm 3.8$ million $\Upsilon(4 S) \rightarrow B \bar{B}$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $e^{+} e^{-}$collider, located at the Stanford Linear Accelerator Center. The BABAR detector [14] provides charged-particle tracking through a combination of a five-layer double-sided silicon microstrip detector (SVT) and a 40-layer central drift chamber, both operating in a 1.5 T magnetic field. Charged kaon and pion identification is achieved through measurements of particle energy-loss in the tracking system and Cherenkov cone angle in a detector of internally reflected Cherenkov light. A segmented $\operatorname{CsI}(\mathrm{Tl})$ electromagnetic calorimeter (EMC) provides photon detection and electron identification. Finally, the instrumented flux return of the magnet allows discrimination of muons from pions.

## $3 \quad B_{C P}$ candidates selection

The $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ candidate $\left(B_{C P}\right)$ is reconstructed by combining three $K_{S}^{0}$ candidates. Two subsamples of $B_{C P}$ candidates are formed. One subsample contains candidates formed by three $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$candidates in an event $\left(B_{C P(+-)}\right)$, while the other subsample is made by candidates formed by two $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$candidates and a third $K_{S}^{0}$ reconstructed in the $\pi^{0} \pi^{0}$ mode ( $B_{C P(00)}$ ). The two subsamples have different signal to background ratios and therefore different analysis requirements were applied to obtain optimal selections.

For the $B_{C P(+-)}$ subsample we reconstruct $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$candidates from pairs of oppositely charged tracks. The two-track composites must originate from a common vertex with a $\pi^{+} \pi^{-}$ invariant mass within $12 \mathrm{MeV} / c^{2}$ (about $4 \sigma$ ) of the nominal $K_{S}^{0}$ mass, and have a reconstructed flight distance $\left(r_{\text {dec }}\right)$ between 0.2 and 40.0 cm from the beam spot in the plane transverse to the beam. We also require that the reconstructed $K_{S}^{0}$ has an angle between the transverse flight direction and the transverse momentum vector of less than 200 mrad . For each $B_{C P(+-)}$ candidate two nearly independent kinematic variables are computed, the beam-energy-substituted mass $m_{\mathrm{ES}}=\sqrt{\left(s / 2+\mathbf{p}_{i} \cdot \mathbf{p}_{B}\right)^{2} / E_{i}^{2}+p_{B}^{2}}$, and the energy difference $\Delta E=E_{B}^{*}-\sqrt{s} / 2$. Here, $\left(E_{i}, \mathbf{p}_{i}\right)$ is the four-vector of the initial $e^{+} e^{-}$system, $\sqrt{s}$ is the center-of-mass energy, $\mathbf{p}_{B}$ is the reconstructed momentum of the $B^{0}$ candidate, and $E_{B}^{*}$ is its energy calculated in the $e^{+} e^{-}$rest frame. For signal decays, the $m_{\mathrm{ES}}$ distribution peaks near the $B^{0}$ mass with a resolution of about $2.5 \mathrm{MeV} / c^{2}$, and the $\Delta E$ distribution peaks near zero with a resolution of about 14 MeV . We select $B_{C P(+-)}$ candidates within the window $5.22<m_{\mathrm{ES}}<5.30 \mathrm{GeV} / c^{2}$ and $-120<\Delta E<120 \mathrm{MeV}$, which includes the signal peak and a "sideband" region for background characterization.

For the $B_{C P(00)}$ subsample we form $\pi^{0} \rightarrow \gamma \gamma$ candidates from pairs of photon candidates in the EMC. Each photon is required to be isolated from any charged tracks, to carry a minimum energy of 50 MeV , and to have the expected lateral shower shape. We reconstruct $K_{S}^{0} \rightarrow \pi^{0} \pi^{0}$ candidates from $\pi^{0}$ pairs which form an invariant mass $480<m_{\pi^{0} \pi^{0}}<520 \mathrm{MeV} / c^{2}$. $B_{C P(00)}$ candidates are constrained to originate from the $e^{+} e^{-}$interaction point using a geometric fit, based on a Kalman Filter [15]. We make a requirement on the consistency of the $\chi^{2}$ of the fit which retains $93 \%$ of the signal events and rejects about $49 \%$ of other $B$ decays. We extract the $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$decay length $L_{K_{S}^{0}}$ and the invariant mass $\left(m_{\gamma \gamma}\right)$ from this fit and require $100<m_{\gamma \gamma}<141 \mathrm{MeV} / c^{2}$ and $L_{K_{S}^{0}}$ greater than 5 times its uncertainty. For $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$candidates we require $0.15<r_{d e c}<60.0 \mathrm{~cm}$.

For each $B_{C P(00)}$ candidate we compute two kinematic variables, the reconstructed mass $m_{B}$ and the missing mass $m_{\text {miss }}=\sqrt{\left(q_{e^{+} e^{-}}-\tilde{q}_{B}\right)^{2}}$, where $q_{e^{+} e^{-}}$is the four-momentum of the initial $e^{+} e^{-}$system and $\tilde{q}_{B}$ is the four-momentum of the $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ candidate after a mass constraint on the $B^{0}$ is applied. By construction, the linear correlation coefficient between $m_{\text {miss }}$ and $m_{B}$ vanishes. This combination of variables shows smaller correlation ( $0.9 \%$ on reconstructed signal Monte Carlo events and $1.7 \%$ on the final data sample) and better background suppression with respect to the equivalent kinematic variables $\Delta E$ and $m_{\mathrm{ES}}$ used for $B_{C P(+-)}$ candidates. This is more relevant for $B_{C P(00)}$ candidates given the asymmetric resolution on these variables due to $\pi^{0}$ energy reconstruction. We select $B_{C P(00)}$ candidates within the window $5.11<m_{\text {miss }}<$ $5.31 \mathrm{GeV} / c^{2}$ and $-150<m_{B}-m_{B}^{P D G}<150 \mathrm{MeV} / c^{2}$, where $m_{B}^{P D G}$ represents the nominal $B^{0}$ mass, reported by the Particle Data Group [16].

The sample of $B^{0} \rightarrow K_{S}^{0} K_{S}^{0} K_{S}^{0}$ candidates is dominated by random $K_{S}^{0} K_{S}^{0} K_{S}^{0}$ combinations from $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ fragmentation. Monte Carlo (MC) studies show that contributions from other $B$ meson decays are small. We exploit topological observables to discriminate the jet-like $e^{+} e^{-} \rightarrow q \bar{q}$ events from the more uniformly distributed $B \bar{B}$ events. In the $\Upsilon(4 S)$ rest frame we
compute the angle $\theta_{T}^{*}$ between the thrust axis of the $B_{C P(+-)}\left(B_{C P(00)}\right)$ candidate and that of the remaining particles in the event. While $\left|\cos \theta_{T}^{*}\right|$ is highly peaked near 1 for $e^{+} e^{-} \rightarrow q \bar{q}$ events, it is nearly uniformly distributed for $B \bar{B}$ events. We require $\left|\cos \theta_{T}^{*}\right|<0.9$ (0.95), reducing by one order of magnitude the number of background events. The maximum-likelihood fit described below also uses discriminant variables based on the momenta and angles of tracks in the event to discriminate $B_{C P}$ candidates from $q \bar{q}$. They are combined in a Fisher discriminant $(\mathcal{F})$ [12] for $B_{C P(+-)}$ candidates, while in the case of $B_{C P(00)}$ candidates we calculate the ratio $L_{2} / L_{0}$ of two Legendre monomials, defined as $L_{j} \equiv \sum_{i}\left|\mathbf{p}_{i}^{*}\right|\left|\cos \theta_{i}^{*}\right|^{j}$, where $\mathbf{p}_{i}^{*}$ is the momentum of particle $i$ in the $e^{+} e^{-}$rest frame, $\theta_{i}^{*}$ is the angle between $\mathbf{p}_{i}^{*}$ and the thrust axis of the $B$ candidate and the sum runs over all reconstructed particles except for the $B$-candidate daughters.

After all selection requirements are applied, the average $B_{C P}$ candidate multiplicity in events with at least one $B_{C P(00)}$ candidate is approximately 1.67 , coming from multiple $K_{S}^{0} \rightarrow \pi^{0} \pi^{0}$ combinations. In these events, we select the candidate with the smallest $\chi^{2}=\sum_{i}\left(m_{i}-m_{K_{S}^{0}}\right)^{2} / \sigma_{m_{i}}^{2}$, where $m_{i}\left(m_{K_{S}^{0}}\right)$ is the measured (nominal $\left.K_{S}^{0}\right)$ mass and $\sigma_{m_{i}}$ is the estimated uncertainty on the mass of the $i$ th $K_{S}^{0}$ candidate. In simulated events, this selection criterion gives the right answer about $81 \%$ of the time. The remaining misreconstructed events, coming from fake $K_{S}^{0} \rightarrow \pi^{0} \pi^{0}$ candidates, do not affect the determination of $\Delta t$ and have a small impact on the other variables used in the final fit. The largest correlation is $\sim 2.5 \%$. In the case of events with $B_{C P(+-)}$ candidates, only $1.4 \%$ of them have more than one candidate, and we apply the same criterion to select the best combination.

Events coming from $b \rightarrow c \bar{c} s$ would reduce any sensitivity to departures from the Standard Model as this process is characterized by a Standard Model $C P$ asymmetry ( $S \sim \sin 2 \beta$ and $C \sim 0$ ). We therefore remove all $B_{C P(+-)}\left(B_{C P(00)}\right)$ candidates that have a $K_{S}^{0} K_{S}^{0}$ mass combination within $3 \sigma(2 \sigma)$ of the $\chi_{c 0}$ or $\chi_{c 2}$ mass. After these vetoes the average efficiency, including $K_{S}^{0}$ sub branching fractions, is about $6 \%$ for $B_{C P(+-)}$ candidates and about $3 \%$ for $B_{C P(00)}$.

Combinatorics from other $B \bar{B}$ decays constitute a further source of background for $B_{C P(00)}$ events. We take this into account by adding a component in the likelihood fit (see Sec. 5), where the shape of each likelihood variable is determined from a simulation of inclusive $B$ decays. This contribution is found to be negligible in the case of $B_{C P(+-)}$ events, and such a component is not included in the maximum likelihood fit.

## 4 Flavor tagging and $\Delta t$ reconstruction

For each $B_{C P}$ candidate we examine the remaining tracks in the event to determine the decay vertex position and the flavor of the $B_{\text {tag }}$ candidate.

We use a neural network to determine the flavor of the $B_{\text {tag }}$ meson from kinematic and particle-identification information [17]. Each event is assigned to one of six mutually exclusive tagging categories, designed to combine flavor tags with similar performance and $\Delta t$ resolution. We parameterize the performance of this algorithm with a data sample ( $B_{\text {flav }}$ ) of fully reconstructed $B^{0} \rightarrow D^{(*)-} \pi^{+} / \rho^{+} / a_{1}^{+}$decays. The effective tagging efficiency obtained from this sample is $Q \equiv \sum_{c} \epsilon^{c}\left(1-2 w^{c}\right)^{2}=0.305 \pm 0.004$, where $\epsilon^{c}$ and $w^{c}$ are the efficiencies and mistag probabilities, respectively, for events tagged in category $c$.

We compute the proper-time difference $\Delta t=\left(z_{C P}-z_{\text {tag }}\right) / \gamma \beta c$ using the known boost of the $e^{+} e^{-}$system and the measured $\Delta z=z_{C P}-z_{\mathrm{tag}}$, the difference of the reconstructed decay vertex positions of the $B_{C P}$ and $B_{\text {tag }}$ candidates along the boost direction (z). A description of the inclusive reconstruction of the $B_{\text {tag }}$ vertex is given in Ref. [18].

To reconstruct the $B_{C P}$ vertex from the $K_{S}^{0}$ trajectories we exploit the knowledge of the average interaction point (IP), which is determined from the spatial distribution of vertices from two-track events. We compute $\Delta t$ and its uncertainty from a geometric fit to the $\Upsilon(4 S) \rightarrow B^{0} \bar{B}^{0}$ system that takes this IP constraint into account. We further improve the sensitivity to $\Delta t$ by imposing a Gaussian constraint on the sum of the two $B$ decay times $\left(t_{C P}+t_{\mathrm{tag}}\right)$ to be equal to $2 \tau_{B^{0}}$ with an uncertainty of $\sqrt{2} \tau_{B^{0}}$, where $\tau_{B^{0}}$ is the world average on the $B^{0}$ mean life [16], which effectively constrains the two vertices to be near the $\Upsilon(4 S)$ line of flight [12]. The uncertainty on the IP position, which follows from the size of the interaction region, is on the order of $100 \mu \mathrm{~m}$ horizontally and roughly $4 \mu \mathrm{~m}$ vertically. The mean uncertainty on $z_{C P}$, a convolution of the interaction region and the vertex of the $B_{C P}$ decay, is $75 \mu \mathrm{~m}$. The mean uncertainty on $z_{\text {tag }}$ is about $200 \mu \mathrm{~m}$ and thus the uncertainty in $\Delta z$ is dominated by the uncertainty in the vertex of the tagging decay. The resulting resolution is comparable to that in $B^{0} \rightarrow J / \psi K_{S}^{0}[12]$.

Simulation studies show that the procedure we use to determine the vertex for a $B_{C P}$ decay provides an unbiased estimate of $z_{C P}$. The estimate of the $\Delta t$ error in an event reflects the strong dependence of the $z_{C P}$ resolution on the number of SVT layers traversed by the $K_{S}^{0}$ decay daughters. However, essentially all events have at least one $K_{S}^{0}$ candidate for which both tracks have at least one hit in the inner three SVT layers (at radii from 3.2 cm to 5.4 cm ). In this case the mean $\Delta t$ resolution is comparable to that in decays in which the vertex is directly reconstructed from charged particles originating at the $B$ decay point [18]. For a small fraction ( $0.1 \%$ ) of the signal events, at least one $K_{S}^{0}$ has tracks with hits in the outer two SVT layers (at radii 9.1 cm to 14.4 cm ) but none of the three $K_{S}^{0}$ s have hits in the inner three layers. In this case the resolution is nearly two times worse but the event can still be used in the $C P$ fit. For events with $\sigma_{\Delta t}>2.5 \mathrm{ps}$ or $|\Delta t|>20 \mathrm{ps}$, the $\Delta t$ information is not used. However, since $C_{f}$ can also be extracted from flavor tagging information alone, these events still contribute to the measurement of $C_{f}$.

The $\Delta t$ resolution function $\mathcal{R}$ is parameterized as the sum of a 'core' and a 'tail' Gaussian distribution, each with a width and mean proportional to $\sigma_{\Delta t}$, and a third Gaussian with a mean of zero and a width fixed at $8 \mathrm{ps}[18]$. We have verified on data that the parameters of $\mathcal{R}$ for $B_{C P}$ decays are similar to those obtained from the $B_{\text {flav }}$ sample, even when the IP constrained vertexing technique is applied. Therefore, we extract these parameters from a fit to the $B_{\text {flav }}$ sample. We find that the $\Delta t$ distribution of background candidates is well described by a delta function convolved with a resolution function having the same functional form as that for the signal. The parameters of the background function are determined in the fit.

## 5 Maximum Likelihood fit

We extract the results from unbinned maximum-likelihood fits to the kinematic, event shape, and $\Delta t$ variables. For each subsample we consider the logarithm of an extended likelihood function

$$
\mathcal{L}=e^{-\left(\sum_{j}^{n} N_{j}\right)} \times \prod_{i}^{N_{T}} \sum_{j}^{n} N_{j} \mathcal{P}_{j}^{i},
$$

where $\mathcal{P}_{j}$ is the probability density function (PDF) for the $\mathrm{j}^{\text {th }}$ fit component and $N_{j}$ the event yields of each component ( $N_{S}$ signal events, $N_{q \bar{q}} q \bar{q}$ events and, for $B_{C P(00)}$ only, $N_{B \bar{B}} B \bar{B}$ decay events) $N_{T}$ is the total number of events selected. The two $\mathcal{L}$ are then summed and maximized to determine the common $S_{f}$ and $C_{f} C P$ asymmetry parameters and the $N_{j}$ which are specific to each subsample.

The $\Delta t$ PDF for a given tagging category is $\mathcal{P}^{c}\left(\Delta t, \sigma_{\Delta t}\right) \epsilon^{c}$ where $\epsilon^{c}$ is the tagging efficiency for tag category $c$. The total likelihood $\mathcal{L}$ is the product of likelihoods for each tagging category, and the free parameters are determined by maximizing the quantity $\ln \mathcal{L}$. Along with the $C P$ asymmetries $S_{f}$ and $C_{f}$, the fit extracts $\epsilon^{c}$ and other parameters for background. The background PDFs include parameters for the $\Delta t$-resolution function $\mathcal{R}$ and for asymmetries in the rate of $B^{0}$ versus $\bar{B}^{0}$ tags. We extract 43 parameters from the fit.

The observables are sufficiently uncorrelated that we can construct the likelihoods as the products of one-dimensional PDFs. The signal PDFs are parameterized from signal MC events. For background PDFs we determine the functional form from data in the sideband regions of the other observables where backgrounds dominate. We include these regions in the fitted sample and simultaneously extract the parameters of the background PDFs along with the fit results.

There are $786 B_{C P(+-)}$ and $4550 B_{C P(00)}$ candidates that pass all the selection criteria. In Table 1 the events yields obtained in the fit are summarized for the two subsamples separately. Figure 1 shows the $m_{\mathrm{ES}}$ and $\Delta E\left(m_{\text {miss }}\right.$ and $\left.m_{B}\right)$ distributions for $B_{C P(+-)}\left(B_{C P(00)}\right)$ signal events with the sPlot event weighting technique [19]. The results of the fit are plotted as curves.

## 6 Results

The $C P$ parameters extracted from the fit are summarized in Table 1. We obtain

$$
\begin{aligned}
S_{f} & =-0.66 \pm 0.26 \pm 0.08 \\
C_{f} & =-0.14 \pm 0.22 \pm 0.05
\end{aligned}
$$

where the first error is statistical and the second systematic (evaluated as described in the following Section). The correlation between $S_{f}$ and $C_{f}$ is $-8.5 \%$. We evaluate the statistical significance of $C P$ violation to be $2.6 \sigma$ by calculating the $2 \Delta \log \mathcal{L}$ variation when fitting data with $S_{f}$ and $C_{f}$ fixed to zero. Using a Monte Carlo technique, in which we assume that the measured values for the $C P$ parameters on the combined data sample are the true values, we evaluate the probability of measuring the values reported in Table 1 for the two sub-samples. We find that the two sub-samples agree within $1.6 \sigma$.

Figure 2 shows distributions of $\Delta t$ for $B^{0}$-tagged and $\bar{B}^{0}$-tagged events, and the asymmetry $\mathcal{A}(\Delta t)=\left(N_{B^{0}}-N_{\bar{B}^{0}}\right) /\left(N_{B^{0}}+N_{\bar{B}^{0}}\right)$, obtained with the sPlot event weighting technique [19]. This result is in good agreement with the value of $\sin 2 \beta$ from $b \rightarrow c \bar{c} s$ decays [6].

|  | $B_{C P(+-)}$ | $B_{C P(00)}$ | Combined |
| :--- | :---: | :---: | :---: |
| $N_{S}$ | $116 \pm 12$ | $60 \pm 12$ | - |
| $N_{q \bar{q}}$ | $670 \pm 26$ | $4482 \pm 71$ | - |
| $N_{B \bar{B}}$ | - | $8 \pm 25$ | - |
| $S_{f}$ | $-1.044_{-0.17}^{+0.26}$ | $0.37 \pm{ }_{-0.54}^{+0.52}$ | $-0.66 \pm 0.26$ |
| $C_{f}$ | $-0.311_{-0.23}^{+0.25}$ | $0.21 \pm 0.38$ | $-0.14 \pm 0.22$ |

Table 1: Events yields and CP asymmetry parameters obtained in the fit. Statistical errors only are shown.


Figure 1: sPlots of (a) $m_{\mathrm{ES}}$ and (b) $\Delta E$ for $B_{C P(+-)}$ subsample and of (c) $m_{m i s s}$ and (d) $m_{B}$ for $B_{C P(00)}$ subsample.


Figure 2: Distributions of $\Delta t$ for events weighted with the sPlot technique for $B_{\mathrm{tag}}$ tagged as $B^{0}$ (top) or $\bar{B}^{0}$ (center), and the asymmetry $\mathcal{A}(\Delta t)$ (bottom). The points are weighted data and the curves are the corresponding PDF projections.

## 7 Systematic studies

We obtain systematic uncertainties in the $C P$ coefficients $S_{f}$ and $C_{f}$ due to the parameterization of kinematic variables and event shape PDFs in signal and background by varying the parameters within one standard deviation (evaluated from a fit to Monte Carlo simulated events). There might be a contribution to $C_{f}$ and $S_{f}$ from $C P$ violation in the $B \bar{B}$ background. In the fit, the values of the effective $C P$ parameters ( $S_{B \bar{B}}$ and $C_{B \bar{B}}$ ) for the $B \bar{B}$ background are fixed to zero. They are varied within the whole physical allowed range $S_{B \bar{B}}^{2}+C_{B \bar{B}}^{2} \leq 1$ and we take the largest variation on signal $S_{f}$ and $C_{f}$ as systematic uncertainty.

We evaluate the uncertainties associated with the assumed parameterization of the $\Delta t$ resolution function for signal and $B \bar{B}$-background by varying the parameters within one standard deviation (extracted from a fit to the $B_{\text {flav }}$ sample). The uncertainties due to knowledge of efficiencies and dilutions of flavor tagging and possible difference in the efficiency between $B^{0}$ and $\bar{B}^{0}$ are evaluated in the same way. The mass difference between the two $B^{0}$ mass eigenstates, $\Delta m_{d}$, and the $B^{0}$ mean life, $\tau_{B^{0}}$, values held fixed in the fit are varied within their uncertainties determined in world averages [16].

We also estimate different uncertainties associated with vertexing. We take the largest value of $S_{f}\left(C_{f}\right)_{\text {fit }}-S_{f}\left(C_{f}\right)_{\text {true }}$ from fits to signal Monte Carlo events where realistic misalignments of the SVT silicon wafers have been introduced. Here the $S_{f}\left(C_{f}\right)_{\text {fit }}$ represents the result of the fit to these simulated events, while $S_{f}\left(C_{f}\right)_{\text {true }}$ represents input values in the Monte Carlo generation. We include an additional contribution from the comparison of the description of the resolution function (RF) between IP-constrained vertexing and nominal vertexing in the case of $B^{0} \rightarrow J / \psi K_{S}^{0}$ events.

We assign a systematic uncertainty on our knowledge of the beam spot position by shifting the beam position in the simulation by $\pm 20 \mu m$ in the vertical direction. The sensitivity due to any calibration problems or time-dependent effects is evaluated by smearing the beam-spot position by an additional $\pm 20 \mu m$ in the vertical direction. The effect of neglecting possible correlations between the variables in the fit is estimated with a Monte Carlo technique. We also estimate the errors due to the effect of doubly CKM-suppressed decays on the tag side by varying the value of the rate of such decays and the strong and weak phase within conservative limits [20].

We add these contributions in quadrature to obtain the total systematic uncertainty. The summary is reported in Table 2. The largest contributions are related to the knowledge of the PDF parameters and the $C P$ content of the $B \bar{B}$ background.

## 8 Conclusions

In summary, we have measured the time-dependent $C P$-violating asymmetries for the decay $B^{0} \rightarrow$ $K_{S}^{0} K_{S}^{0} K_{S}^{0}: S_{f}=-0.66 \pm 0.26 \pm 0.08$ and $C_{f}=-0.14 \pm 0.22 \pm 0.05$. Within the current experimental uncertainties, these measurements are in good agreement with the SM expectation.

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|  | $\Delta S(+)$ | $\Delta S(-)$ | $\Delta C(+)$ | $\Delta C(-)$ |
| :---: | :---: | :---: | :---: | :---: |
| $(+-)$ pdf parameters | 0.007 | 0.010 | 0.020 | 0.018 |
| $(00)$ pdf parameters | 0.025 | 0.022 | 0.027 | 0.022 |
| $B \bar{B} C P$ | 0.077 | 0.077 | 0.026 | 0.026 |
| data RF | 0.004 | 0.006 | 0.008 | 0.006 |
| flavor tagging | 0.007 | 0.010 | 0.017 | 0.012 |
| $\tau_{B}$ and $\Delta m_{d}$ | 0.004 | 0.003 | 0.005 | 0.009 |
| SVT alignment | 0.016 | 0.016 | 0.008 | 0.008 |
| vertexing method | 0.016 | 0.016 | 0.003 | 0.003 |
| beam-spot | 0.004 | 0.004 | 0.001 | 0.001 |
| fit correlations | 0.004 | 0.004 | 0.025 | 0.025 |
| tag side interference | 0.001 | 0.001 | 0.011 | 0.011 |
| total errors | 0.085 | 0.085 | 0.055 | 0.051 |

Table 2: Summary of systematic uncertainties on S and C.

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