

# Cosmological Constant and Axions in String Theory

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String theory axions appear to be promising candidates for explaining cosmological constant via quintessence. In this paper, we study conditions on the string compactifications under which axion quintessence can happen. For sufficiently large number of axions, cosmological constant can be accounted for as the potential energy of axions that have not yet relaxed to their minima. In compactifications that incorporate unified models of particle physics, the height of the axion potential can naturally fall close to the observed value of cosmological constant.

## 1. Introduction

Ever since the observation of nonzero cosmological constant

$$\Lambda \sim e^{-280} M_P^4, \tag{1.1}$$

where  $M_P = 1/\sqrt{8\pi G_N} = 2.4 \times 10^{18} \text{GeV}$ , it has been a puzzle why the cosmological constant is so small compared to the Planck scale and yet nonzero. Theoretically, one would expect the cosmological constant to be around the Planck scale. It has been argued that even if we were able to set the cosmological constant to its present value at low orders in perturbation theory, higher order radiative corrections generate cosmological constant of the order of the Planck scale  $\delta\Lambda \sim M_P^4$ . This would give cosmological constant which is 120 orders of magnitude larger than the observed value.

In one approach to this problem, the cosmological constant is attributed to the vacuum energy of a scalar field, called quintessence, that has not yet relaxed to its vacuum [3,4]. The vacuum is assumed to have zero or vanishingly small cosmological constant due to an unknown physical principle. To account for the cosmological constant this way, it is necessary to explain why the scalar potential takes values close to the measured cosmological constant and why the scalar field is lighter than the Hubble scale  $H \sim e^{-140} M_P$ , so that it contributes to dark energy rather than dark matter.

Such light scalars are expected to have Planck suppressed couplings to matter fields that would lead to observable long-range forces and to variation of constants of nature [5]. Stringent tests of equivalence principle and of variation of constants of nature constrain the scalar couplings to matter to anomalously small values, imposing fine-tuning on the quintessence models. Furthermore, even if these bounds could be evaded, additional fine-tuning is necessary since radiative corrections are expected to spoil the flatness of the scalar potential and to generate large mass for the scalar field [6].

To evade these problems, it has been suggested to consider pseudoscalar fields called axions. These are protected by global symmetry  $a \rightarrow a + c$  where  $a$  is the axion and  $c$  is an arbitrary constant. The shift symmetry suppresses the couplings of axion to matter which relaxes the above observational constraints. Effects that break the shift symmetry generate axion mass and potential. In string theory, the shift symmetry is broken only on nonperturbative level. The nonperturbative effects that break the shift symmetry are exponentially suppressed and can naturally lead to potentials many orders of magnitude below Planckian energy densities.

In string compactifications that incorporate unified models of particle physics, matching to the observed value of gauge couplings and the unification scale leads to further constraints. These fix some parameters of the compactification and hence allow us to estimate the scale of the instanton generated axion potential. We perform this estimate and find that in some string compactifications, such as of heterotic M-theory, the potential comes out roughly in the range preferred for quintessence. In others, such as weakly coupled heterotic string, the potential comes out too large, hence disfavoring axion quintessence. Thus axion quintessence may relate the hierarchy between cosmological constant and the Planck scale to the hierarchy between the unification scale and the Planck scale and can differentiate between different compactifications.

In summary, string theory axions can both be very light so that they contribute to dark energy and can evade the observational bounds coming from long range forces and the variation of constants of nature [7]. Besides getting the correct value of cosmological constant, one also has to make sure that the axion is slow rolling down its potential so that it acts like dark energy rather than like dark matter. As we will discuss below, this puts further constraints on parameters of the axion in addition to the above requirement that the axion has potential energy comparable to the cosmological constant.

In this paper, we study whether these conditions are met by string theory axions. We find that characteristics of string theory axions are uniform across different compactifications. In the case of one axion, the axion coupling parameter  $F_a$  has to be comparable or larger than the Planck mass  $M_P$ . We will argue that in string theory, there is an upper bound on  $F_a$ , which for very light axions restricts  $F_a$  below the Planck scale. Hence, in string theory a single axion quintessence does not seem to be natural. Quintessence could be achieved only by fine-tuning the initial conditions of the axion to make it sit near the top of its potential for a long time to simulate cosmological constant. This gives an illustration of a situation, in which a mechanism that is consistent as an effective field theory does not seem to work in string theory [9,10,8].

Although single axion quintessence seems to be excluded in string theory, quintessence with many axions is possible in some string compactifications. Whether this happens depends on the axion parameters and on the number of axions that one gets in a string compactification. In the following, we will determine these conditions. We find that with at least roughly  $10^{4-5}$  axions, the quintessence could happen without fine-tuning the initial conditions of the axions and thus explain why the cosmological constant is nonzero.

While  $10^{4-5}$  axions may seem as a large number, compactifications with such number of axions are known. To assess whether this many axions could lead to problems, such as the species problem, we estimate the radiative corrections to the Planck mass due to the large number of light axions. Requiring that the tree level contribution to Planck mass dominates the loop contributions leads to an upper bound on the number of axions and hence an upper bound on the number of Hubble times of cosmic acceleration. We perform this estimate and find that the axion quintessence is well in the range allowed by the species problem.

Many axions have been previously discussed in the related problem of achieving inflation in string theory in [11]. The many axion quintessence as an effective field theory was studied in [12]. In a sense, some of the focus of the present paper is to determine whether this effective field theory works when embedded into string theory. The physics of many axion quintessence and inflation is the same, although the two mechanisms happen in a different regions of axion parameter space. Hence, it could be that some of the string theory axions have driven inflation while others are currently responsible for cosmological constant. For recent proposals of other dynamical mechanisms to solve the cosmological constant problem, see [13,14].

This paper is organized as follows. In section 2, we discuss the generation of axion potential in string theory and the conditions under which this potential is comparable to the cosmological constant. In section 3, which is the main part of the paper, we discuss the cosmology of the axion quintessence, determining the conditions under which it can be realized in string theory. In this section we borrow results for string theory axions that are derived for various string compactifications in sections 4-7. In these sections we also estimate the height of the axion potential and give an upper bound on the duration of axion quintessence from consideration of radiative corrections.

## 2. Very Light Axions in String Theory

In string theory there are fields with naturally very flat potential. These are pseudoscalar fields, called axions, that have the shift symmetry  $a \rightarrow a + c$ . If this symmetry was exact, it would set the potential to be independent of  $a$  and render the axion massless. Effects that break this symmetry generate potential for  $a$ . In string theory, the shift symmetry is exact to all orders in perturbation theory so the axions receive potential only from nonperturbative instanton effects. These effects generate potentials that are exponentially

suppressed by the instanton action. Hence, if the instantons have large actions, they can give rise to a potential many orders of magnitude below the Planck scale. In this section, we will estimate how large the instanton actions have to be to lead to an axion potential with magnitude around the present value of cosmological constant.

The instantons break the shift symmetry down to a discrete shift symmetry  $a \rightarrow a + 2\pi n$  where  $n$  is arbitrary integer. They generate a superpotential

$$W = M^3 e^{-S_{inst} + ia}, \quad (2.1)$$

where  $S_{inst}$  is the instanton action and  $M$  is the scale of the instanton physics which could be around the Planck scale or the string scale. One can estimate the axion potential by substituting (2.1) into the formula for the potential of low-energy effective  $\mathcal{N} = 1$  supergravity

$$V = e^{\frac{K}{M_P^2}} \left( K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - \frac{3}{M_P^2} |W|^2 \right), \quad (2.2)$$

where  $K$  is the Kähler potential and  $W$  is the superpotential.

If we do not assume low energy supersymmetry breaking, then we estimate

$$V \sim M^4 e^{-S_{inst}} (1 - \cos(a)) + V_0, \quad (2.3)$$

where  $V_0$  represents other contributions to vacuum energy. Since, we assume that axions are the only very light scalars, all the other fields will have rolled down to their vacuum well before the dark energy has started to dominate the density of the universe. It follows that they contribute only a constant term to the potential, given by the sum of their vacuum energies. Hence, for our purposes  $V_0$  is a constant.

If we take  $M \sim M_P$ , the axion contribution to the vacuum energy is comparable to the present value of dark energy

$$\rho_{vac} \sim e^{-280} M_P^4, \quad (2.4)$$

if  $S_{inst} \sim 280$ .

Low energy supersymmetry breaking can further suppress the axion potential. To estimate the axion potential, we add to the superpotential a term  $W_0$  from the supersymmetry breaking sector

$$W = M^3 e^{-S_{inst} + ia} + W_0. \quad (2.5)$$

The axion potential gets a contribution  $V \sim M^2 \partial_i W_0 e^{-S_{inst} + ia} + c.c.$  from interference between the one-instanton term and the supersymmetry breaking term  $\partial_i W \sim m_S^2$ . Here  $m_S$  is the scale of supersymmetry breaking. Hence, the axion potential is

$$V \sim m_S^2 M^2 e^{-S_{inst}} (1 - \cos(a)) + V_0. \quad (2.6)$$

If supersymmetry is broken at low energy  $m_S \sim 1\text{TeV}$ , we get  $V \sim e^{-280} M_P^4$  for instantons that have actions  $S_{inst} \sim 200$ . In summary contribution of one axion to vacuum energy is comparable to present dark energy density if the instantons breaking the shift symmetry have actions in the range  $S_{inst} \sim 200 - 280$ .

With such such large instanton actions, the instanton effects that would stabilize the moduli that are scalar partners of the axions are negligible, so other effects are needed to stabilize them. These could be for example perturbative effects that leave the axions massless. A concrete example that uses tree level potential from RR and NS-NS fluxes to stabilize the moduli has been recently discussed by Kachru et al. [15] in the context of type IIA string. We should also point out that supersymmetry breaking is expected to give the moduli heavy masses. If the SUSY breaking mass of the moduli comes from perturbative effects that do not break the shift symmetry, the axions remain massless. Hence the stabilization of these moduli could be tied to supersymmetry breaking. For sufficiently large supersymmetry breaking scale, the moduli are heavy enough to evade various constrains coming from dark matter density, fifth force and other experiments. Some models where moduli are stabilized along these lines were recently discussed in [16,17].

### 3. Axions and the Vacuum Energy

As discussed in the introduction, observational constraints favor a pseudoscalar axion quintessence that is protected by shift symmetry. We will now study the cosmology of such axions, with emphasis on the conditions that lead to quintessence.

We describe the universe in the flat FRW metric

$$ds^2 = -dt^2 + R(t)^2(dx^2 + dy^2 + dz^2), \quad (3.1)$$

where  $R(t)$  is the expansion factor of the universe. The axion is a four-dimensional pseudoscalar field that is periodic with period  $2\pi$ . We assume that it has an instanton generated potential

$$V(a) = \mu^4 (1 - \cos(a)) + V_0, \quad (3.2)$$

where  $\mu$  parametrizes the scale of the axion potential and  $V_0$  represents other contributions to vacuum energy. As discussed below eq. (2.3), we take  $V_0$  to be a constant. The action of the axion in flat FRW universe described with the metric (3.1) is

$$S = \int d^4x R^3 \left( -\frac{F_a^2}{2} \partial_\mu a \partial^\mu a - \mu^4 (1 - \cos(a)) - V_0 \right), \quad (3.3)$$

where  $F_a$  is the axion decay constant. We note that (3.3) has nonstandard normalization of the kinetic term. Rescaling the axion  $a \rightarrow a/F_a$  restores canonical normalization and rescales the period of the axion to  $2\pi F_a$ .

We assume that the scalar field is homogeneous in space so that its expectation value depends only on time  $a = a(t)$ . Varying the action (3.3) with respect to  $a$  gives the equation of motion of the axion

$$\ddot{a} + 3H\dot{a} + \frac{\mu^4}{F_a^2} \sin(a) = 0, \quad (3.4)$$

where  $H = \dot{R}/R$  is the Hubble parameter. In a universe whose density is dominated by dark energy

$$H = \sqrt{\frac{\rho_{vac}}{3M_P^2}}. \quad (3.5)$$

The equation (3.4) describes a particle moving in one-dimensional potential  $V(a)$  with friction force  $-3H\dot{a}$ . If the initial conditions of the axions are generic, the situation is easy to summarize. For  $H \lesssim m_a$ , where  $m_a^2 = V''/F_a^2 = \mu^4/F_a^2$  is the mass of the axion particles, the axion is under-damped. The axion oscillates around the minimum of its potential. These oscillations describe a Bose-Einstein condensate of axion particles at zero momentum with mass  $m_a = \mu^2/F_a$ . In this case, the axions contribute to cold dark matter [18,1,2]. For  $H \gtrsim m_a$ , the axion is overdamped. Due to the Hubble friction, the axion is slowly rolling down its potential and contributes to dark energy density. Hence, the condition for an axion with generic initial conditions to contribute to dark energy instead of dark matter is

$$H \gtrsim \frac{\mu^2}{F_a}. \quad (3.6)$$

The axion contribution to dark energy is at most  $\mu^4$ . With the help of (3.5) and (3.6), this becomes

$$\delta\rho_{vac} \lesssim \mu^4 \lesssim \rho_{vac} \frac{F_a^2}{M_P^2}. \quad (3.7)$$

The most important point to notice about (3.7) is that the relative contribution of the axion to dark energy depends only on the ratio of the  $F_a$  and  $M_P$ . In particular, a single

axion that starts with generic initial conditions can account for all of dark energy only if it has Planck size axion decay constant. Hence, to find out whether an axion can account for all of dark energy, it is crucial to understand what are the possible values of  $F_a$  in string theory.

As we will demonstrate below in various string compactifications, there is an upper bound on the axion decay constant

$$F_a \lesssim \frac{xM_P}{S_{inst}}, \quad (3.8)$$

where  $x$  is of order one and  $S_{inst}$  is the action of the instantons that break the axionic shift symmetry and generate axion potential. It is interesting to notice that this bound depends only on  $M_P$  and  $S_{inst}$ . All the dependence on  $g_s, \ell_s$  and the type of string theory enters the formula only through  $M_P, S_{inst}$ . This simplicity suggests that the formula (3.8) is true in any four-dimensional string compactification<sup>1</sup>.

For moderately large  $S_{inst}$ , (3.7) together with (3.8) implies a strong bound on the contribution of one axion to the dark energy

$$\delta\rho_{vac} \lesssim \frac{\rho_{vac}}{S_{inst}^2}. \quad (3.9)$$

For  $S_{inst} \sim 200 - 280$  this gives  $\delta\rho_{vac} \lesssim \rho_{vac}/10^{4-5}$ . This bound originates from the fact that only axions whose potential is about  $10^{4-5}$  times smaller than the current vacuum energy are lighter than the Hubble scale and hence contribute to dark energy. These axions have instanton actions in the range

$$S_{inst} \sim 210 - 290. \quad (3.10)$$

This is slightly larger than the actions we estimated in previous section because of the extra suppression of the axion potential by a factor of  $10^{4-5}$  compared to the cosmological constant. Axions with larger potential are under-damped so they oscillate around the minimum of their potential. They contribute to dark matter density instead. These axions could potentially generate too much dark matter, thus over-closing the universe.

Hence, a single axion with generic initial conditions can account only for a fraction of the present dark energy density. However, string compactifications can have many axions

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<sup>1</sup> This bound was conjectured in [10] to hold in any quantum theory of gravity while this work was in gestation.



in their four-dimensional spectrum. Each of these axions, if it satisfies the condition (3.6), gives an additive contribution to the vacuum energy. For simplicity, let us assume that all axions have the same axion decay constant  $F_a \sim M_P/S_{inst}$ . If there are  $N$  axions, their contribution to vacuum energy adds up to

$$\delta\rho_{vac,N} \sim \rho_{vac} \frac{\sum_i^N F_{a,i}^2}{M_P^2} \lesssim \frac{N\rho_{vac}}{S_{inst}^2}. \quad (3.11)$$

So in compactifications that have

$$N \gtrsim S_{inst}^2 \quad (3.12)$$

axions, the cosmological constant could be entirely due to axion potential energy. In string compactifications we expect the instanton actions  $S_{inst}$  of different instantons to vary by a factor of order one. Hence only some of them will fall into the preferred range (3.10) and the actual number of axions necessary for quintessence is somewhat larger than the estimate (3.12). For an extensive discussion of this issue in the related context of axion inflation, see [19].

Known string theory Calabi-Yau three-fold compactifications have up to  $\sim 10^{3-4}$  axions in their four-dimensional spectrum. There are examples of F-theory compactified on Calabi-Yau four-folds that lead to  $\sim 10^{5-6}$  axions [20]. Hence, the quintessence with many axions could explain cosmological constant in some string compactifications.

If we assume that the entire vacuum energy is due to the axions, so that  $V_0 = 0$  in (3.2), the axion quintessence lasts only for a finite number of Hubble times. As an illustration of this, let us assume that all axions have the same potential (3.2). A simple calculation shows that the number of e-foldings grows as [11,19]

$$N_{e-fold} \sim \frac{\sum_i^N F_{a,i}^2}{M_P^2} \lesssim \frac{N}{S_{inst}^2}, \quad (3.13)$$

where we used the estimate (3.8) for  $F_{a,i}$ . So in compactifications with large number of axions, the axion quintessence could last for several Hubble times.

It is expected that  $F_a$ 's of different axions are not equal but rather differ by factors of order one. Let us discuss how this variation affects (3.13). The number of e-foldings of quintessence due to axions (3.13) depends on the average of the squares of axion decay constants, hence it is insensitive to the individual variation of  $F_a$ . We expect that the estimate (3.8) gives the average  $F_a$  correct up to a factor of order one. Hence we can trust the estimate (3.13) for the number of e-foldings up to a factor or order one.

In string compactifications with a small number of axions, (3.12) leads us to conclude that only a small fraction of vacuum energy is due to axions, if the axion initial conditions are generic. In these compactifications, the cosmological constant is due to the constant piece  $V_0$  of vacuum energy (2.3) that represents the energy of the “true vacuum” in which all scalars have already reached the minima of their potentials.

To close this section, let us return back to the heavier axions. Under generic initial conditions these axions contribute to dark matter. However, for special initial conditions the axions could contribute to dark energy density. If their initial conditions are fine-tuned so that the axions stay at the top of their potential for at least one Hubble time, they will behave like dark energy instead. Let us estimate the necessary fine-tuning. For simplicity, we assume that there is only one such heavy axion. The axion is slow-rolling on the flat portion of its potential near the top so that it contributes to dark energy. The equation of motion for the deviation  $\delta a = \pi - a$  of the axion from the top of its potential is (3.4)

$$3H\dot{a} + \frac{\mu^4}{F_a^2}\delta a = 0. \quad (3.14)$$

The axion stays close to the top,  $|\delta a| \lesssim 1$  for at least one Hubble time  $t_H \sim 1/H$  if the axion speed is less than the Hubble scale  $\dot{a} \lesssim H$ . Combining this with (3.5) and (3.14) gives a bound on the initial axion displacement

$$\delta a_{init} \lesssim \frac{F_a^2}{M_P^2} \frac{\rho_{vac}}{\mu^4} \sim \frac{1}{S_{inst}^2} \frac{\rho_{vac}}{\mu^4}. \quad (3.15)$$

If we assume that the entire cosmological constant is due to potential energy of a single axion then  $\rho_{vac} \sim \mu^4$  so the axion initial conditions have to be fine-tuned to one part in  $S_{inst}^2 \sim 10^{4-5}$

$$\delta a_{init} \lesssim \frac{1}{S_{inst}^2}. \quad (3.16)$$

In [12] it has been argued that the quantum fluctuations could perturb the axion away from the maximum of the potential, thus spoiling the fine-tuning of the initial conditions.

In summary, the cosmological constant could be due to axion quintessence if there are at least  $S_{inst}^2 \sim 10^{4-5}$  axions. In compactifications with fewer axions, the cosmological constant could still be due to axions, modulo issues with quantum fluctuations, if the initial conditions of say one of them are fine-tuned to one part in  $S_{inst}^2 \sim 10^{4-5}$ .

In the following four sections, we discuss the parameters of axions in different string compactifications. Among other results, we derive the upper bound (3.8) on the axion decay constant in these compactifications.

## 4. Heterotic String Theory

In heterotic string theory, the axions come from zero modes of the NS-NS  $B$ -field. The components of  $B$ -field polarized along the four noncompact dimensions are dual to a pseudoscalar conventionally called the model independent axion. The model independent axion has axion decay constant [21,22]  $F_a = \alpha_G M_P / 2\sqrt{2}\pi$ , where  $\alpha_G \sim 1/25$  is the unified gauge coupling. We recall that in four-dimensional compactifications of heterotic string theory on a six-manifold  $X$  the gauge coupling and the Planck scale are

$$M_P^2 = \frac{4\pi V_X}{g_s^2 \ell_s^8} \quad \alpha_G = \frac{g_s^2 \ell_s^6}{V_X}. \quad (4.1)$$

This can be deduced by dimensional reduction of the ten-dimensional gauge action  $-1/4(2\pi)g_s^2 \ell_s^6 \int \text{tr} F \wedge \star F$  and the gravity action  $2\pi/g_s^2 \ell_s^8 \int d^{10}x \sqrt{-g} R$ . The instantons that break the shift symmetry of the model-independent axion are NS5-branes wrapped around  $X$ . Their action is  $S_{inst} = 2\pi V_X / g_s^2 = 2\pi / \alpha_G$ . For  $\alpha_G \sim 1/25$  this gives  $S_{inst} \sim 157$ . In terms of  $S_{inst}$ , the axion decay constant becomes

$$F_a = \frac{M_P}{\sqrt{2} S_{inst}} \quad (4.2)$$

in agreement with the general conjecture (3.8). Numerically, (4.2) gives  $F_a \sim 1.1 \times 10^{16} \text{ GeV}$ . The axion decay constant is two orders of magnitude below  $M_P$  so the model independent axion alone does not lead to quintessence unless the initial conditions of the axion are fine-tuned.

Hence, we turn our attention to model dependent axions. The model dependent axions come from zero modes of the NS-NS  $B$ -field polarized within the compactification manifold. If  $\omega_i, i = 1, \dots, b_2(X)$  are harmonic two-forms normalized so that

$$\int_{C_i} \omega_j = \delta_{ij}, \quad (4.3)$$

where  $C_i$  is a basis of  $H_2(X, \mathbb{Z})$  modulo torsion, then the axions are the four-dimensional fields  $a_i$  in the ansatz

$$B = \sum_i a_i \frac{\omega_i}{2\pi}. \quad (4.4)$$

Dimensional reduction of the kinetic term of the  $B$ -field  $-\frac{2\pi}{g_s^2 \ell_s^4} \int H \wedge \star H$  to four dimensions gives the kinetic energy of the axions  $S_{kin} = -\frac{1}{2} \sum_{i,j} \int d^4x \gamma_{ij} \partial_\mu a_i \partial^\mu a_j$ , where

$$\gamma_{ij} = \frac{1}{2\pi g_s^2 \ell_s^4} \int_X \omega_i \wedge \star \omega_j. \quad (4.5)$$

Let us estimate the axion decay constant of a generic model dependent axion in heterotic string theory. The axion comes from a zero mode of a harmonic two-form  $\beta$  that is dual to a two-cycle  $C$ , so  $\int_C \omega = 1$ . If  $R = V_C^{1/2}$  is the size of  $C$ , then the integral  $\int_X \omega_i \wedge \star \omega_j$  scales as  $R^2$  so we estimate the axion decay constant to be

$$F_a = \frac{xR}{\sqrt{2\pi}g_s\ell_s^2}, \quad (4.6)$$

where  $x$  is a dimensionless model dependent constant of order one. The instantons that violate the shift symmetry  $a \rightarrow a + c$ , are worldsheet instantons wrapping  $C$ . The action of these instantons is

$$S_{inst} = \frac{2\pi R^2}{\ell_s^2}. \quad (4.7)$$

Using this we express the axion decay constant in terms of  $S_{inst}$  and  $M_P$

$$F_a = \frac{xM_P}{S_{inst}} \sqrt{\frac{R^6}{2V_X}} \lesssim \frac{xM_P}{\sqrt{2}S_{inst}}. \quad (4.8)$$

The last inequality comes from  $R^6 \lesssim V_X$ , since the size of the curve  $C$  cannot be greater than the size  $R_X \sim V_X^{1/6}$  of the compactification manifold  $X$ .

### *Constraints from Particle Physics*

Heterotic string theory leads naturally to four-dimensional models of particle physics based on unified gauge groups. Here, we will use the additional constraints coming from matching to four-dimensional gauge couplings to see that axions quintessence does not work in weakly coupled heterotic string theory. Similar argument has been advanced in [23].

For heterotic string, the volume of the compactification manifold is determined in terms of the gauge coupling and string parameters (4.1)

$$V_X = \frac{g_s^2 \ell_s^6}{\alpha_G} = 25g_s^2 \ell_s^6. \quad (4.9)$$

The weakly coupled perturbative description is valid for  $g_s \lesssim 1$  which gives an upper bound on the volume of the compactification manifold  $V_X \lesssim 25\ell_s^6$ . This has severe implications for the flatness of the axion potential. The worldsheet instantons breaking the axionic shift symmetry have actions that are proportional to the area of the curve they wrap. For small  $V_X$ , the areas of the curves are small and the axion potential due to worldsheet instantons

is not sufficiently suppressed. For a generic axion the instanton action (4.7) is bounded above because  $R \lesssim V_X^{1/6}$ , so

$$S_{inst} \lesssim \frac{2\pi V_X^{1/3}}{\ell_s^2} = \frac{2\pi g_s^{2/3}}{\alpha_G^{1/3}} = 18g_s^{2/3}, \quad (4.10)$$

where we used (4.9) to reexpress  $V_X$  in terms of  $\alpha_G$ . In weakly coupled heterotic string we take  $g_s \lesssim 1$  which gives<sup>2</sup>  $S_{inst} \lesssim 18$ . The worldsheet instanton actions are one order of magnitude below the range  $210 \sim 290$  necessary for getting sufficiently flat axion potential for quintessence. Hence the axions get large instanton generated potential and are not suitable for explaining present day cosmic acceleration.

## 5. Heterotic M-theory

In weakly coupled heterotic string theory we found that the instanton actions (4.10) of instantons that generate axion potential are too small. In strongly coupled heterotic string, one can achieve larger instanton actions [24,23], since the instanton action (4.10) grows as a positive power of  $g_s$ . When the string coupling gets large, the heterotic string is better described using a dual description as M-theory compactified on  $X \times I$  where  $X$  is the compactification manifold of the heterotic string and  $I$  is an interval. The length of the interval grows with the string coupling. The  $E_8 \times E_8$  gauge symmetry lives on the boundaries of  $I$ . The world-sheet instantons of heterotic string become open M2-branes stretched across the interval. Their action is proportional to the length of the interval. Hence, for sufficiently long  $I$ , these actions are large and the axion potential can be exponentially suppressed down to current dark energy density. For simplicity we will consider the case when  $X \times I$  has the product metric. In more general heterotic M-theory compactifications, the metric along  $I$  is warped. We do not expect the warping to significantly affect our results.

Let us discuss some features of these compactifications. For further details on axions in heterotic M-theory and discussion of the conventions, see [22]. We define the M-theory length  $\ell_{11} = (4\pi\kappa^2)^{1/9}$  and mass  $M_{11} = \ell_{11}^{-1}$ . We normalize  $G$ -form field so that it has

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<sup>2</sup> Larger values of  $S_{inst}$  are possible for some axions for example in anisotropic CY manifolds [22]. However, here we are interested in generic axions because, as discussed in section 3, a large number of axions is necessary for quintessence.

integer periods. With these conventions, the action of eleven-dimensional supergravity becomes

$$S_{11} = 2\pi \int \left( \frac{1}{\ell_{11}^9} d^{11}x \sqrt{-g} R - \frac{1}{2\ell_{11}^3} G \wedge \star G - \frac{1}{6} C \wedge G \wedge G \right). \quad (5.1)$$

The gauge fields live on the boundary of the interval. Their action is

$$S_{YM} = -\frac{1}{4(2\pi)\ell_{11}^6} \int \text{tr} F \wedge \star F. \quad (5.2)$$

Dimensional reduction of (5.1) and (5.2) leads to four-dimensional gauge coupling and Planck mass

$$\alpha_G = \frac{\ell_{11}^6}{V_X}, \quad M_P^2 = \frac{4\pi V_X L}{\ell_{11}^9}, \quad (5.3)$$

where  $L$  is the length of the interval  $I$ . Note that in eleven-dimensional Planck units, the volume of the Calabi-Yau manifold  $X$  is determined by the unified gauge coupling  $V_X = \alpha_G^{-1} \ell_{11}^6 \sim 25 \ell_{11}^6$ .

In heterotic M-theory, there is a model independent axion and several model dependent axions. As with weakly coupled heterotic string, we concentrate on the model dependent axions. These receive potential from membrane instantons stretched between the two boundaries and wrapping a curve in the Calabi-Yau manifold. The number and the properties of model dependent axions depends on the compactification. The model dependent axions come from modes of the three-form field with one index along  $I$  and two along  $X$

$$C = \sum_i a_i \omega_i \frac{dx^{11}}{2\pi L}. \quad (5.4)$$

For an axion coming from a generic curve  $C$  with  $\int_C \omega = 1$ , dimensional reduction of the  $G$ -field kinetic energy (5.1) gives the axion decay constant

$$F_a = \frac{M_P}{2\sqrt{2}\pi} \frac{\ell_{11}^3}{V_C L} = \frac{M_P}{\sqrt{2} S_{inst}}, \quad (5.5)$$

where  $V_C$  is the volume of the curve  $C$  and  $L$  is the length of the M-theory interval.  $S_{inst} = 2\pi V_C L / \ell_{11}^3$  is the action of the M2-brane instanton wrapping  $C \times I$ . These instanton breaks the shift symmetry of the axion to  $a \rightarrow a + 2\pi$  and give the axion a nonzero mass.

### 5.1. Unification and the Cosmological Constant

Compactifications of heterotic M-theory naturally incorporate unification of gauge couplings. Matching to the experimental values of the unified gauge coupling and the unification scale determines some parameters of the compactification. This allows us to estimate the scale of the axion potential by estimating the instanton actions of the membrane instantons that break the axionic shift symmetry.

Recall that in heterotic M-theory, the gauge fields live on the boundary of  $I$ . The unified gauge group is embedded into one of the two boundary  $E_8$  gauge theories. In the usual approach to phenomenology, the unified gauge group is broken to standard model gauge group using discrete Wilson lines. Hence, the unification scale is set by inverse radius of the Wilson lines which is roughly the inverse radius of  $X$

$$M_{GUT} \sim \frac{1}{R_X} \sim \frac{\alpha_G^{1/6}}{\ell_{11}}. \quad (5.6)$$

Consider generic axion corresponding to  $C \times I$  where  $C$  is a curve in  $X$ . For a generic curve  $C$ ,  $V_C \sim V_X^{1/3} = \alpha_G^{-1/3} \ell_{11}^2$ , so the action of an M2-brane wrapping  $C \times I$  is

$$S_{inst} = \frac{2\pi V_C L}{\ell_{11}^3} \sim \frac{2\pi}{\alpha_G^{1/3}} \frac{L}{\ell_{11}}. \quad (5.7)$$

This becomes, in terms of the unification scale (5.6) and the Planck scale (5.3),

$$S_{inst} \sim \frac{\alpha_G}{2} \frac{M_P^2}{M_{GUT}^2}. \quad (5.8)$$

If we take for the unification scale  $M_{GUT} = 2 \times 10^{16} \text{GeV}$ , we find that the instantons have actions around  $S_{inst} \sim 290$ , which is at the upper end of the range  $S_{inst} \sim 210 - 290$  preferred for axion quintessence. If the metric is warped along the interval  $I$ , the instanton actions can be somewhat different. As discussed below (3.12), in generic compactifications we expect the instanton actions  $S_{inst}$  of different membrane instantons to vary by factor of order one. Hence only some of them will fall into the range  $210 \sim 290$  and the number of axions necessary for quintessence is somewhat larger than the estimate  $10^{4-5}$  (3.12). This remark applies also to axions in other string compactifications that are considered in the following sections.

Hence in the context of axion quintessence in heterotic M-theory, the hierarchy between the cosmological constant and the Planck scale is related to the hierarchy between

the GUT scale and the Planck scale. The estimated value of the cosmological constant can be close to the observed value. Whether that is the case, depends on the details of supersymmetry breaking and the generation of the axion potential that were discussed in section 2.

One can derive similar estimates for cosmological constant and the unification scale in other string compactifications like  $G_2$  holonomy and type II string compactifications that we study in the following sections. In these compactifications, the cosmological constant gets again related to the hierarchy between the unification scale and the Planck scale. Intuitively, this is because the hierarchy between  $M_{GUT}$  and  $M_P$  comes from increasing the size of the compactification manifold which increases the instanton actions, since these are given by volumes of various submanifolds of  $X$ .

### 5.2. Axion Quintessence and the Species Problem

$10^{4-5}$  axions may seem quite a large number of light scalars. The light axions radiatively induce corrections to the Planck scale

$$\delta M_P^2 \simeq \pm \frac{N \Lambda_{UV}^2}{16\pi^2}, \quad (5.9)$$

where  $N$  is the number of axions and  $\Lambda_{UV}$  is a high energy cut-off scale. For large number of axions  $N$ , this could be comparable to the tree level value of Planck scale (5.3). A good measure of whether we can trust our tree-level estimates of axion parameters is whether the radiative corrections (5.9) to the Planck scale are suppressed compared to the tree-level contribution. Requiring that these corrections are suppressed leads to an upper bound on the number of axions and hence an upper bound on the duration of quintessence.

In the context of the heterotic M-theory, we take the high-energy cut-off scale to be the M-theory scale  $\Lambda_{UV} \sim 1/\ell_{11}$ . Requiring that the radiative corrections are smaller than the tree-level expression for Planck mass gives  $N \lesssim 16\pi^2 (M_P \ell_{11})^2$ , which with the help of (5.3) and (5.7) becomes

$$N \lesssim \frac{32\pi^2}{\alpha_G^{2/3}} S_{inst}. \quad (5.10)$$

This leads to an upper bound on the duration of the axion quintessence (3.13)

$$N_{e-fold} \lesssim \frac{32\pi^2}{\alpha_G^{2/3} S_{inst}}. \quad (5.11)$$



One thing to notice about (5.11) is that the number of Hubble times of accelerated expansions decreases with  $S_{inst}$  and hence with the energy of the axion potential. Similar formulas are valid in other string compactifications considered in the following sections. For instanton actions  $S_{inst} \sim 210 - 290$ , this gives  $N_{e-fold} \lesssim 10 - 13$ . Hence quintessence could last for a few Hubble times in heterotic M-theory.

### *Axion Inflation and the Species Problem*

The bound (5.11) also gives a limit on the number of Hubble times of inflation in the axion model of inflation of Dimopoulos et al. [11]. In that model, the axion potential is used to explain the inflation in the early universe. If we assume that the inflationary potential has height around the GUT scale  $V \sim (10^{16} \text{ GeV})^4$ , then the actions of instantons that generate this potential are roughly  $S_{inst} \sim \ln(M_P^4/V) \sim 20$ . Here (5.11) gives an upper bound on the duration of axion inflation  $N_{e-fold} \lesssim 130$  which is above the minimum of 60 e-folds required by present day cosmological experiments.

## **6. M-Theory on $G_2$ Holonomy Manifolds**

We assume that M-theory is compactified on a manifold  $X$  of  $G_2$  holonomy with volume  $V_X$ . The four-dimensional effective theory has  $\mathcal{N} = 1$  supersymmetry and can lead to semi-realistic models of particle physics [25,26].

The axions come from zero modes of the three-form field  $C$ . If  $D_i, i = 1, \dots, b_3(X)$  is a basis of three-cycles of  $X$  and  $\omega_i, i = 1, \dots, b_3(X)$  is the dual basis of harmonic three-forms on  $X$  such that  $\int_{D_i} \omega_j = \delta_{ij}$ , then the axions  $a_i$  are four-dimensional fields coming from the ansatz

$$C = \frac{1}{2\pi} \sum_i a_i \omega_i. \quad (6.1)$$

The kinetic energy of the axions comes from dimensional reduction of (5.1)

$$S_{kin} = -\frac{1}{2\pi\ell_{11}^3} \int d^4x \frac{1}{2} \partial_\mu a_i \partial^\mu a_j \int \omega_i \wedge \star \omega_j. \quad (6.2)$$

The generic axion  $a$  has axion decay constant roughly

$$F_a^2 = \frac{x^2 R}{2\pi \ell_{11}^3}, \quad (6.3)$$

where  $R = V_D^{1/3}$  is the size of the three-cycle  $D$  and  $x$  is a dimensionless model dependent number of order one.

The axion gets potential from M2-brane instanton wrapping three-cycle  $D$ . The membrane instanton has action

$$S_{inst} = 2\pi \frac{R^3}{\ell_{11}^3}. \quad (6.4)$$

For quintessence, we need  $S_{inst} \sim 210 - 290$ , which determines the size of the three-cycles

$$R = \ell_{11} \left( \frac{S_{inst}}{2\pi} \right)^{1/3}, \quad (6.5)$$

or numerically  $R \sim 3\ell_{11}$ . Combining (6.3) and (6.5) we express the axion decay constant in terms of  $M_P$  and  $S_{inst}$

$$F_a = \frac{xM_P}{\sqrt{2}S_{inst}} \sqrt{\frac{R^7}{V_X}} \lesssim \frac{xM_P}{\sqrt{2}S_{inst}}, \quad (6.6)$$

since  $R \lesssim V_X^{1/7}$  as the size of the cycle  $D$  is bounded by the size of  $X$ . In (6.6) we used that the Planck scale is  $M_P^2 = 4\pi V_X / \ell_{11}^9$  which follows from dimensional reduction of the eleven-dimensional supergravity action (5.1).

### 6.1. Constraints from Unification

$G_2$  holonomy compactifications naturally implement models of particle physics based on unified gauge groups [25,26]. Matching to the observed value of gauge coupling and unification scale gives us further constraints on the compactifications that help estimate the size of the instanton actions. This gives a measure of what is the likely value of axion potential in  $G_2$  holonomy compactifications.

In these models, gauge symmetry comes from an orbifold singularity that is fibered along a three-cycle  $Q$  in  $X$ . The unified gauge coupling is [26]

$$\alpha_G = \frac{\ell_{11}^3}{V_Q}. \quad (6.7)$$

If we assume that the unified gauge group is broken down to the standard model gauge group using discrete Wilson lines, the unification scale is given roughly by the inverse radius of  $Q$ . After taking into account threshold corrections, the relation between the GUT scale and the volume  $V_Q$  is [26]

$$M_{GUT}^3 = \frac{L(Q)}{V_Q}, \quad (6.8)$$

where  $L(Q)$  is Reidemeister or Ray-Singer torsion, which is a topological invariant that depends on  $Q$  and on the Wilson line on  $Q$  that breaks  $SU(5)$  down to the standard model gauge group. For example  $Q$  could be a lens space  $S^3/Z_q$ , with Wilson line on  $Q$  that has eigenvalues  $\exp(2\pi i \Delta_i/q)$  with  $\Delta_i = (2w, 2w, 2w, -3w, -3w)$ , with  $w, q$  coprime integers,  $L(Q) = 4q \sin^2(5\pi w/q)$ . For the minimal choice  $q = 2, w = 1$ , one has  $L(Q) = 8$ . Combining (6.7) and (6.8), we find the M-theory scale in terms of low-energy parameters

$$M_{11} = \frac{M_{GUT}}{(\alpha_G L(Q))^{1/3}}. \quad (6.9)$$

A generic three-cycle  $D$  has volume roughly  $V_D \sim V_X^{3/7}$ . The action of a membrane instanton that wraps  $D$  is  $S_{inst} = 2\pi V_D/\ell_{11}^3$ . In terms of the Planck scale and the unification scale this is

$$S_{inst} \sim 2\pi \left( \frac{(\alpha_G L(Q))^{2/3} M_P^2}{4\pi M_{GUT}^2} \right)^{3/7} \quad (6.10)$$

or numerically  $S_{inst} \sim 90$ . This is two to three times below the preferred range  $S_{inst} \sim 210 - 290$ .

## 7. Type II Compactifications

Let us briefly review the calculation of the decay constant of the axions in type II string compactifications [22]. The axions come from dimensional reduction of  $q$ -form RR-field  $C_q$ . The axions are the four-dimensional fields  $a_i$  in the ansatz

$$C_q = \sum_i \frac{a_i}{2\pi} \omega_i, \quad i = 1, \dots, b_q(X), \quad (7.1)$$

where  $\omega_i$  are harmonic forms normalized so that  $\int_{C_i} \omega_j = \delta_{ij}$ , where  $C_i$  is a basis of  $H_q(X)$ . The kinetic energy of axions comes from dimensional reduction of the kinetic term of the  $q$ -form field

$$-\frac{2\pi}{\ell_s^{8-2q}} \int_{R^4 \times X} F_{q+1} \wedge \star F_{q+1}. \quad (7.2)$$

Substituting (7.1) into (7.2) one finds  $S_{kin} = -\frac{1}{2} \sum_{i,j} \int d^4x \gamma_{ij} \partial_\mu a_i \partial^\mu a_j$ , where

$$\gamma_{ij} = \frac{1}{2\pi \ell_s^{8-2q}} \int_X \omega_i \wedge \star \omega_j. \quad (7.3)$$

If the axion comes from cycle  $C$ , the integral  $\int_X \omega \wedge \star \omega$  scales as  $R^{6-2q}$ , where  $R = V_C^{1/q}$  is the size of  $C$ . Hence the axion decay constant is roughly

$$F_a^2 = \frac{x^2 R^{6-2q}}{2\pi \ell_s^{8-2q}}, \quad (7.4)$$

where  $x$  is a dimensionless number of order one. The axionic shift symmetry  $a \rightarrow a + c$  is preserved to all orders in string perturbation theory. Nonperturbatively, the shift symmetry gets broken to a discrete symmetry  $a \rightarrow a + 2\pi$  by Euclidean  $D(q-1)$  instantons that wrap  $C$ . The action of the  $D(q-1)$ -brane instanton that generates the axion potential is

$$S_{inst} = \frac{2\pi}{g_s} \left( \frac{R}{\ell_s} \right)^q. \quad (7.5)$$

One gets instanton actions in the preferred range  $S_{inst} \sim 210 - 290$  if the size of  $C$  is roughly a few string lengths

$$R = \ell_s \left( \frac{g_s S_{inst}}{2\pi} \right)^{1/q}. \quad (7.6)$$

The four-dimensional Planck scale  $M_P^2 = 4\pi V_X / g_s^2 \ell_s^8$  follows by dimensional reduction of the gravity action  $\frac{2\pi}{g_s^2 \ell_s^8} \int d^{10}x \sqrt{-g} R$ . In terms of  $M_P$ , the axion decay constant is

$$F = \frac{x M_P}{\sqrt{2} S_{inst}} \sqrt{\frac{R^6}{V_X}} \lesssim \frac{x M_P}{\sqrt{2} S_{inst}} \quad (7.7)$$

since the size of the cycle  $C$  is bounded by the size of the compactification manifold  $R \lesssim V_X^{1/6}$ .

### 7.1. Constraints from Unification

To assess the scale of the axion potential, we estimate the action of a generic D-brane instanton that breaks the axion shift symmetry. Assuming that the string compactification implements unification leads to a more precise estimate of the parameters of the string compactification and hence of the instanton actions. The gauge symmetry in Type II D-brane models lives on a stack of D-branes wrapping a cycle  $Q$  in  $X$ . The four-dimensional gauge coupling is  $\alpha_G = g_s \ell_s^q / V_Q$ . If the gauge symmetry is broken down to standard model gauge symmetry using discrete Wilson lines, the unification scale is given roughly by the inverse radius of  $Q$

$$M_{GUT} \sim \frac{1}{V_Q^{1/q}} = \frac{\alpha_G^{1/q}}{g_s^{1/q} \ell_s}. \quad (7.8)$$

A D-brane instanton that wraps a generic cycle  $C$  has action  $S_{inst} = 2\pi V_C/g_s \ell_s^q$ , which for  $V_C \sim V_X^{q/6}$  becomes in terms of the Planck scale and the unification scale

$$S_{inst} \sim 2\pi \frac{g_s^{\frac{q-4}{3}} \alpha_G^{1/3}}{(4\pi)^{\frac{q}{6}}} \left( \frac{M_P}{M_{GUT}} \right)^{q/3}. \quad (7.9)$$

For type IIA D6-brane models,  $q = 3$  gives

$$S_{D2} \sim \frac{\pi^{1/2} \alpha_G^{1/3}}{g_s^{1/3}} \frac{M_P}{M_{GUT}}, \quad (7.10)$$

which numerically evaluates to  $S_{inst} \sim 73g_s^{-1/3}$ . Hence for moderately small string coupling, the instanton action is in the preferred range  $S_{inst} \sim 210 - 290$ . For type IIB models with gauge symmetry coming from D7-branes, we get

$$S_{D3} \sim \left( \frac{\pi \alpha_G}{2} \right)^{1/3} \left( \frac{M_P}{M_{GUT}} \right)^{4/3}. \quad (7.11)$$

Note that this answer depends only on the phenomenologically observed parameters. In particular it does not depend on the string coupling. Numerically, we have  $S_{inst} \sim 240$  which is in the range preferred for axion quintessence.

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