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The Two-loop Anomalous Dimension Matrix for Soft Gluon Exchange

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The resummation of soft gluon exchange for QCD hard scattering requires a matrix of anomalous dimensions. We compute this matrix directly for arbitrary $2 \to n$ massless processes for the first time at two loops. Using color generator notation, we show that it is proportional to the one-loop matrix. This result reproduces all pole terms in dimensional regularization of the explicit calculations of massless $2 \to 2$ amplitudes in the literature, and it predicts all poles at next-to-next-to-leading order in any $2 \to n$ process that has been computed at next-to-leading order. The proportionality of the one- and two-loop matrices makes possible the resummation in closed form of the next-to-next-to-leading logarithms and poles in dimensional regularization for the $2 \to n$ processes.

The calculation of high-energy cross sections in perturbative quantum chromodynamics (QCD) for hadronic collisions involves the factorization of long- and short-distance effects. Sensitivity to long-distance dynamics is enhanced by powers of logarithms whenever there is an incomplete cancellation of parton emission and virtual corrections. In such situations, it is useful to organize, or resum, these corrections to all orders in perturbation theory. Correspondingly, in partonic scattering or production amplitudes, it is necessary to organize poles in ε that arise in dimensional regularization (with $D = 4 - 2\varepsilon$). The resummation of these poles and related logarithmic enhancements is well-understood for inclusive reactions mediated by electroweak interactions, such as the Sudakov form factor [1, 2] and in Drell-Yan processes [3]. With recent advances in the computation of splitting functions [4], many such corrections can be resummed explicitly to next-to-next-to-leading level. Their structure at arbitrary level is known to be determined by a handful of anomalous dimensions.

The situation for QCD hard scattering processes containing four or more partons — critical to understanding many types of backgrounds to new physics at hadron colliders [5] — is more complex. Resummation beyond leading logarithms or poles requires a matrix of additional anomalous dimensions [6–9]. These matrices are found in turn from the renormalization of the vacuum matrix elements of products of Wilson lines, one for each external parton in the underlying process [7]. In this paper, we investigate the structure of the two-loop anomalous dimension matrix. We will find that, remarkably, for every hard-scattering process involving only massless partons, this matrix is proportional to the one-loop matrix. We will concentrate below on the role that the matrix plays in partonic amplitudes. The full calculation of the two-loop matrix will be given elsewhere [10]. In this paper, we provide the simple calculation that is at the heart of the main result. We will show that certain color correlations due to two-loop diagrams that couple three Wilson lines vanish identically. We will also provide an explicit expression in terms of color generators [11, 12] for all single-pole terms in massless $2 \to n$ amplitudes.

We consider a general process involving the scattering of massless partons, which we denote by "f":

$$\begin{aligned} \mathrm{f}: \quad & f_1(p_1, r_1) + f_2(p_2, r_2) \\ & \to f_3(p_3, r_3) + \cdots + f_{n+2}(p_{n+2}, r_{n+2})\,. \end{aligned} \tag{1}$$

The f_i are the flavors of the participating partons, which carry momenta $\{p_i\}$ and color $\{r_i\}$. Adopting the color-state notation of Ref. [12], we represent the amplitude for this process as $|\mathcal{M}_{\mathrm{f}}\rangle$.

It is convenient to express these amplitudes as vectors with C elements in the space of color tensors, for some choice of basis tensors $\{(c_I)_{\{r_i\}}\}$ [7, 12, 13],

$$\begin{aligned} & \left| \mathcal{M}_{\mathrm{f}} \left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu), \varepsilon \right) \right\rangle \equiv \\ & \quad \sum_{L=1}^{C} \mathcal{M}_{\mathrm{f},L} \left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu), \varepsilon \right) (c_L)_{\{r_i\}} \\ & \quad . \end{aligned} \tag{2}$$

We will analyze these amplitudes at fixed momenta p_i for the participating partons, which we represent as $p_i = Q\beta_i\,, \quad \beta_i^2 = 0\,,$ where the β_i are four-velocities, and where Q is an overall momentum scale.

In dimensional regularization, on-shell amplitudes may be factorized into jet, soft and hard functions that describe the dynamics of partons collinear with the external lines, soft exchanges between those partons, and the short-distance scattering process, respectively. This factorization follows from the general space-time structure of long-distance contributions to elastic processes [6, 14]. The general form of the factorized amplitude, for equal factorization and renormalization scales μ, is [13]

$$\begin{aligned} \left| \mathcal{M}_{\mathrm{f}} \left(\beta_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu), \varepsilon \right) \right\rangle \; = \; & \prod_{i=1}^{n+2} J^{[i]} \left(\alpha_s(\mu), \varepsilon \right) \\ \times \mathbf{S}_{\mathrm{f}} \left(\beta_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu), \varepsilon \right) & \left| H_{\mathrm{f}} \left(\beta_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu) \right) \right\rangle \,, \end{aligned} \tag{3}$$

where $J^{[i]}$ is the jet function for external parton i, $\mathbf{S}_{\mathrm{f}}$ is the soft function, and H_{f} is the hard (short-distance) function.

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The jet function for parton i can be expressed to all orders in terms of three anomalous dimensions, $\mathcal{K}^{[i]}$, $\mathcal{G}^{[i]}$ and $\gamma_K^{[i]}$, of which the first is determined order-by-order from the third. The general form of the jet function, and its expansion to second order is given by (expanding any function as $f(\alpha_s) = \sum_n (\alpha_s/\pi)^n f^{(n)}$) [2],

$$\ln J^{[i]}\left(\alpha_s(\mu),\varepsilon\right) = \frac{1}{2}\int_0^{\mu}\frac{d\xi}{\xi}\left[\mathcal{K}^{[i]}(\alpha_s(\mu),\varepsilon) + \mathcal{G}^{[i]}\left(-1,\bar{\alpha_s}\left(\xi,\varepsilon\right),\varepsilon\right) + \int_{\xi}^{\mu}\frac{d\tilde{\mu}}{\tilde{\mu}}\gamma_K^{[i]}\left(\bar{\alpha_s}\left(\tilde{\mu},\varepsilon\right)\right)\right] \tag{4}$$
$$= -\left(\frac{\alpha_s}{\pi}\right)\left(\frac{1}{8\varepsilon^2}\gamma_K^{[i](1)} + \frac{1}{4\varepsilon}\mathcal{G}^{[i](1)}(\varepsilon)\right) + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{\beta_0}{32}\frac{1}{\varepsilon^2}\left(\frac{3}{4\varepsilon}\gamma_K^{[i](1)} + \mathcal{G}^{[i](1)}(\varepsilon)\right) - \frac{1}{8}\left(\frac{\gamma_K^{[i](2)}}{4\varepsilon^2} + \frac{\mathcal{G}^{[i](2)}(\varepsilon)}{\varepsilon}\right)\right] + \dots\,.$$

In the expansion we use the D-dimensional running-coupling, evaluated at one-loop order,

$$\bar{\alpha_s}\left(\tilde{\mu},\varepsilon\right) = \alpha_s(\mu)\left(\frac{\mu^2}{\tilde{\mu}^2}\right)^{\varepsilon}\sum_{n=0}^{\infty}\left[\frac{\beta_0}{4\pi\varepsilon}\left(\left(\frac{\mu^2}{\tilde{\mu}^2}\right)^{\varepsilon} - 1\right)\alpha_s(\mu)\right]^n, \tag{5}$$

with the one-loop coefficient $\beta_0 = 11C_A/3 - 4T_F n_F/3$. The corresponding expression for the soft matrix is

$$\mathbf{S}_{\mathrm{f}}\left(\frac{\beta_i\cdot\beta_j}{u_0},\alpha_s(\mu),\varepsilon\right) = \mathrm{P}\,\exp\left[-\int_0^{\mu}\frac{d\tilde{\mu}}{\tilde{\mu}}\mathbf{\Gamma}_{S_{\mathrm{f}}}\left(\frac{\beta_i\cdot\beta_j}{u_0},\bar{\alpha_s}\left(\tilde{\mu},\varepsilon\right)\right)\right]$$
$$= 1 + \frac{1}{2\varepsilon}\left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(1)} + \frac{1}{8\varepsilon^2}\left(\frac{\alpha_s}{\pi}\right)^2\left(\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(1)}\right)^2 - \frac{\beta_0}{16\varepsilon^2}\left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(1)} + \frac{1}{4\varepsilon}\left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(2)} + \dots\,, \tag{6}$$

where $u_0 = \mu^2/Q^2$, so that $\beta_i\cdot\beta_j/u_0 = s_{ij}/\mu^2$.

Expanding $\mathcal{G}^{[i]} = \mathcal{G}_0^{[i]} + \varepsilon\mathcal{G}^{[i]\prime} + \dots$, one finds from Eq. (4) the single pole in ε in the logarithm of the jet function at two loops. For the quark case this term is

$$-\frac{\mathcal{G}_0^{[q](2)}}{8} + \frac{\beta_0\,\mathcal{G}^{[q](1)\prime}}{32} = -\frac{3}{8}C_F^2\left[\frac{1}{16} - \frac{1}{2}\zeta(2) + \zeta(3)\right]$$
$$-\frac{1}{16}C_A C_F\left[\frac{961}{216} + \frac{11}{4}\zeta(2) - \frac{13}{2}\zeta(3)\right]$$
$$+\frac{1}{16}C_F T_F n_F\left[\frac{65}{54} + \zeta(2)\right], \tag{7}$$

using values of $\mathcal{G}^{[q]}(\varepsilon)$ from ref. [15]. Notice the contribution from the running of the finite term at one loop, which appears as an $\mathcal{O}(\varepsilon)$ contribution in $\mathcal{G}^{[i](1)}$.

The one-loop soft anomalous dimension in color-generator form is

$$\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(1)} = \frac{1}{2}\sum_{i\in\mathrm{f}}\sum_{j\neq i}\mathbf{T}_i\cdot\mathbf{T}_j\,\ln\left(\frac{\mu^2}{-s_{ij}}\right), \tag{8}$$

where $s_{ij} = (p_i + p_j)^2$, with all momenta defined to flow into (or out of) the amplitude. The $\mathbf{T}_i$ are given explicitly by color generators in the representation of parton i, multiplied by ± 1: plus one for an outgoing quark or gluon, or incoming antiquark; minus one for an incoming quark or gluon, or outgoing antiquark. The color generator form for the anomalous dimension matrix is more flexible, but less explicit, than the corresponding matrix expressions in a chosen basis of color tensors for the amplitude. An example of the latter for $q\bar{q}\to q\bar{q}$ scattering, in an s-channel t-channel singlet basis, is

$$\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(1)} = \begin{pmatrix} \frac{1}{N_c}\left(\mathcal{U}-\mathcal{T}\right) + 2\,C_F\,\mathcal{S} & \left(\mathcal{S}-\mathcal{U}\right) \\ \left(\mathcal{T}-\mathcal{U}\right) & \frac{1}{N_c}\left(\mathcal{U}-\mathcal{S}\right) + 2\,C_F\,\mathcal{T}\end{pmatrix}, \tag{9}$$

where $\mathcal{T}\equiv\ln\left(\frac{-t}{\mu^2}\right)$, and so on for the other Mandelstam invariants, defined by $s = s_{12}$, $t = s_{13}$, $u = s_{14}$. Resummed cross sections are determined by the eigenvalues and eigenvectors of these matrices [8, 16].

We are now ready to provide our result for the full two-loop soft anomalous dimension matrix,

$$\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(2)} = \frac{K}{2}\,\mathbf{\Gamma}_{S_{\mathrm{f}}}^{(1)}\,. \tag{10}$$

Here $K = C_A(67/18 - \zeta(2)) - 10T_F n_F/9$ is the same constant appearing in the relation between the one- and two-loop Sudakov, or "cusp" anomalous dimensions [17, 18]: $\gamma_K^{[i]} = 2C_i(\alpha_s/\pi)[1 + (\alpha_s/\pi)K/2]$. Remarkably, relationship (10) holds for an arbitrary $2\to n$ process, even though the two-loop diagrams shown in Fig. 1 apparently couple together the color factors of three eikonal (Wilson) lines coherently. We derive Eq. (10) using the color generator formalism; however, the result is completely general, and applies to explicit matrix representations such as Eq. (9).

Following the method described in detail at one loop in Ref. [7], and extended to two loops in Ref. [10], the

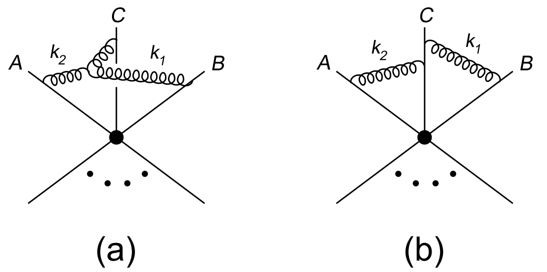

FIG. 1: Two-loop diagrams involving three eikonal lines.

two-loop anomalous dimension is found from the residue of single-pole terms in suitable combinations of Wilson lines computed at two loops. The simplicity of the result (10) follows from the special properties of the diagrams of Fig. 1, which connect three different eikonal lines. First consider Fig. 1a, where a three-gluon coupling ties together three eikonals labelled v_A, v_B and v_C, in an otherwise arbitrary eikonal process. We shall prove that this integral is zero, as long as we take the eikonals v_A and v_B to be lightlike.

Since v_A and v_B are lightlike, we choose a frame in which $v_A^\mu = \delta^{\mu -}$ and $v_B^\mu = \delta^{\mu +}$. Components transverse to v_A and v_B carry the subscript T. The eikonal integral in momentum space is then

$$
\begin{aligned}
F_{1a}(v_A, v_B, v_C) \;&=\; \int d^D k_1 d^D k_2 \, \frac{1}{k_1^2 + i\epsilon} \, \frac{1}{k_2^2 + i\epsilon} \, \frac{1}{(k_1 + k_2)^2 + i\epsilon} \, \frac{1}{k_1^- + i\epsilon} \, \frac{1}{k_2^+ + i\epsilon} \\
&\times \frac{\left[v_C^- \left(k_1^+ - k_2^+\right) + v_C^+ \left(k_1^- - k_2^-\right) - v_{C,T} \cdot (k_{1,T} - k_{2,T}) + v_C^+ \, \left(k_1^- + 2k_2^-\right) + v_C^- \, \left(-2k_1^+ - k_2^+\right) \right]}{v_C^- \left(k_1^+ + k_2^+\right) + v_C^+ \left(k_1^- + k_2^-\right) - v_{C,T} \cdot (k_{1,T} + k_{2,T}) + i\epsilon} \, , \qquad (11)
\end{aligned}
$$

where the term in square brackets is the three-gluon vertex momentum factor. We now introduce a change of variables (with unit Jacobean) from momenta k_i^μ to $\bar{k}_i^\mu$,

$$
\begin{aligned}
\left(k_1^+, k_1^-, k_{1,T}\right) \;&=\; \left(\zeta \, \bar{k}_2^- \, , \, \zeta^{-1} \, \bar{k}_2^+ \, , \, \bar{k}_{2,T}\right) \, , \\
\left(k_2^+, k_2^-, k_{2,T}\right) \;&=\; \left(\zeta \, \bar{k}_1^- \, , \, \zeta^{-1} \, \bar{k}_1^+ \, , \, \bar{k}_{1,T}\right) \, , \qquad (12)
\end{aligned}
$$

where $\zeta = v_C^+ / v_C^-$. It provides an expression identical to $F_{1a}(v_A, v_B, v_C)$, Eq. (11), but of the opposite sign. The integral corresponding to Fig. 1a therefore vanishes.

Regarding Fig. 1b, the same change of variables yields $1/[(v_C \cdot (k_1 + k_2))(v_C \cdot k_1)] = 1/[(v_C \cdot (\bar{k}_1 + \bar{k}_2))(v_C \cdot \bar{k}_2)]$, from which it is easy to show that this diagram reduces to the product of one-loop diagrams, and so does not contribute to the two-loop anomalous dimension. Indeed, the only nontrivial contributions to $\mathbf{\Gamma}_{S_\mathrm{f}}^{(2)}$ at two loops involve only two eikonal lines. Using results from refs. [17, 18], the color structure of these contributions reduces to that of a single gluon exchange. The sum of the diagrams then modifies the one-loop result by the same multiplicative factor as for the cusp anomalous dimension, which gives Eq. (10).

The explicit expression for single poles in $2 \to n$ amplitudes is easily found from Eqs. (4) and (6) using the explicit form of the two-loop matrix (10),

$$
\begin{aligned}
&\left| \mathcal{M}_\mathrm{f}^{(2)} \right\rangle^{\text{(single pole)}} \;= \qquad (13) \\
&\frac{1}{\varepsilon} \left[\sum_{i \in \mathrm{f}} \left(- \frac{\mathcal{G}_0^{[i](2)}}{8} + \frac{\beta_0 \, \mathcal{G}^{[i](1)\prime}}{32} \right) + \frac{K}{8} \, \mathbf{\Gamma}_{S_\mathrm{f}}^{(1)} \right] \, \left| \mathcal{M}_\mathrm{f}^{(0)} \right\rangle \\
&- \frac{1}{4\varepsilon} \sum_{i \in \mathrm{f}} \mathcal{G}_0^{[i](1)} \, \left| \mathcal{H}_\mathrm{f}^{(1)}(0) \right\rangle - \sum_{i \in \mathrm{f}} \frac{1}{8\varepsilon} \, \gamma_K^{[i](1)} \, \left| \mathcal{H}_\mathrm{f}^{(1)\prime}(0) \right\rangle \, .
\end{aligned}
$$

Here we normalize the one-loop hard scattering by absorbing into it all finite terms from the jet functions, order by order, and $|\mathcal{H}_\mathrm{f}^{(1)}(0)\rangle$, $|\mathcal{H}_\mathrm{f}^{(1)\prime}(0)\rangle$ are this function and its derivative with respect to ε, respectively, evaluated at $\varepsilon = 0$. (This absorption is possible to any loop order because the jets are diagonal in color.) Explicit comparison [10] shows that this simple result agrees with all single-pole terms found at $2 \to 2$ in the literature, as summarized for example in Refs. [19, 20]. It also predicts all such poles in a $2 \to n$ process, once the one-loop hard part $\left| \mathcal{H}_\mathrm{f}^{(1)} \right\rangle$ is known.

We remark that an analogous anomalous dimension matrix for Wilson lines has been computed in Ref. [18], in the forward limit $t \to 0$. This limit is a singular one, with respect to our arguments regarding Fig. 1; thus our results and theirs are not directly comparable.

It is also worth remarking on the relationship between our results and the influential alternative formalism of Ref. [12], in which both pole and finite terms are put into an exponential form to two loops. We may think of these as alternative schemes for organizing infrared poles. When explicit calculations are organized according to the scheme of Ref. [12], more complex color products are found, namely $i f_{abc} \mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_k^c = -[\mathbf{T}_i \cdot \mathbf{T}_j \, , \, \mathbf{T}_j \cdot \mathbf{T}_k]$, appearing in the matrix $\hat{\mathbf{H}}^{(2)}$ at order $1/\varepsilon$ [19, 20]. Such products are not encountered in the resummation scheme described above. These differences in color structure, however, are by no means disagreements. They arise from a particular commutator, between the one-loop finite terms that are exponentiated in the formalism of Ref. [12], and the one-loop soft anomalous dimension matrix $\mathbf{\Gamma}_{S_\mathrm{f}}^{(1)}$. The result of performing the commutator [10] agrees with the form of $\hat{\mathbf{H}}^{(2)}$ in Ref. [19] for $2 \to 2$ processes, and with that proposed in Ref. [21] for 2 gluon $\to$ n gluon processes, based on consistency of collinear limits.

Given an explicit two-loop amplitude, the strategy described here may be reversed, and $\mathbf{\Gamma}^{(2)}_{S_{\rm f}}$ extracted directly from the amplitude. This approach was adopted in Ref. [22] for the case of quark-quark elastic scattering, in the context of electroweak Sudakov corrections. The original version of Ref. [22] differs from ours due to omission of the commutator contribution described above. The authors have informed us that a revision is in preparation.

Similar remarks apply to the color-diagonal single poles given in Eq. (13). These coefficients do not equal the corresponding coefficients $H_i^{(2)}$ in the formalism of Ref. [12], but they are connected [23]. The difference can be related precisely to the different treatment of finite terms in the two approaches [10].

In addition to clarifying the structure of singular terms in calculations of $2 \to 2$ processes at two loops, the results outlined here have potentially useful consequences and suggest further directions of research. Eq. (13) predicts the two-loop pole structure for any $2 \to n$ process, in color-generator form, for any process whose one-loop hard function is known to $\mathcal{O}(\varepsilon)$.

Another practical consequence is that, because the one- and two-loop anomalous dimensions are proportional, all terms in the expansion of the soft function commute to next-to-next-to-leading level (NNLL), and in this approximation, the ordering operator P can be dropped in Eq. (6). Thus, once the color eigenstates of the one-loop matrix are known, the same states will diagonalize the two-loop matrix. A semi-numerical approach, bypassing diagonalization, is to simply exponentiate the relevant matrices in any convenient basis [9]. Given the relation (10), this is now possible at NNLL as well as NLL.

The study of these matrices for processes beyond $2 \to 2$, already begun in Ref. [24], is clearly an important challenge. Another intriguing question is whether the proportionality (10) might extend beyond two loops, whether in QCD or any of its allied gauge theories. If so, it could have consequences for the interpretation of infrared diverences in the relevant theory. The extension, and/or modification of the results above for the production of massive colored particles is another important direction for research.

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