

Higher Curvature Effects in ADD and RS Models * †

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Abstract

Over the last few years several extra-dimensional models have been introduced in attempt to deal with the hierarchy problem. These models can lead to rather unique and spectacular signatures at Terascale colliders such as the LHC and ILC. The ADD and RS models, though quite distinct, have many common feature including a constant curvature bulk, localized Standard Model(SM) fields and the assumption of the validity of the EH action as a description of gravitational interactions.

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1 Introduction

Over the last few years several extra-dimensional models have been introduced in attempt to deal with the hierarchy problem. These models can lead to rather unique and spectacular signatures at Terascale colliders such as the LHC and ILC. The ADD and RS models, though quite distinct, have many common feature including a constant curvature bulk, localized Standard Model(SM) fields and the assumption of the validity of the EH action as a description of gravitational interactions.

It is well known that the EH action can only be regarded as an effective theory so we may on general grounds expect that as one probes energies approaching the fundamental scale, M , (as one will do at the LHC/ILC for both models) significant deviations from EH expectations are likely to appear. From a bottom-up approach we can claim ignorance of what a more complete gravitational theory may be like but we would expect that the leading deviations from EH would appear as higher dimensional operators involving various invariants constructed from the curvature tensor. This motivates us to consider a more general class of gravitational actions of the $D = 4 + n$ dimensional form

$$S_g = \frac{M^{D-2}}{2} \int d^D x \sqrt{g} F(R, P, Q), \quad (1)$$

where $P = R_{AB}R^{AB}$ and $Q = R_{ABCD}R^{ABCD}$. Various actions of this generic type have been examined in the literature within a variety of contexts including black holes, cosmology, modifications of gravity within the solar system as well as at short-distances scales of interest to us here[1]. It will be assumed in what follows that at low energies $F \rightarrow R$ so that the EH action, with all its successes, is recovered.

In both the ADD and RS models we wish to know the graviton KK spectrum and wave-functions as well as their couplings to SM fields. We also need to know how \bar{M}_{Pl} and the more fundamental scale M are related. To this end it can be shown that for spaces with constant curvature the above action is the most general one; furthermore it can be shown that we can obtain all of the desired information by expanding this action around the constant background metric to

quadratic order[1]. Performing this expansion then yields an effective action of the form

$$S_{eff} = \frac{M^{D-2}}{2} \int d^D x \sqrt{g} [\Lambda + a_1 R + a_2 R^2 + a_3 C + a_4 G], \quad (2)$$

where $G = R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} = R^2 - 4P + Q$ is the Gauss-Bonnet invariant and $C = C_{ABCD}C^{ABCD} = Q - 4P/(n+2) + 2R^2/(n+3)(n+2)$ is the square of the Weyl tensor. This simplification is obtainable from the direct expansion $F = F_0 + (R - R_0)F_R + (P - P_0)F_P + (Q - Q_0)F_Q +$ quadratic terms, where F_0 is a constant corresponding to the evaluation of F itself in the fixed curvature background metric and $F_X = \partial F / \partial X|_0$; a quantity X_0 here means that X is to be evaluated in terms of the background metric which we here assume to be a space of constant curvature as is the case in both the ADD and RS models. Thus the quantities R_0, P_0, Q_0, F_X and F_{XY} are just numbers which depend on the explicit form of the metric and the number of extra dimensions. This procedure directly gives us the constants Λ, a_i above[1] in terms of F and its derivatives evaluated in the background metric as we will see below.

2 ADD

For the action F , the D-dimensional field content consists of a massless tensor field (*i.e.*, the ‘usual’ graviton) as well as a massive scalar field and a massive tensor ghost field with bulk masses directly calculable from F itself. This can be seen most clearly in, *e.g.*, the ADD model where the full exchange amplitude between two localized SM sources (before KK summation) can be written as $A \sim [T_{\mu\nu}T^{\mu\nu} - T^2/(n+2)]/(k^2 - m_n^2) - [T_{\mu\nu}T^{\mu\nu} - T^2/(n+3)]/(k^2 - (m_T^2 + m_n^2)) + T^2/(n+2)(n+3)[k^2 - (m_S^2 + m_n^2)]$. Here $T_{\mu\nu}$ is the localized SM stress-energy tensor and T is its trace. Ghost fields are potentially very dangerous in perturbation theory and can cause unitarity violations so we may want to rid ourselves of this massive tensor field. To remove this ghost state from the spectrum it is sufficient to force the bulk tensor mass m_T to infinity; this will make all of the fields in its entire KK tower decomposition have infinite mass. In this simple ADD case one finds that

$m_T^2 = -F_R/(n+2)(F_P + 4F_Q)$ and thus we see that to make m_T infinitely large it is sufficient to demand that F be only a function of R and the combination $P - 4Q$. These results can also be shown to hold qualitatively in the case of the RS model and the $P - 4Q$ dependence of F will be assumed in the discussion below. Of course, we still need to insure that the usual KK gravitons and the new KK scalars are non-tachyonic so that $F_R > 0$ will also be required. For either ADD or RS model, one now in general finds that $a_3 = 0$ and

$$\begin{aligned}
\Lambda &= F_0 - R_0 F_R + F_{RR} R_0^2/2 + \lambda R_0^2 [F_Q - R_0 F_{QQ} + \lambda F_{QQ} R_0^2/2] \\
a_1 &= F_R - R_0 F_{RR} + \lambda R_0^2 F_{RQ} \\
a_2 &= -F_Q + F_{RR}/2 + R_0 F_{RQ} - \lambda R_0^2 F_{QQ} \\
a_4 &= -a_2 + F_{RR}/2.
\end{aligned} \tag{3}$$

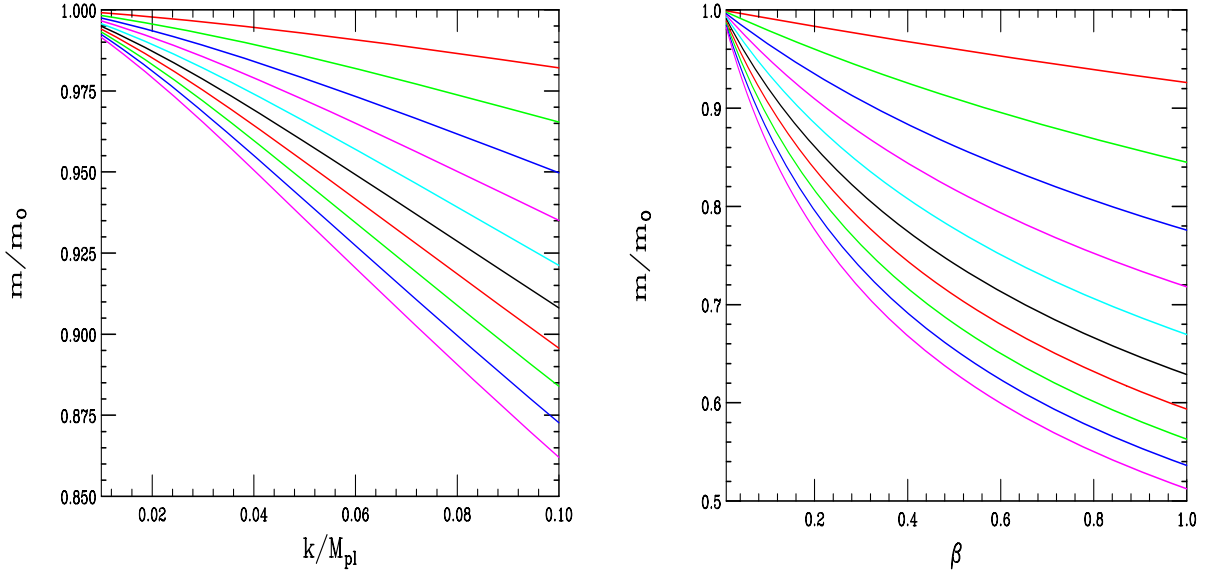


Figure 1: Shifts in the RS KK graviton masses due to finite β corrections: (Left) As a function of c with $\beta = 0.1$ to 1 in steps of 0.1 from top to bottom and (Right) as a function of β for $c = 0.01 - 0.1$ in steps of 0.01 from top to bottom.

The ADD case is particularly easy to analyze as the space is flat and there is no bulk cosmological constant. Here the expansion of F to quadratic order is quite simple: $F \rightarrow F_R R + [-F_Q + F_{RR}/2]R^2 + F_Q G$.

In ADD there are two major modifications due to $R \rightarrow F$. First, due to the existence of the D-dimensional bulk scalar which produces its own KK tower there are new exchanges between SM sources and new emissions that can lead to missing energy signatures. However, the effect of these new states on these conventional signatures is quite suppressed as scalars couple to the trace of the energy momentum tensor. For SM fields the ratio $T^2/T_{\mu\nu}T^{\mu\nu} \sim (m^2/s)^k$, with m a SM particle mass and $k = 1, 2$. Further suppression occurs since the KK spectrum of the scalars begins at the value of the bulk mass m_S ; for most reasonable choices of F it can be shown that m_S is naturally $\sim M \sim \text{TeV}$. For example, if $F = R + \beta R^2/M^2$, we find that the value of m_S^2 is $(n+2)M^2/(4\beta(n+3))$; thus we see that with β of $\mathcal{O}(1)$, $m_S \sim M$.

The second effect is to modify the usual ADD relation to

$$\bar{M}_{Pl}^2 = V_n M^{n+2} F_R, \quad (4)$$

where we expect F_R is $\mathcal{O}(1)$ and $V = (2\pi R)^n$ for a torus. Since \bar{M}_{Pl} is fixed, for a given M the effect of $F_R \neq 1$ is to shift the value of R which leads to a modification of the KK spectrum as $m_{KK} \sim 1/R$, *i.e.*, $m_{KK} \rightarrow m_{KK} F_R^{1/n}$. Similar arguments lead to rescalings of the graviton emission amplitude, $d\sigma_{ADD} \rightarrow F_R^{-1} d\sigma_{ADD}$ as well as the amplitude for graviton KK exchange, $A_{KK} \rightarrow F_R^{-1} A_{KK}$, both of which can lead to important experimental consequences, *i.e.*, $\mathcal{O}(1)$ modifications to anticipated ADD search reaches at colliders.

3 RS

In RS the bulk is of constant background curvature with $R_0 = -20k^2$, k being defined via the metric $ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$ induced by a bulk cosmological constant Λ . For general $F(R, P-4Q)$, the equations of motion lead to a relationship between F, k, M and Λ : $224k^4 F_Q + 8k^2 F_R + F_0 = 2\Lambda/M^3$ from which k as a function of Λ (or the other way around) can be extracted. It is important to remember here that $F_{0,R,Q}$ are themselves functions of k . For example, if we assume that

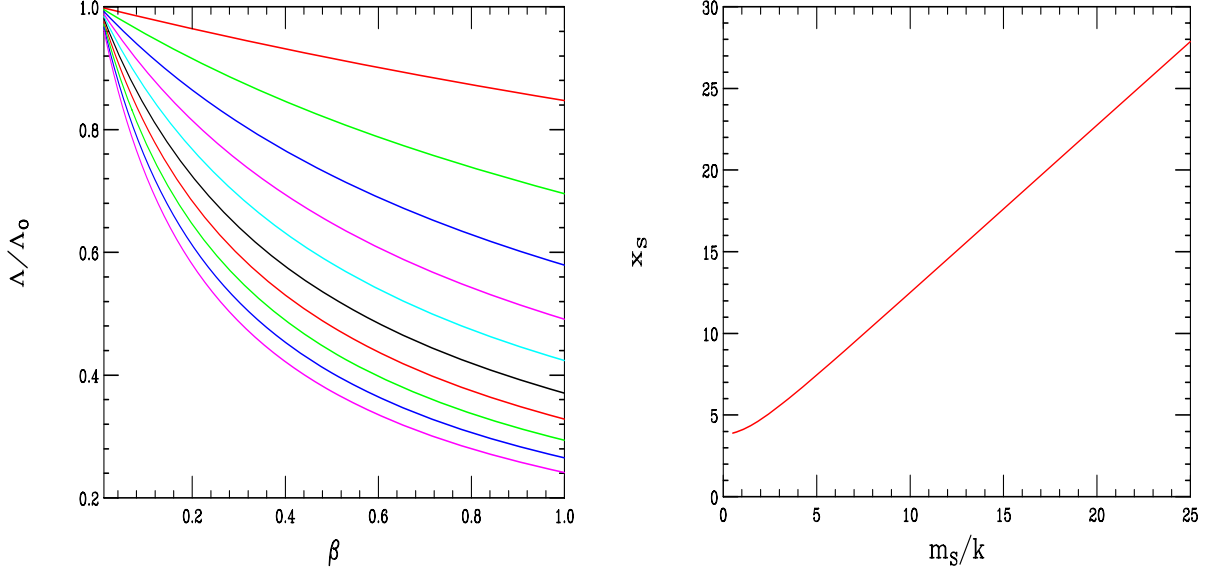


Figure 2: (Left) Shift in the cosmological constant in RS due to finite β for $c = 0.01 - 0.10$ from top to bottom in 0.01 steps. (Right) Root for the determination of mass of the lightest KK state corresponding to the new RS scalar as a function of the scaled bulk mass. As a point of comparison the first KK graviton has a root of $\simeq 3.83$.

$F = R + R^2/M^2$, we obtain

$$k^2 = \frac{3M^2}{40\beta} \left[1 \pm \left(1 + \frac{40\Lambda}{9M^5}\beta \right)^{1/2} \right], \quad (5)$$

for which the negative root goes over to the usual EH result as the parameter $\beta \rightarrow 0$. Further analysis of the equations of motion shows that the basic RS relationship between the 5d fundamental scale and \bar{M}_{Pl} is also altered by $F \neq R$; we now find that

$$H(k) \frac{M^3}{k} = \bar{M}_{Pl}^2, \quad (6)$$

where in general $H = F_R + 36k^2 F_Q + 1000k^4 F_{RQ} + 10080k^6 F_{QQ}$. If, for example, $F = R + R^2/M^2$, then $H = 1 - 40\beta k^2/M^2$. Since \bar{M}_{Pl} is known, for a fixed value of M the two expressions above lead to shifts in the values of both k and Λ as compared to standard RS expectations and, consequently, to a shift in the KK graviton spectrum. This is shown in Fig. 1 for our simple example of $F =$

$R + \beta R^2/M^2$ where the KK mass shift is explicitly given by

$$\frac{m}{m_0} = (80\beta c^{4/3})^{-1}[-1 + (1 + 160\beta c^{4/3})^{1/2}]. \quad (7)$$

Here we have defined $c = k_0/\overline{M}_{Pl}$, where k_0 is the value obtained in the usual RS model employing the EH action. The corresponding shift in Λ is shown in Fig. 2.

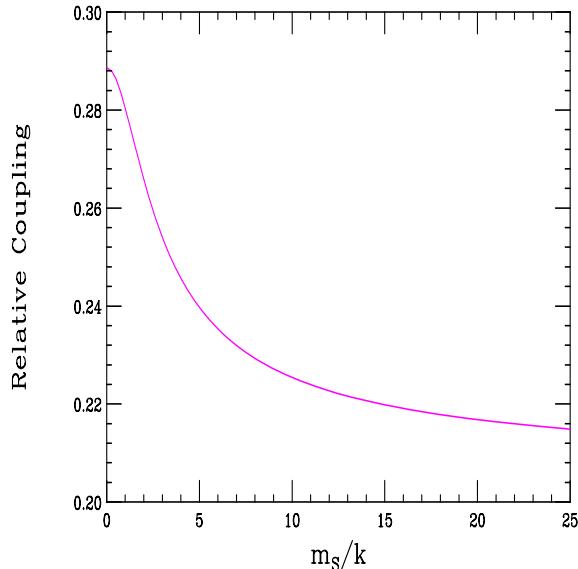


Figure 3: Estimate of the relative wavefunction suppression for the lightest scalar KK coupling to matter in comparison to the corresponding KK graviton. Recall that scalars couple to T whereas gravitons couple to $T_{\mu\nu}$.

As in ADD, the new scalar obtains a bulk mass which in RS case is given by

$$m_S^2 = \frac{3}{8} \frac{F_R + 20k^2 F_{RR} + 280k^4 F_{RQ}}{F_{RR} - 2F_Q - 40k^2 F_{RQ} - 560k^4 F_{QQ}}. \quad (8)$$

Within a given model this value can be used to obtain the scalar KK spectrum by solving for the roots of the equation $(2 - \nu)J_\nu(x_n) + x_n J_{\nu-1}(x_n) = 0$ with J the usual Bessel function and $\nu^2 = 4 + m_S^2/k^2$ in the usual RS fashion; the KK masses are then given by $m_n = x_n k e^{-\pi k r c}$. Fig. 2 shows the root for the lightest KK scalar state, x_S , as a function of m_S/k . We thus naturally expect these scalar KK states to be more massive than the corresponding more familiar graviton

excitations. Since their couplings are suppressed as in the ADD case they might be difficult to detect experimentally; this is made even more so by the small value of the scalar KK wavefunction evaluated at the TeV brane where the SM matter is assumed to be localized. This further relative suppression is shown in Fig. 3.

Hopefully new dimensions will be discovered at the Terascale.

Acknowledgments

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References

- [1] For more details and original references, see T. G. Rizzo, arXiv:hep-ph/0603242.