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NEW PERSPECTIVES FOR QCD FROM AdS/CFT*

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Abstract

The AdS/CFT correspondence between conformal field theory and string states in an extended space-time has provided new insights into not only hadron spectra, but also their light-front wavefunctions. We show that there is an exact correspondence between the fifth-dimensional coordinate of anti-de Sitter space z and a specific impact variable ζ which measures the separation of the constituents within the hadron in ordinary space-time. This connection allows one to predict the form of the light-front wavefunctions of mesons and baryons, the fundamental entities which encode hadron properties and scattering amplitudes. A new relativistic Schrödinger light-cone equation is found which reproduces the results obtained using the fifth-dimensional theory.

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1 The Conformal Approximation to QCD

One of the most interesting recent developments in hadron physics has been the use of Anti-de Sitter space holographic methods in order to obtain a first approximation to nonperturbative QCD. The essential principle underlying the AdS/CFT approach to conformal gauge theories is the isomorphism of the group of Poincare' and conformal transformations $SO(4,2)$ to the group of isometries of Anti-de Sitter space. The AdS metric is

$$ds^2 = \frac{R^2}{z^2}(\eta^{\mu\nu} dx_\mu dx^\mu - dz^2)$$

which is invariant under scale changes of the coordinate in the fifth dimension $z \to \lambda z$ and $dx_\mu \to \lambda dx_\mu$. Thus one can match scale transformations of the theory in $3+1$ physical space-time to scale transformations in the fifth dimension z. The amplitude $\phi(z)$ represents the extension of the hadron into the fifth dimension. The behavior of $\phi(z) \to z^\Delta$ at $z \to 0$ must match the twist dimension of the hadron at short distances $x^2 \to 0$. As shown by Polchinski and Strassler[1], one can simulate confinement by imposing the condition $\phi(z = z_0 = \frac{1}{\Lambda_{QCD}})$. This approach, has been successful in reproducing general properties of scattering processes of QCD bound states[1, 2], the low-lying hadron spectra[3, 4], hadron couplings and chiral symmetry breaking[4, 5], quark potentials in confining backgrounds[6] and pomeron physics[7].

It was originally believed that the AdS/CFT mathematical tool could only be applicable to strictly conformal theories such as $\mathcal{N} = 4$ supersymmetry. However, if one considers a semi-classical approximation to QCD with massless quarks and without particle creation or absorption, then the resulting β function is zero, the coupling is constant, and the approximate theory is scale and conformal invariant. Conformal symmetry is of course broken in physical QCD; nevertheless, one can use conformal symmetry as a *template*, systematically correcting for its nonzero β function as well as higher-twist effects. For example, "commensurate scale relations"[8] which relate QCD observables to each other, such as the generalized Crewther relation[9], have no renormalization scale or scheme ambiguity and retain a convergent perturbative structure which reflects the underlying conformal symmetry of the classical theory. In general, the scale is set such that one has the correct analytic behavior at the heavy particle thresholds[10].

In a confining theory where gluons have an effective mass, all vacuum polarization corrections to the gluon self-energy decouple at long wavelength. Theoretical[11] and phenomenological[12] evidence is in fact accumulating that QCD couplings based on physical observables such as τ decay[13] become constant at small virtuality; *i.e.*, effective charges develop an infrared fixed point in contradiction to the usual assumption of singular growth in the infrared. The near-constant behavior of effective couplings also suggests that QCD can be approximated as a conformal theory even at relatively small momentum transfer. The importance of using an analytic effective charge[14] such as the pinch scheme[15, 16] for unifying the electroweak and strong couplings and forces is also important[17]. Thus conformal symmetry is a useful first approximant even for physical QCD.

2 Hadronic Spectra in AdS/QCD

Guy de Teramond and I[18, 3] have recently shown how a holographic model based on truncated AdS space can be used to obtain the hadronic spectrum of light quark $q\bar{q}, qqq$ and gg bound states. Specific hadrons are identified by the correspondence of the amplitude in the fifth dimension with the twist dimension of the interpolating operator for the hadron's valence Fock state, including its orbital angular momentum excitations. An interesting aspect of our approach is to show that the mass parameter μR which appears in the string theory in the fifth dimension is quantized, and that it appears as a Casimir constant governing the orbital angular momentum of the hadronic constituents analogous to $L(L+1)$ in the radial Schrödinger equation.

As an example, the set of three-quark baryons with spin 1/2 and higher is described in AdS/CFT by the Dirac equation in the fifth dimension[18]

$$\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] \psi_\pm(z) = 0. \tag{1}$$

The constants $\mathcal{L}_+ = L+1$, $\mathcal{L}_- = L+2$ in this equation are Casimir constants which are determined to match the twist dimension of the solutions with arbitrary relative orbital angular momentum. The solution is

$$\Psi(x,z) = Ce^{-iP\cdot x}\left[\psi(z)_+\; u_+(P) + \psi(z)_-\; u_-(P)\right], \tag{2}$$

with $\psi_+(z) = z^2 J_{1+L}(z\mathcal{M})$ and $\psi_-(z) = z^2 J_{2+L}(z\mathcal{M})$. The physical string solutions have plane waves and chiral spinors $u(P)_\pm$ along the Poincaré coordinates and hadronic invariant mass states given by $P_\mu P^\mu = \mathcal{M}^2$. A discrete four-dimensional spectrum follows when we impose the boundary condition $\psi_\pm(z = 1/\Lambda_{\rm QCD}) = 0$: $\mathcal{M}^+_{\alpha,k} = \beta_{\alpha,k}\Lambda_{\rm QCD}$, $\mathcal{M}^-_{\alpha,k} = \beta_{\alpha+1,k}\Lambda_{\rm QCD}$, with a scale-independent mass ratio[3]. Figure 1(a) shows the predicted orbital spectrum of the nucleon states and Fig. 1(b) the Δ orbital resonances. The spin 3/2 trajectories are determined from the corresponding Rarita-Schwinger equation. The data for the baryon spectra are from S. Eidelman *et al.*[19] The internal parity of states is determined from the SU(6) spin-flavor symmetry. Since only one parameter, the QCD mass scale Λ_{QCD}, is introduced, the agreement with the pattern of physical states is remarkable. In particular, the ratio of Δ to nucleon trajectories is determined by the ratio of zeros of Bessel functions. The predicted mass spectrum in the truncated space model is linear $M \propto L$ at high orbital angular momentum, in contrast to the quadratic dependence $M^2 \propto L$ in the usual Regge parametrization.

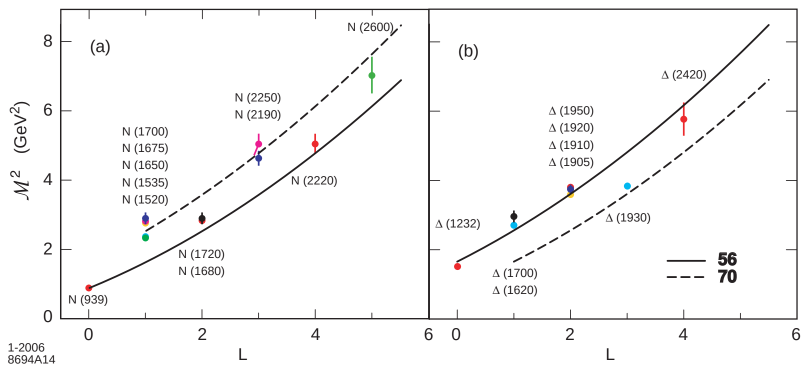

Figure 1: Predictions for the light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

3 Hadron Wavefunctions in AdS/QCD

One of the important tools in atomic physics is the Schrödinger wavefunction; it provides a quantum mechanical description of the position and spin coordinates of nonrelativistic bound states at a given time t. Similarly, it is an important goal in hadron and nuclear physics to determine the wavefunctions of hadrons in terms of their fundamental quark and gluon constituents. The dynamics of higher Fock states such as the $|uudq\overline{Q}\rangle$ fluctuation of the proton is nontrivial, leading to asymmetric $s(x)$ and $\overline{s}(x)$ distributions, $\overline{u}(x) \neq \overline{d}(x)$, and intrinsic heavy quarks $c\overline{c}$ and $b\overline{b}$ which have their support at high momentum[20] . Color adds an extra element of complexity: for example there are five-different color singlet combinations of six 3_C quark representations which appear in the deuteron's valence wavefunction, leading to the hidden color phenomena[21].

An important example of the utility of light-front wavefunctions in hadron physics is the computation of polarization effects such as the single-spin azimuthal asymmetries in semi-inclusive deep inelastic scattering, representing the correlation of the spin of the proton target and the virtual photon to hadron production plane: $\vec{S}_p \cdot \vec{q} \times \vec{p}_H$. Such asymmetries are time-reversal odd, but they can arise in QCD through phase differences in different spin amplitudes. In fact, final-state interactions from gluon exchange between the outgoing quarks and the target spectator system lead to single-spin asymmetries in semi-inclusive deep inelastic lepton-proton scattering which are not power-law suppressed at large photon virtuality Q^2 at fixed x_{bj}.[22] In contrast to the SSAs arising from transversity and the Collins fragmentation function, the fragmentation of the quark into hadrons is not necessary; one predicts a correlation with the production plane of the quark jet itself. Physically, the final-state interaction phase arises as the infrared-finite difference of QCD Coulomb phases for hadron wave functions with differing orbital angular momentum. The same proton matrix element which determines the spin-orbit correlation $\vec{S} \cdot \vec{L}$ also produces the anomalous magnetic moment of the proton, the Pauli form factor, and the generalized parton distribution E which is measured in deeply virtual Compton scattering. Thus the contribution of each quark current to the SSA is proportional to the contribution $\kappa_{q/p}$ of that quark to the proton target's anomalous magnetic moment $\kappa_p = \sum_q e_q \kappa_{q/p}$.[22, 23] The HERMES collaboration has recently measured the SSA in pion electroproduction using transverse target polarization.[24] The Sivers and Collins effects can be separated using planar correlations; both contributions are observed to contribute, with values not in disagreement with theory expectations.[24, 25]

We have recently shown that the amplitude $\Phi(z)$ describing the hadronic state in AdS_5 can be precisely mapped to the light-front wavefunctions $\psi_{n/h}$ of hadrons in physical space-time[18], thus providing a relativistic description of hadrons in QCD at the amplitude level. The light-front wavefunctions are relativistic and frame-independent generalizations of the familiar Schrödinger wavefunctions of atomic physics, but they are determined at fixed light-cone time $\tau = t + z/c$—the "front form" advocated by Dirac—rather than at fixed ordinary time t.

Formally, the light-front expansion is constructed by quantizing QCD at fixed light-cone time [26] $\tau = t + z/c$ and forming the invariant light-front Hamiltonian: $H^{QCD}_{LF} = P^+P^- - \vec{P}^2_\perp$ where $P^\pm = P^0 \pm P^z$.[27] The momentum generators P^+ and $\vec{P}_\perp$ are kinematical; *i.e.*, they are independent of the interactions. The generator $P^- = i\frac{d}{d\tau}$ generates light-cone time translations, and the eigen-spectrum of the Lorentz scalar H^{QCD}_{LF} gives the mass spectrum of the color-singlet hadron states in QCD together with their respective light-front wavefunctions. For example, the proton state satisfies: $H^{QCD}_{LF} \mid \psi_p\rangle = M^2_p \mid \psi_p\rangle$.

Our approach shows that there is an exact correspondence between the fifth-dimensional coordinate of anti-de Sitter space z and a specific impact variable ζ in the light-front formalism which measures the separation of the constituents within the hadron in ordinary space-time. We derived this correspondence by noticing that the mapping of $z \to \zeta$ analytically transforms the expression for the form factors in AdS/CFT to the exact Drell-Yan-West expression in terms of light-front wavefunctions. In the case of a two-parton constituent bound state the correspondence between the string amplitude $\Phi(z)$ and the light-front wave function $\widetilde{\psi}(x, \mathbf{b})$ is expressed in closed form[18]

$$\left|\widetilde{\psi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x)\, e^{3A(\zeta)}\, \frac{|\Phi(\zeta)|^2}{\zeta^4}, \tag{3}$$

where $\zeta^2 = x(1-x)\mathbf{b}^2_\perp$. Here $b_\perp$ is the impact separation and Fourier conjugate to $k_\perp$. The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{\rm QCD}$, represents the invariant separation between point-like constituents, and it is also the holographic variable z in AdS; *i.e.*, we can identify $\zeta = z$. The prediction for the meson light-front wavefunction is shown in Fig. 2. We can also transform the equation of motion in the fifth dimension using the z to ζ

mapping to obtain an effective two-particle light-front radial equation

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \tag{4}$$

with the effective potential $V(\zeta) \to -(1-4L^2)/4\zeta^2$ in the conformal limit. The solution to (4) is $\phi(z) = z^{-\frac{3}{2}}\Phi(z) = Cz^{\frac{1}{2}} J_L(z\mathcal{M})$. This equation reproduces the AdS/CFT solutions. The lowest stable state is determined by the Breitenlohner-Freedman bound[28] and its eigenvalues by the boundary conditions at $\phi(z = 1/\Lambda_{\rm QCD}) = 0$ and given in terms of the roots of the Bessel functions: $\mathcal{M}_{L,k} = \beta_{L,k}\Lambda_{\rm QCD}$. Normalized LFWFs follow from (3)

$$\widetilde{\psi}_{L,k}(x,\zeta) = B_{L,k}\sqrt{x(1-x)}J_L\left(\zeta\beta_{L,k}\Lambda_{\rm QCD}\right)\theta(z \le \Lambda^{-1}_{\rm QCD}), \tag{5}$$

where $B_{L,k} = \pi^{-\frac{1}{2}}\Lambda_{\rm QCD}\ J_{1+L}(\beta_{L,k})$. The resulting wavefunctions (see: Fig. 2) display confinement at large inter-quark separation and conformal symmetry at short distances, reproducing dimensional counting rules for hard exclusive processes and the scaling and conformal properties of the LFWFs at high relative momenta in agreement with perturbative QCD. The hadron form factors can be predicted from overlap

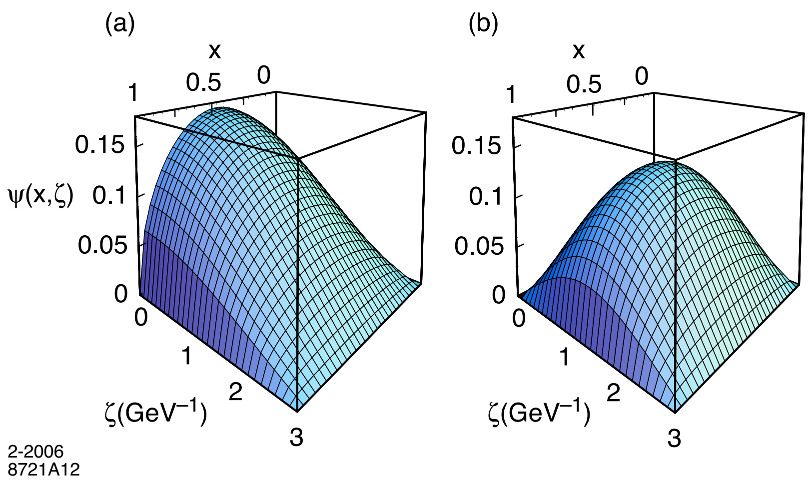

Figure 2: AdS/QCD Predictions for the light-front wavefunctions of a meson.

integrals in AdS space or equivalently by using the Drell-Yan West formula in physical space-time. The prediction for the pion form factor is shown in Fig. 3.

Since they are complete and orthonormal, these AdS/CFT model wavefunctions can be used as an initial ansatz for a variational treatment or as a basis for the diagonalization of the light-front QCD Hamiltonian. We are now in fact investigating this possibility with J. Vary and A. Harindranath. The wavefunctions predicted by AdS/QCD have many phenomenological applications ranging from exclusive B and D decays, deeply virtual Compton scattering and exclusive reactions such as form

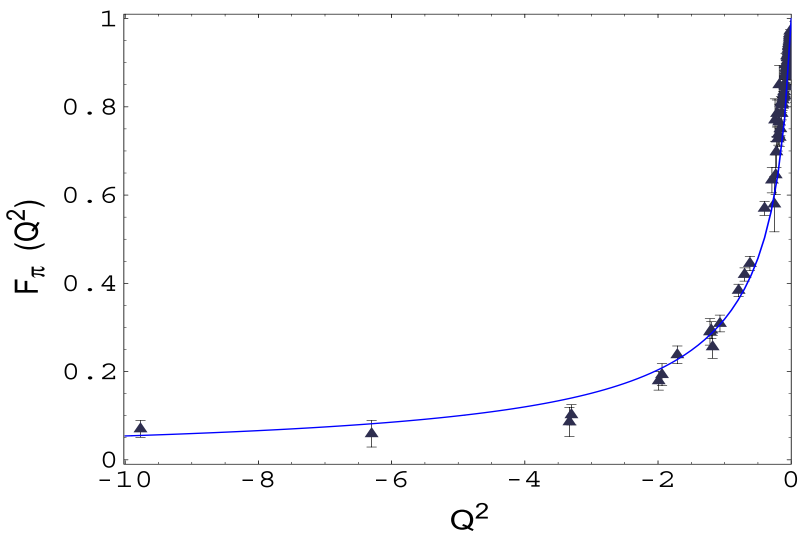

Figure 3: AdS/QCD Predictions for the pion form factor.

factors, two-photon processes, and two body scattering. A connection between the theories and tools used in string theory and the fundamental constituents of matter, quarks and gluons, has thus been found.

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