SLAC-PUB-11958
BABAR-PROC-06/062
hep-ex/0607031
July 18, 2006

Measurement of the CKM Matrix Elements $|V_{cb}|$ and $|V_{ub}|$ at the B-factories

W. Menges
Department of Physics, Queen Mary, University of London
Mile End Road, London, E1 4NS, UK
(from the *BABAR* Collaboration)

Abstract

Recent results on inclusive and exclusive semileptonic B decays from B-factories are presented. The impact of these measurements on the determination of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ is discussed.

Contributed to the Proceedings of the
14^{th} International Workshop On Deep Inelastic Scattering (DIS 2006),
20-24 Apr 2006, Tsukuba, Japan

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

Work supported in part by Department of Energy contract DE-AC02-76SF00515

1 Introduction

Semileptonic B decays provide direct access to the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$, whose ratio measures the side of the Unitarity Triangle opposite the angle β. Semileptonic B decays also probe the structure of B mesons. Inclusive decays are sensitive to quantities such as the mass and momentum distribution of the b quark inside the B meson, whereas exclusive decays depend on form factors for specific final states. The theory is quite advanced and the measurements can be optimised to minimise the dominant theoretical uncertainties entering in the determination of $|V_{ub}|$ and $|V_{cb}|$.

2 Semileptonic Decays with Charm

The rate for inclusive $\overline{B} \to X_c \ell \bar{\nu}$ decays can be calculated in terms of the Heavy Quark Expansion (HQE), which is in powers of α_s and $1/m_b$:

$$\Gamma_{sl} = \frac{G_F m_b^5}{192\pi^3} |V_{cb}|^2 (1 + A_{EW}) A_{pert}(\alpha_s) A_{non-pert}(1/m_b, 1/m_c, a_i), \tag{1}$$

where A_{EW} and A_{pert} represent electroweak and QCD perturbative corrections, respectively. The non-perturbative QCD part, $A_{non-pert}$, is expanded in terms of the heavy quark masses (m_b, m_c), with a_i as coefficients. Buchmüller and Flächer[1] combined the measurements of moments of lepton energy (E_ℓ) and hadronic final state invariant mass (m_X) spectra from various experiments. In addition they also included moments of photon energy spectrum from radiative $b \to s\gamma$ decays to increase the sensitivity to some parameters such as m_b. A global fit to heavy-quark parameters gives $|V_{cb}| = (41.96 \pm 0.23 \pm 0.35 \pm 0.59) \cdot 10^{-3}$ and $m_b = (4.59 \pm 0.04)\,\text{GeV}$.

The measurement of exclusive $\overline{B} \to X_c \ell \bar{\nu}$ decays is a good cross check of the inclusive measurements and an important measurement by itself. In this case, the main theoretical uncertainty comes from the assumptions about the form factors. The technique of determining $|V_{cb}|$ by using $\overline{B} \to D^* \ell \bar{\nu}$ decays is well established. The differential distribution can be written in terms of w, the D^* boost in the B rest frame, as

$$\frac{d\Gamma(\overline{B} \to D^* \ell \bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} \mathcal{F}(w)^2 \mathcal{G}(w) \tag{2}$$

where $\mathcal{G}(w)$ is a phase space factor and $\mathcal{F}(w)$ is a form factor (FF). $\mathcal{F}(1) = 1$ in the heavy quark limit, but lattice QCD[2] can be used to compute effects due to finite quark masses, leading to $\mathcal{F}(1) = 0.919^{+0.030}_{-0.035}$. The shape of $\mathcal{F}(w)$ cannot be predicted by theory, and is parameterised in terms of a slope ρ^2 and FF ratios R_1 and R_2, independent of w. The helicity amplitudes entering in the $\overline{B} \to D^* \ell \bar{\nu}$ decay are functions of the above parameters. These amplitudes can be determined by fitting the fully differential rate of $\overline{B} \to D^* \ell \bar{\nu}$ decays as a function of w and three angles. *BABAR* [3] measured the FF parameters following this approach, giving $R_1 = 1.396 \pm 0.074$, $R_2 = 0.885 \pm 0.048$ and $\rho^2 = 1.145 \pm 0.075$, where the errors are a factor 5 better than in previous determinations[5]. Using the improved values of R_1 and R_2 the *BABAR* exclusive $|V_{cb}|$ measurement[6] gives $|V_{cb}| = (37.6 \pm 0.3_{stat} \pm 1.3_{syst} \pm 1.4_{theory}) \cdot 10^{-3}$.

3 Charmless Semileptonic Decays

$|V_{ub}|$ is related to the full rate of inclusive $\overline{B} \to X_u \ell \bar{\nu}$ decays by an expression equivalent to Eq. 1, giving a theory uncertainty of $\sim 5\%$. In practise, the accessible rate is much reduced and the theoretical uncertainty increases considerably, since the overwhelming background from $\overline{B} \to X_c \ell \bar{\nu}$ decays must be suppressed by stringent kinematic requirements. These cuts are all based on the mass difference between u and c quark. The distributions of E_ℓ and q^2, the squared invariant mass of the lepton pair, extends to higher values for signal, whereas the m_X spectrum and the light-cone momentum $P_+ = E_X - |\vec{p}_X|$ are concentrated at lower values. Therefore regions of the phase space can be selected where the signal over background ratio is adequate. However, in some regions HPE breaks down and a so-called shape function is needed to resum non-perturbative contributions. It depends on m_b and heavy quark parameters, and most of the theoretical uncertainty in inclusive $|V_{ub}|$ determinations is due to our imperfect knowledge of them. $|V_{ub}|$ is determined from the measurement of the charmless semileptonic partial branching fraction, $\Delta\mathcal{B}(\overline{B} \to X_u \ell \bar{\nu})$, the B meson lifetime, τ_b, and the rate $\zeta(\Delta\Phi)$, which is predicted by theory (BLNP[7], DGE[8]) and depends on the phase space region, $\Delta\Phi$, defined by kinematic cuts:

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(\overline{B} \to X_u \ell \bar{\nu})}{\tau_b \cdot \zeta(\Delta\Phi)}}. \quad (3)$$

The B-factories have studied several kinematic variables to measure $|V_{ub}|$. The endpoint of the lepton energy spectrum for charmless decays is well above the one for charm decays ($E_\ell > 2.3\,\mathrm{GeV}$). The good knowledge of the charm background allows to push this cut below the charm threshold, thereby increasing the acceptance and decreasing theory uncertainty. The results[9, 10, 11] are summarised in Tab. 1. Reconstructing unambiguously other variables involving either the neutrino or the X system is experimentally challenging and requires more knowledge of the whole event. This can be achieved by reconstructing one B in a pure hadronic mode and studying the recoiling B, whose momentum and flavour are then known. This technique provides signal over background ratios of about one or higher, at the expense of a very small signal efficiency ($\mathcal{O}(10^{-3})$). Belle has measured $|V_{ub}|$ for three different combinations of kinematical variables, shown in Tab. 1. The *BABAR* result for m_x-q^2 agrees within errors with the Belle measurement. The latest average from HFAG[12] using BLNP[7] gives $|V_{ub}| = (4.45 \pm 0.20 \pm 0.26) \cdot 10^{-3}$. Using the alternative approach by DGE[8] gives $|V_{ub}| = (4.41 \pm 0.20 \pm 0.20) \cdot 10^{-3}$. Both results agree very well and the total uncertainty on $|V_{ub}|$ is 7.4%.

The differential rate for exclusive $\overline{B} \to X_u \ell \bar{\nu}$ decays in terms of q^2 is proportional to $|V_{ub}|^2 F(q^2)^2$. In the simple case of $B \to \pi \ell \nu$ and massless leptons only one FF is needed. The absolute value of this FF is predicted by several theoretical frameworks (light-cone sum rules (LCSR)[15], lattice QCD (LQCD)[16, 17], and quark models); the dependence on q^2 can be checked experimentally, thereby allowing to discriminate different theoretical calculations. The *BABAR* measurement[18] agrees with LCSR and LQCD but disfavours the quark model ISGWII[19]. The total branching ratio is $(1.38{\pm}0.10{\pm}0.16{\pm}0.08){\cdot}10^{-4}$, where the errors are statistical, systematic, and due to FF shape uncertainties. Using LQCD[17], this translates into $|V_{ub}| = (3.82 \pm 0.14 \pm 0.22 \pm 0.11^{+0.88}_{-0.52}) \cdot 10^{-3}$, where the fourth error reflects the uncertainty of the FF normalisation. Belle performed a similar measurement[20] using $\overline{B} \to D^* \ell \bar{\nu}$ tagged events giving consistent results.

4 Summary

Inclusive measurements of $|V_{cb}|$ give a precision of 2% dominated by HQE theory uncertainties. Inclusive measurements of $|V_{ub}|$ have reached a precision of 7% dominated by HQE parameters. The exclusive measurements provide important cross checks and give consistent results within the still large FF uncertainties.

accepted region	f_u	$\Delta\mathcal{B}[10^{-4}]$	$\lvert V_{ub}\rvert[10^{-3}]$
BABAR ($E_e > 2.0$ GeV)[9]	0.26	$5.3 \pm 0.3 \pm 0.5$	$4.41 \pm 0.29 \pm 0.31$
BELLE ($E_e > 1.9$ GeV)[10]	0.34	$8.5 \pm 0.4 \pm 1.5$	$4.82 \pm 0.45 \pm 0.30$
CLEO ($E_e > 2.1$ GeV)[11]	0.19	$3.3 \pm 0.2 \pm 0.7$	$4.09 \pm 0.48 \pm 0.36$
BABAR ($m_X < 1.7$ GeV, $q^2 > 8$ GeV^2)[13]	0.34	$8.7 \pm 0.9 \pm 0.9$	$4.75 \pm 0.35 \pm 0.32$
BELLE ($m_X < 1.7$ GeV, $q^2 > 8$ GeV^2)[14]	0.34	$8.4 \pm 0.8 \pm 1.0$	$4.68 \pm 0.37 \pm 0.32$
BELLE ($P_+ < 0.66$ GeV)[14]	0.57	$11.0 \pm 1.0 \pm 1.6$	$4.14 \pm 0.35 \pm 0.29$
BELLE ($m_X < 1.7$ GeV)[14]	0.66	$12.4 \pm 1.1 \pm 1.2$	$4.06 \pm 0.27 \pm 0.24$

Table 1: Measurements of partial branching fractions $\Delta\mathcal{B}$ for inclusive $\overline{B} \to X_u \ell \bar{\nu}$ decays and $|V_{ub}|$, adjusted by HFAG to common input parameters. f_u is the space phase acceptance. The errors on $|V_{ub}|$ refer to experimental and theoretical uncertainties, respectively.

References

[1] O. Buchmüller and H. Flächer, *Phys. Rev.* D **73**, 073008 (2006).

[2] S. Hashimoto *et al.* , *Phys. Rev.* D **66**, 014503 (2002).

[3] B. Aubert *et al.* (BABAR Collab.), hep-ex/0602023.

[4] I. Caprini, L. Lellouch, and M. Neubert, *Nucl. Phys.* B **530**, 153 (1998).

[5] J.E. Duboscq *et al.* (CLEO Collab.), *Phys. Rev. Lett.* **76**, 3898 (1996).

[6] B. Aubert *et al.* (BABAR Collab.), *Phys. Rev.* D-RC **71**, 051502 (2005).

[7] B.O. Lange, M. Neubert, G. Paz, *Phys. Rev.* D **72**, 073006 (2005).

[8] J.R. Andersen and E. Gardi, *JHEP* **0601**, 097 (2006).

[9] B. Aubert *et al.* (BABAR Collab.), *Phys. Rev.* D **73**, 012006 (2006).

[10] A. Limosani *et al.* (BELLE Collab.) *Phys. Lett.* B **621**, 28 (2005).

[11] A. Bornheim *et al.* (CLEO Collab.), *Phys. Rev. Lett.* **88**, 231803 (2002).

[12] E. Barberio *et al.* (Heavy Flavor Averaging Group), hep-ex/0603003.

[13] B. Aubert *et al.* (BABAR Collab.), hep-ex/0507017.

[14] I. Bizjak *et al.* (BELLE Collab.), *Phys. Rev. Lett.* **95**, 241801 (2005).

[15] P. Ball and R. Zwicky, *Phys. Rev.* D **71**, 014015 (2005).

[16] J. Shigemitsu *et al.*, *Nucl. Phys. Proc. Suppl.* **140**, 464 (2005).

[17] M. Okamoto *et al.*, *Nucl. Phys. Proc. Suppl.* **140**, 461 (2005).

[18] B. Aubert *et al.* (BABAR Collab.), *Phys. Rev.* D **72**, 051102 (2005).

[19] D. Scora and N. Isgur, *Phys. Rev.* D **52**, 2783 (1995).

[20] K. Abe *et al.* (BELLE Collab.), hep-ex/0508018.