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Supersymmetry with Small μ: Connections between Naturalness, Dark Matter, and (Possibly) Flavor

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Abstract

Weak scale supersymmetric theories often suffer from several naturalness problems: the problems of reproducing the correct scale for electroweak symmetry breaking, the correct abundance for dark matter, and small rates for flavor violating processes. We argue that the first two problems point to particular regions of parameter space in models with weak scale supersymmetry: those with a small μ term. This has an interesting implication on direct dark matter detection experiments. We find that, if the signs of the three gaugino mass parameters are all equal, we can obtain a solid lower bound on the spin-independent neutralino-nucleon cross section, $\sigma_{\rm SI}$. In the case that the gaugino masses satisfy the unified mass relations, we obtain $\sigma_{\rm SI} \gtrsim 4 \times 10^{-46}$ cm^2 (1×10^{-46} cm^2) for fine-tuning in electroweak symmetry breaking no worse than 10% (5%). We also discuss a possibility that the three problems listed above are all connected to the hierarchy of fermion masses. This occurs if supersymmetry breaking and electroweak symmetry breaking (the Higgs fields) are coupled to matter fields with similar hierarchical structures. The discovery of $\mu \to e$ transition processes in near future experiments is predicted in such a framework.

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1 Introduction

Weak scale supersymmetry provides an elegant framework to solve the naturalness problem of the standard model as well as to explain the dark matter of the universe. An extreme sensitivity of the weak scale to ultraviolet physics in the standard model is softened due to the existence of superparticles at this scale, and the stable lightest supersymmetric particle (LSP) left over from the early history of the universe composes dark matter today. These qualitative successes, however, should now be reviewed much more carefully. On one hand, non-discovery of both superparticles and a light Higgs boson at LEP II [1] raises the overall mass scale for the superparticles, leading to a tension with naturalness of electroweak symmetry breaking. On the other hand, the accurate measurement of the dark matter density by WMAP [2] gives a precise constraint on the spectrum of superparticles. It is plausible that a careful study of these issues provides strong hints on a possible realization of supersymmetry at the weak scale.

Phenomenology of supersymmetric theories depends strongly on how fundamental supersymmetry breaking is mediated to the supersymmetric standard model sector. What is the underlying mechanism of the mediation? A promising possibility arises if mediation occurs through gravitationally suppressed interactions. This has a virtue that the supersymmetric mass term for the Higgs doublets (μ term) is naturally generated with the weak scale size, because it can arise as a sort of supersymmetry breaking term in this case [3]. This provides an elegant "solution" to the μ problem, which plagues many other scenarios for supersymmetry breaking. Another virtue of mediation by gravitational strength interactions is that the gravitino is likely to be heavier than the lightest neutralino, giving a possibility of weakly interacting massive particle (WIMP) dark matter. Such a mediation is also minimal in the sense that it does not require any other physics than that at the gravitational scale, which we know exists. We thus mainly focus on this class of mediation – gravity mediation broadly defined – in this paper, and study what current experimental data imply on the structure of superparticle spectra. We argue that the current data strongly suggest that the μ term is small, $|\mu| \lesssim (200\sim400)$ GeV, regardless of any details of supersymmetry breaking. This in turn has a striking consequence in the context of the minimal supersymmetric standard model (MSSM) — the cross section for direct dark matter detection is large and can naturally be in the range where the CDMS II experiment will explore in the next two years. In fact, we can obtain a solid lower bound on the cross section if the signs of the gaugino masses are universal. In the case that the gaugino masses satisfy the unified mass relations, we obtain $\sigma \gtrsim 4 \times 10^{-46}$ cm^2 (1×10^{-46} cm^2) for fine-tuning in electroweak symmetry breaking no worse than 10% (5%).

An important consequence of mediation by gravitational scale interactions is that it opens up a window to connect physics at the weak scale to that at high energies, such as the Planck or unification scale, since the low energy scalar (squark, slepton and Higgs boson) masses carry all information from the Planck to the weak scales through their renormalization group evolutions. The possibility of perturbative extrapolations of physics across a vast energy interval, in fact, is a unique feature of theories with weak scale supersymmetry and supported by the successful unification of gauge couplings at a scale of $\approx 10^{16}$ GeV [4]. We thus explore possible implications of low energy spectra suggested by the current data (small μ term) on physics at the gravitational scale. We find that a desired patten of superparticle spectra is obtained if physics of supersymmetry breaking mediation is intimately related to that of flavor. This has interesting implications on low energy experiments exploring flavor violation. We present an explicit scheme incorporating these ideas, which we call next to minimal supergravity, and estimate rates of various flavor violating processes in this framework. We find that some of these processes, such as $\mu \to e\gamma$, are naturally close to the current experimental bounds, so that they are expected to be within the reach of near future experiments.

The organization of the paper is as follows. In section 2 we consider a connection between naturalness in electroweak symmetry breaking and physics of dark matter. We see that a small value for the μ parameter, required from naturalness of electroweak symmetry breaking, leads quite naturally to a thermal relic abundance of the lightest neutralino consistent with the WMAP data. We also perform a "model independent" analysis of the direct detection cross section for such dark matter, and find that it is generically large. In section 3 we explore possible high energy theories that lead to superparticle spectra identified in section 2. As one of such theories, we consider a scenario in which physics of supersymmetry breaking is intimately related to that of flavor. We estimate various flavor violating processes, and find that lepton flavor violating processes in the first two generations are generically large and close to the current experimental bounds. Finally, conclusions are given in section 4.

2 Small μ Term: Connection between Electroweak Naturalness and Dark Matter

In this section we see a close connection between naturalness of electroweak symmetry breaking and physics of dark matter, suggested by the data from LEP II and WMAP. These data both seem to suggest a certain parameter space in weak scale supersymmetry. We find that this has an interesting consequence on the detection of dark matter in the context of the MSSM.

Let us begin our discussion by reviewing the situation in minimal supergravity (mSUGRA) [5]: a popular scenario for gravitational mediation, in which certain flavor universal interactions between the supersymmetry breaking sector and the supersymmetric standard model sector are assumed to be responsible for the mediation. This scenario provides a simple parameterization of relevant physics at the Planck scale. In the simplest case, the supersymmetry breaking masses at the gravitational/unification scale are parameterized by five free parameters: the universal gaugino mass $M_{1/2}$, the universal scalar squared mass m_0^2, the universal scalar trilinear interaction A_0, the supersymmetric Higgs mass μ, and the holomorphic supersymmetry breaking Higgs squared mass μB. While this setup leads to qualitatively correct low energy physics, it has become gradually clearer that it does not seem to give a very good description of our world at the quantitative level. In particular, given the current experimental constraints on the superparticle and the Higgs boson masses, a typical parameter region of mSUGRA leads to too large electroweak symmetry breaking and too large relic abundance for the dark matter. The five independent parameters must be finely tuned to reproduce the observed electroweak symmetry breaking scale as well as the correct amount of the dark matter, determined precisely by the recent WMAP data [2]. In view of these unpleasant situations, it seems clear that we must deviate from the simplest mSUGRA scenario to account for latest observations in a natural manner.

What direction should we take? Looking at carefully the problems of the simplest mSUGRA described above, we find that these seemingly unrelated problems are in fact somewhat correlated. Let us first consider the issue of electroweak symmetry breaking. For reasonably large values of $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$, e.g. $\tan\beta \gtrsim 3$, which is suggested by the LEP II lower bound on the Higgs boson mass [1], the equation determining the electroweak symmetry breaking scale is given by

$$\frac{M_{\rm Higgs}^2}{2} \simeq -m_{H_u}^2 - |\mu|^2, \tag{1}$$

where $M_{\rm Higgs}$ represents the mass of the lightest CP-even Higgs boson, and $m_{H_u}^2$ the soft supersymmetry breaking squared mass for the up-type Higgs boson H_u ($m_{H_u}^2 < 0$). In the MSSM, $M_{\rm Higgs}$ cannot be larger than $\simeq 120$ GeV without significant fine-tuning of parameters. This implies that each term in the right-hand-side of Eq. (1) cannot be much larger than $\simeq (90~{\rm GeV})^2$ if we want to avoid fine-tuning between two independent parameters $m_{H_u}^2$ and μ. In fact, this is not so straightforward to achieve, because $m_{H_u}^2$ generically receives large radiative corrections from top-stop loops, which are logarithmically enhanced for the case of gravity mediation. Indeed, in the simplest mSUGRA, $|m_{H_u}^2|$ is generically quite large at the weak scale, leading to fine-tuning in electroweak symmetry breaking.

While lowering $|m_{H_u}^2|$ is an issue, which has been one of the main focuses in efforts trying to reduce fine-tuning, there is a general consequence of Eq. (1) which applies to any theories that do not extend the Higgs sector drastically at the weak scale — in order for a theory to be natural, the μ term should not be larger than 100 GeV by more than a factor of a few *no matter what the mechanism of reducing* $|m_{H_u}^2|$ *is.* Requiring that the cancellation between the two terms in Eq. (1) is no worse than Δ^{-1}, we obtain

$$|\mu| \lesssim (270~{\rm GeV}) \left(\frac{10\%}{\Delta^{-1}} \right)^{1/2}, \tag{2}$$

for $M_{\rm Higgs} \simeq 120$ GeV. This has a striking consequence on physics of dark matter. Since the lightest neutralino, which is assumed to be the LSP, contains a non-negligible mixture of the Higgsino, the relic abundance is reduced compared with a typical mSUGRA parameter region, reproducing the observed dark matter abundance quite naturally. A large Higgsino fraction in the lightest neutralino also dramatically increases the possibility of detecting the dark matter, as we will see shortly. A connection between naturalness of electroweak symmetry breaking and the detectability of dark matter has been emphasized in Ref. [6] in the context of a model solving the fine-tuning problem (for earlier work, see e.g. [7]), and neutralino dark matter with non-negligible Higgsino mixture has been studied recently in various contexts, e.g., in [8 – 19] (see e.g. [20 – 22] for earlier work).[1]

The argument described above suggests that a key point of making supersymmetric theories natural is to lower the value of μ, and thus the value of $|m_{H_u}^2|$. This has the following far reaching implications in the context of minimal supersymmetric models [24, 25]. First, barring the possibility of accidental cancellations, small values of $|m_{H_u}^2|$ imply that the top squarks should be light because $|m_{H_u}^2|$ receives radiative corrections proportional to the squared masses of the top squarks, $m_{\tilde{t}}^2$, through the $O(1)$ top Yukawa coupling. Then, to satisfy the LEP II bound on the physical Higgs boson mass $M_{\rm Higgs} \gtrsim 114$ GeV, the scalar trilinear interaction for the top squarks, A_t, should be large at the weak scale:

$$\left| \frac{A_t}{m_{\tilde{t}}} \right| \approx (1.5 \sim 2.5). \tag{3}$$

In gravity mediation, this leads to the following two requirements on the soft supersymmetry breaking parameters at the unification scale M_U: the value of A_t should be non-zero and negative, $A_t < 0$, and the gluino mass M_3 should be reasonably large, $M_3 \gtrsim 150$ GeV. (Throughout the paper, our sign convention for μ and the soft supersymmetry breaking parameters follows that of SUSY Les Houches Accord [26]. For the case of non-universal gaugino masses, we take the convention that the gluino mass parameter, M_3, is positive.) These requirements come from the fact that we cannot obtain large enough $|A_t/m_{\tilde{t}}|$ at the weak scale for $A_t(M_U) = 0$, and yet the sensitivity of low energy A_t to $A_t(M_U)$ is so weak that we also need a renormalization group contribution to A_t from M_3, which drives A_t negative at the infrared. This has a consequence

[1] In fact, smallness of $|\mu|$ should generally be true in any theory (not necessarily the MSSM) which does not have severe fine-tuning. For example, this is true for μB-driven electroweak symmetry breaking discussed in Ref. [23], where the equation determining the electroweak scale differs from that in Eq. (1). (The bound in this case is somewhat weaker and is given by $|\mu| \lesssim \lambda\,(390 \text{ GeV})(10\%/\Delta^{-1})^{1/2}$, where $v \simeq 174$ GeV and $\lambda \lesssim (2 \sim 3)$ in general.) If there is a singlet field whose vacuum expectation value (VEV) contributes to the Higgsino mass, we should simply replace $|\mu|$ by $|\mu_{\rm eff}|$ involving the singlet VEV. Implications on dark matter physics, discussed in this paper, then mostly persist in these extended models, unless the LSP contains a significant amount of the singlino. In particular, the correct thermal relic abundance is naturally obtained for small $\mu_{\rm eff}$, since it does not depend very strongly on the value of $\tan\beta$.

that

$$A_t < 0, \tag{4}$$

at the weak scale. Another important constraint on the top squark sector is that the top squarks should be light. From consideration of infrared contribution to $m_{H_u}^2$ from $m_{\tilde{t}}^2$ alone, we find that $m_{\tilde{t}}^2$ should not be much larger than $\approx (300\ \mathrm{GeV})^2$. With these light top squarks, the lightest Higgs boson can evade the LEP II bound only if the tree-level contribution is sizable. In the MSSM, this leads to the bound

$$\tan\beta \gtrsim 5. \tag{5}$$

Note that the conditions of Eqs. (3 – 5) should be satisfied for any minimal and natural supersymmetric theories in which supersymmetry breaking is mediated to the supersymmetric standard model sector through gravitationally suppressed interactions.

With the sign of A_t given by Eq. (4), the constraint from the $b \to s\gamma$ process prefers the sign of μ to be positive:

$$\mu > 0. \tag{6}$$

This is because, for $A_t < 0$, the contributions from chargino and charged Higgs boson loops interfere destructively (constructively) for $\mu > 0$ (< 0) in the amplitude, so that the case of negative μ is almost excluded. With $\mu > 0$, the constraint from the muon anomalous magnetic moment prefers

$$M_2 > 0. \tag{7}$$

It is interesting that the same sign is suggested for M_2 as M_3, so that the simple assumption of universal gaugino masses is not disfavored by these considerations.

We now consider the implication of Eqs. (2 – 7) on the detectability of neutralino dark matter. To naturally reproduce the correct abundance as a thermal relic, we assume that the lightest neutralino χ is mostly the bino, but containing sizable fractions of the Higgsino components:

$$\chi = N_{\chi 1}\,\tilde{B} + N_{\chi 2}\,\tilde{W}^0 + N_{\chi 3}\,\tilde{h}_d^0 + N_{\chi 4}\,\tilde{h}_u^0, \tag{8}$$

where $\tilde{B}$ represents the bino, $\tilde{W}^0$, $\tilde{h}_d^0$ and $\tilde{h}_u^0$ represent the neutral components of the wino, down-type Higgsino and up-type Higgsino, respectively, and the coefficients $N_{\chi i}$ ($i = 1, \cdots, 4$) are given by

$$N_{\chi 1} \simeq 1, \tag{9}$$

$$N_{\chi 2} \simeq 0, \tag{10}$$

$$N_{\chi 3} \simeq \frac{m_Z \sin\theta_W (\mu \sin\beta + M_1 \cos\beta)}{\mu^2 - M_1^2}, \tag{11}$$

$$N_{\chi 4} \simeq -\frac{m_Z \sin\theta_W (\mu \cos\beta + M_1 \sin\beta)}{\mu^2 - M_1^2}. \tag{12}$$

Here, m_Z is the Z boson mass, θ_W the Weinberg angle, and M_1 the bino mass parameter. The mass for χ is then given by

$$m_\chi \simeq |M_1|. \tag{13}$$

Note that M_1 can in principle take either sign, although the sign is positive if the three gaugino mass parameters carry the same sign, as in the case of universal gaugino masses.

In the parameter region relevant to us, the cross sections between the neutralino dark matter, χ, and nuclei are dominated by the t-channel Higgs boson exchange diagrams. There are two contributions coming from the lighter and heavier neutral Higgs boson exchanges, which are generically comparable in size. We then find that the spin-independent cross section between χ and nuclei, normalized to the nucleon [27], is approximately given by

$$\sigma_{\rm SI} \simeq \frac{g'^4 m_N^4}{4\pi} \frac{\mu^2}{(\mu^2 - M_1^2)^2} \left[\frac{X_d \tan\beta}{m_H^2} + \frac{(X_u + X_d)\left(\frac{2}{\tan\beta} + \frac{M_1}{\mu}\right)}{m_h^2} \right]^2, \tag{14}$$

where $g' \simeq 0.36$ is the $U(1)_Y$ gauge coupling, $m_N \simeq 1$ GeV is the nucleon mass, and X_u and X_d are linear combinations of nucleon matrix elements, which we conservatively take as $X_u \simeq 0.14$ and $X_d \simeq 0.24$ [28]. Here, we have used the decoupling approximation for the Higgs sector, $\alpha \simeq \beta - \pi/2$ with α the neutral Higgs boson mixing angle, and m_H and m_h represent the masses of the heavier and lighter CP-even Higgs bosons, respectively. Comparing with the results of more precise numerical computations, we find that the above simple formula reproduces the numerical results within about a factor of two in most of the parameter space. These errors are comparable or smaller than those coming from uncertainties for the matrix elements.

The formula of Eq. (14) has several interesting implications. First, together with Eq. (6), it implies that the contributions from the heavy and light Higgs bosons always interfere constructively if the sign of M_1 is positive, as in the case of universal gaugino masses. This allows us to give a solid lower bound on $\sigma_{\rm SI}$ as a function of μ, M_1, $\tan\beta$, m_H and m_h. Since $\tan\beta \gtrsim 5$ (see Eq. (5)) and $m_h \lesssim 120$ GeV, we can then obtain a *model independent* lower bound on $\sigma_{\rm SI}$ as a function of $m_\chi \simeq M_1$, once we fix the values of μ and m_H. The value of μ is directly constrained from above for a given value of allowed fine-tuning Δ^{-1} (from Eq. (2)). The mass of the heavy Higgs boson, m_H, does not have a definite upper bound available from the current data. However, since we are mainly interested in the case where all the superparticle masses are of order a few hundred GeV, it is natural to expect that m_H is also not much larger than these values. Motivated by this, we have plotted in Fig. 1 the allowed region of $\sigma_{\rm SI}$ in the case of (a) $\mu = 270$ GeV and (b) $\mu = 380$ GeV, corresponding to $\Delta^{-1} = 10\%$ and $\Delta^{-1} = 5\%$ respectively, for $m_A = 250$ GeV and 400 GeV. Here, m_A is the mass of the pseudo-scalar Higgs boson, which is related to m_H by $m_H^2 \simeq m_A^2 + m_Z^2 \sin^2 2\beta$ at tree level, and we have used the exact formula for the Higgs-boson mediated dark-matter detection cross section, rather than

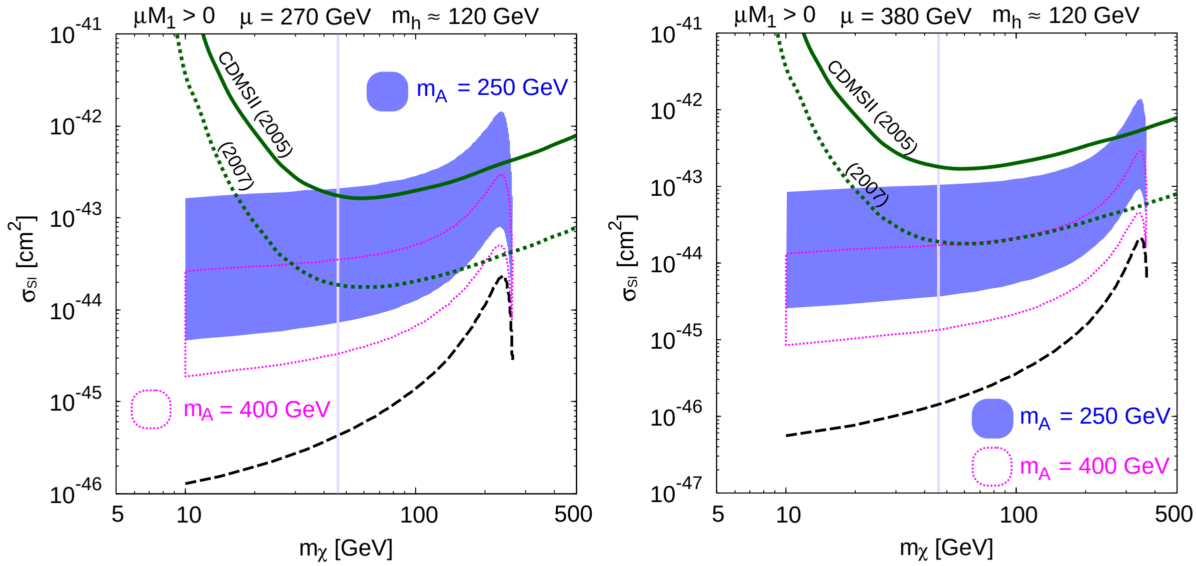

Figure 1: The allowed range of the spin-independent cross section between the dark matter and nucleon, $\sigma_{\rm SI}$, for (a) $\mu = 270$ GeV ($\Delta^{-1} = 10\%$) and (b) $\mu = 380$ GeV ($\Delta^{-1} = 5\%$) in the case of $\mu M_1 > 0$. The two regions in each plot correspond to $m_A = 250$ GeV (shaded) and 400 GeV (region inside the dotted lines), and $\tan\beta$ is varied within $5 < \tan\beta < 50$. The long dashed lines correspond to the smallest possible cross section, obtained for $m_A, \tan\beta \to \infty$.

the approximate formula of Eq. (14). In the figure, we have also depicted the vertical line at $m_\chi = 46$ GeV, which is the lower bound on the mass of χ in the case that the gaugino masses satisfy the unified mass relations. The value of $\tan\beta$ is varied between 5 and 50 for each value of m_A. While the regions close to the upper edges (very large $\tan\beta$) are constrained by the $B_s \to \mu^+\mu^-$ process [14, 17], we find that it does not significantly affect the allowed region of $\sigma_{\rm SI}$. (The constraint from $b \to s\gamma$ can be satisfied depending on other parameters.) In the figure, we have also drawn the exclusion curve from the latest CDMS II data [29] by a solid line, and an estimate for the expected future sensitivity (an order of magnitude improvement of the current bound by the end of 2007 [30]) by a dashed line.

From the figure, we find that the prospect for dark matter detection at CDMS II is rather promising for $\mu = 270$ GeV, corresponding to $\Delta^{-1} = 10\%$. For $m_A = 250$ GeV, the CDMS II covers the allowed parameter region almost entirely (especially if $m_\chi \gtrsim 46$ GeV due to the unified gaugino mass relations), although we must be somewhat fortunate for larger values of m_A. For $\mu = 380$ GeV, corresponding to $\Delta^{-1} = 5\%$, the discovery prospect is not as good as the case of $\mu = 270$ GeV, but there is still a room that the dark matter is discovered at CDMS II for $m_A \lesssim 400$ GeV. It is interesting to point out that the cross section of Eq. (14) takes the smallest

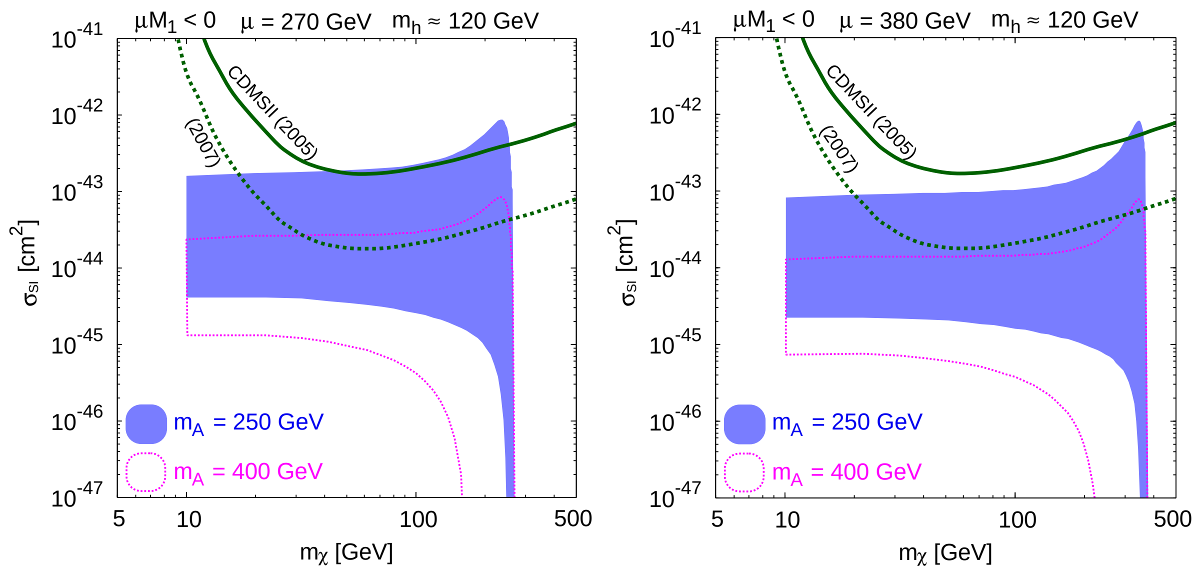

Figure 2: The allowed range of the spin-independent cross section between the dark matter and nucleon, $\sigma_{\rm SI}$, for (a) $\mu = 270$ GeV ($\Delta^{-1} = 10\%$) and (b) $\mu = 380$ GeV ($\Delta^{-1} = 5\%$) in the case of $\mu M_1 < 0$. The two regions in each plot correspond to $m_A = 250$ GeV (shaded) and 400 GeV (region inside the dotted lines), and $\tan\beta$ is varied within $5 < \tan\beta < 50$.

value at $m_H, \tan\beta \to \infty$ for fixed values of μ and M_1. These values are depicted by long dashed lines, as a function of m_χ, in Fig. 1 (a), (b). We find that, for $\mu M_1 > 0$, the dark matter-nucleon spin-independent cross section, $\sigma_{\rm SI}$, due to Higgs boson exchange has an absolute lower bound:

$$\sigma_{\rm SI} \gtrsim 1 \times 10^{-46}\ {\rm cm}^2 \quad (5 \times 10^{-47}\ {\rm cm}^2), \tag{15}$$

for $\mu \leq 270$ GeV (380 GeV), corresponding to $\Delta^{-1} \gtrsim 10\%$ (5%). In the case that the gaugino masses satisfy the unified mass relations ($m_\chi \gtrsim 46$ GeV), the bound becomes

$$\sigma_{\rm SI} \gtrsim 4 \times 10^{-46}\ {\rm cm}^2 \quad (1 \times 10^{-46}\ {\rm cm}^2). \tag{16}$$

Barring the possibility of accidental cancellations with other contributions, such as the top squark exchange contribution, these provide *the naturalness lower bounds on the dark matter detection cross section* (for $\mu M_1 > 0$). These ranges of $\sigma_{\rm SI}$ can be explored by future ton-scale detector experiments, such as XENON [31]. In fact, the allowed regions can be almost entirely covered if $m_\chi \gtrsim 46$ GeV.

The case of $\mu M_1 < 0$, on the other hand, admits a possibility of cancellation between the two contributions from heavy and light Higgs boson exchange. For relatively large $\tan\beta$, an excessive cancellation occurs for $m_h^2/m_H^2 \sim 1.5\, M_1/\mu \tan\beta$. In Fig. 2, we have plotted the allowed region of

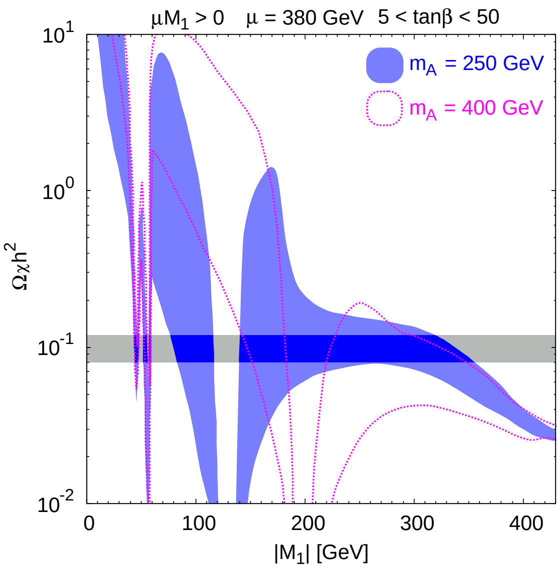

Figure 3: The thermal relic abundance of the lightest neutralino $\Omega_\chi h^2$ for $\mu = 380$ GeV for both signs of M_1: $\mu M_1 > 0$ (left) and $\mu M_1 < 0$ (right). The two regions in each plot correspond to $m_A = 250$ GeV (shaded) and 400 GeV (region inside the dotted lines), and $\tan\beta$ is varied within $5 < \tan\beta < 50$. The WMAP data for the dark matter energy density $0.08 < \Omega_{\rm DM}h^2 < 0.12$ is also indicated.

$\sigma_{\rm SI}$ for $\mu M_1 < 0$ in the case of (a) $\mu = 270$ GeV ($\Delta^{-1} = 10\%$) and (b) $\mu = 380$ GeV ($\Delta^{-1} = 5\%$) for $m_A = 250$ GeV and 400 GeV. Compared with the case of $\mu M_1 > 0$ in Fig. 1, we find that the cross section can be much smaller, especially when the cancellation takes place, although the naturalness lower bound of Eq. (15) can still provide a rough guide on a typical range of the direct detection cross section.

Another important consequence of a small μ term is the reduction of the relic abundance of the lightest neutralino. It is well known that the bino-dominated neutralino is over abundant unless either coannihilation with a slepton/stop is possible or s-channel diagrams mediated by the pseudo-scalar Higgs boson are resonantly enhanced. This is, however, a prediction of the "finely tuned" MSSM, such as the simplest mSUGRA, which typically gives a relatively large μ parameter. Once we have a small μ term, the annihilation cross section into, for example, Zh is significantly enhanced because it depends on μ by M_1^2/μ^4 (W^+W^- mode is proportional to $m_Z^4 M_1^2/\mu^8$ for $\mu \gg M_1$). Therefore, in models with *natural* electroweak symmetry breaking, the correct size of the dark matter abundance $\Omega_\chi h^2 \simeq 0.1$ can be *naturally* obtained without living in somewhat fortunate parameter regions such as $M_1 \simeq m_{\tilde{l}}$ or $M_1 \simeq m_A/2$.

As an example, we show in Fig. 3 the thermal relic abundance $\Omega_\chi h^2$ of the neutralino for

$\mu = 380$ GeV for both signs of M_1: $\mu M_1 > 0$ (left) and $\mu M_1 < 0$ (right). The relic abundance has been calculated using the DarkSUSY package [32]. The value of $\tan\beta$ is varied within $5 < \tan\beta < 50$. In the calculation, we have neglected the effect of the slepton/squark mediated annihilation processes as well as coannihilation effects, which depend on additional parameters. This does not affect the value of $\Omega_\chi h^2$ significantly, unless sleptons/squarks are very light or degenerate with the lightest neutralino. For each value of m_A, we can see the effect of the A-pole resonance when $|M_1| \sim m_A/2$. (We also see the effects of the h-pole and Z-pole resonances at around $|M_1| \sim 50$ GeV.) We find that values of $\Omega_\chi h^2$ consistent with the WMAP data ($0.08 < \Omega_{\rm DM} h^2 < 0.12$ at the 2σ level) are obtained for M_1 that are not necessarily close to the pole. In fact, we find that quite wide ranges of M_1 accommodate the observed value of $\Omega_\chi h^2$, due to the smallness of the μ parameter. (For larger values of m_A, the regions with the A-pole resonance disappear from the range of the figure, but we can still reproduce the observed value of $\Omega_{\rm DM} h^2$ naturally with $|M_1| \lesssim 400$ GeV. For example, for $m_A = 800$ GeV, the WMAP range of $\Omega_{\rm DM} h^2$ can be reproduced for $|M_1| \approx (300 \sim 400)$ GeV ($\approx (300 \sim 350)$ GeV) for $\mu M_1 > 0$ (< 0).)

Once we assume that the mechanism of the dark matter production is (dominantly) the thermal one, we can further constrain the predicted regions for the dark matter detection cross section, given in Figs. 1 and 2. In Fig. 4, we have shown the allowed regions in the m_χ-$\sigma_{\rm SI}$ plane, under the assumption that the correct dark matter abundance, $0.08 < \Omega_\chi h^2 < 0.12$, is obtained thermally ($\mu M_1 > 0$ in the left panel and $\mu M_1 < 0$ in the right). In drawing these regions, we have scanned $5 < \tan\beta < 50$, 270 GeV $< \mu <$ 380 GeV, and 250 GeV $< m_A <$ 400 GeV. In each case of $\mu M_1 > 0$ and < 0, we can clearly see that only a part of the region in Fig. 1 (or Fig. 2) survives. In the case of $\mu M_1 > 0$, for example, the region with small m_χ disappears, and we obtain a lower bound of $\sigma_{\rm SI}$: $\sigma_{\rm SI} \gtrsim 3 \times 10^{-45}$ cm^2 for $m_A < 400$ GeV.

A simple analysis presented above shows a remarkably close connection between naturalness of electroweak symmetry breaking and physics of dark matter. This interplay can be used to narrow down parameter space of weak scale supersymmetry effectively. Suppose, for example, that the mass and the cross section with a nucleon are measured for the dark matter by a direct detection experiment. This will give information on the Higgs sector parameters, such as m_A and $\tan\beta$, which are not easy to measure at the LHC. On the other hand, in the parameter region under consideration, the LHC can be very powerful in determining the structure of the neutralino sector, as was shown e.g. in [33]. Indirect detection of dark matter, for example at GLAST [34], is also very promising in this parameter region. The key point is relatively small values for the μ parameter. As we have seen, this is required both from naturalness of electroweak symmetry breaking and dark matter physics. It is interesting that this parameter region is also favorable for the LHC and for future dark matter detection experiments.

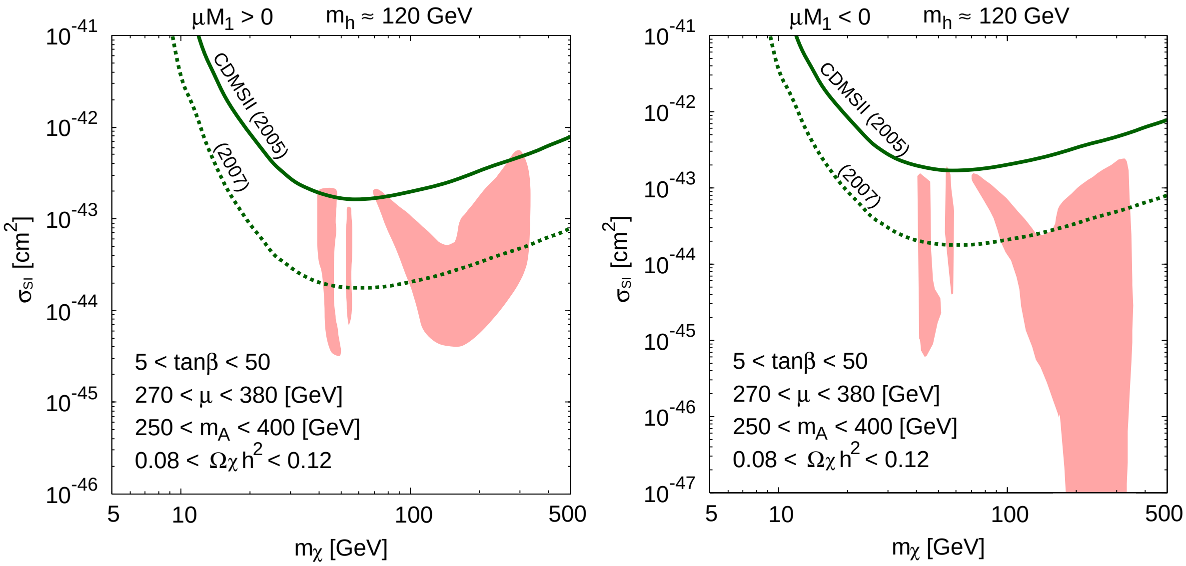

Figure 4: The spin-independent cross section between the dark matter and nucleon, $\sigma_{\rm SI}$, for $\mu M_1 > 0$ (left) and $\mu M_1 < 0$ (right). The parameters are scanned in the range $5 < \tan\beta < 50$, $270~{\rm GeV} < \mu < 380~{\rm GeV}$, and $250~{\rm GeV} < m_A < 400~{\rm GeV}$, and only the parameter sets consistent with $0.08 < \Omega_\chi h^2 < 0.12$ are used to draw the regions.

In the next section, we present a scenario that naturally leads to the parameter region identified in this section, in the framework of gravity mediation (broadly defined). We find that the scenario has nontrivial implications on physics at the gravitational scale as well as on low-energy flavor violating processes.

3 Next to Minimal Supergravity: Connection between Supersymmetry Breaking and Flavor

In this section we consider possible implications of the observations made in the last section. A key point for making gravity mediation work better is to render the low-energy value of the μ parameter small. It is well known that this can be achieved by making $m_{H_u}^2$ larger than the other scalar squared masses at the gravitational (or unification) scale. This is because the low-energy value of $m_{H_u}^2$ then becomes less negative, compared with the simplest mSUGRA case, which in turn leads to smaller values for the μ parameter (see Eq. (1)). Non-universal gaugino masses also help in this respect. For M_3 smaller than M_1 and M_2 at the high energy threshold, the low-energy values of the top squark masses are smaller than in the mSUGRA case, leading to less negative $m_{H_u}^2$, and thus smaller μ, at the weak scale.

We note that while making μ small is a *necessary* condition to reduce fine-tuning, it is certainly not a *sufficient* condition. For instance, if we need a cancellation to make a low-energy value of $m_{H_u}^2$ less negative (which leads to small μ), then it means that we simply moved the "place" where a cancellation/fine-tuning takes place. The improvement of fine-tuning in the case of large $m_{H_u}^2$ at a high scale is thus nontrivial. It arises from the fact that the contribution from top-stop loop is effectively reduced in renormalization group evolutions from high to low scales. While the resulting reduction of fine-tuning is mild in this case, here we take it as an example of theories achieving (partially) the goals envisioned in the previous section and study its possible implications on low energy physics.[2]

What could the underlying reason be, leading to $m_{H_u}^2$ larger than the other scalar squared masses? An interesting possibility is that supersymmetry breaking and electroweak symmetry breaking (the Higgs fields) reside "at the same location" in some "space." In this case, the observed Yukawa couplings imply that the third generation matter lives "closer" to this location, while the lighter generations live "far away" from it. Such a setup suppresses the Yukawa couplings for light generations, keeping those for the third generation unsuppressed. This also suppresses direct (possibly flavor violating) contributions in the supersymmetry breaking masses of the light generation sfermions. The masses for these particles are then generated through standard model gauge interactions, avoiding the supersymmetric flavor problem. The Higgs bosons and the third generation sfermions (top squarks), on the other hand, can obtain direct contributions from supersymmetry breaking. Thus, they can have different supersymmetry breaking masses than those of the light generation sfermions. This setup, therefore, reproduces the pattern suggested by the low energy data.

The "space" described above can be real geometrical spacetime. For example, it may be extra space dimensions which are slightly larger than the fundamental scale, e.g. string scale, in which case direct interactions between fields localized in different positions are suppressed. The resulting suppressions are $\approx e^{-d^2}$ (e^{-d}) if the interactions are generated by stringy effects [37] (by exchange of massive fields [38]), although the suppression factors are, in general, arbitrary [39]. Alternatively, the "separations" in "space" may be obtained effectively as a result of strong (nearly conformal) gauge dynamics, giving the Higgs and supersymmetry breaking fields as composite states. The low-energy Yukawa couplings and supersymmetry breaking operators are then obtained through mixings between elementary and composite quark and lepton states. In fact, this latter picture is obtained as a 4D "dual" picture of the former, if the extra dimension

[2]We emphasize, however, that the analysis in the previous section is much more general and applies to any theories in which the low energy value of μ ($> M_1$) is small. For instance, such a spectrum can be obtained by making some of the (third generation) squark squared masses small (or even negative) at the unification scale, as discussed in [24, 35]. In fact, this setup can be trivially accommodated in the framework discussed in this subsection. Alternative possibilities include lowering the effective messenger scale (to some intermediate scales) by mixing moduli and anomaly mediated contributions to supersymmetry breaking [36, 24].

is one dimensional and warped [40].[3] For earlier work on connecting structures of the Yukawa couplings and supersymmetry breaking parameters, see e.g. [41 – 45].

Instead of working out the detailed underlying mechanism, here we adopt a (useful) phenomenological parametrization of the situations described above. This parameterization captures essential features of our general setup and provides a simple stage for phenomenological analyses. Suppose we consider a 4D supergravity theory. We assume that all the interactions in the superspace Kähler density $\mathcal{F}$, the gauge kinetic functions f^a, and the superpotential W are of order unity in units of the fundamental scale $M_* \sim M_{\rm Pl}$, except that the (non-holomorphic) quadratic terms in $\mathcal{F}$ have arbitrary "wavefunction factors." Here, $M_{\rm Pl}$ is the 4D reduced Planck scale, and $\mathcal{F}$ is related to the Kähler potential K by $K = -3M_{\rm Pl}^2 \ln(-\mathcal{F}/3M_{\rm Pl}^2)$. Denoting the supersymmetry breaking field as X, this leads to the following form for $\mathcal{F}$, f^a and W:

$$\mathcal{F} = -3M_{\rm Pl}^2 + \sum_r \mathcal{Z}_r \Phi_r^\dagger \Phi_r + \mathcal{Z}_X X^\dagger X + \sum_r (X + X^\dagger)\Phi_r^\dagger \Phi_r + \sum_r X^\dagger X \Phi_r^\dagger \Phi_r + \cdots, \tag{17}$$

$$f^a = \frac{1}{g_a^2} + X + \cdots, \tag{18}$$

$$\begin{aligned} W = {} & QUH_u + QDH_d + LEH_d \ \ (+LNH_u) \\ & + XQUH_u + XQDH_d + XLEH_d \ \ (+XLNH_u), \end{aligned} \tag{19}$$

where we have set $M_* = 1$ and omitted order one coefficients. The chiral superfields Φ_r represent the MSSM fields: $3 \times \{Q, U, D, L, E\}$, H_u and H_d (and $3 \times N$ if we introduce right-handed neutrinos). The superscript a in f^a denotes $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, with g_a the corresponding gauge coupling, and $\mathcal{Z}_r$ and $\mathcal{Z}_X$ are the "wavefunction factors" for the MSSM fields and the field X, respectively. The generation indices are omitted in the superpotential.

The structure given in Eqs. (17 – 19) leads, after canonically normalizing fields, to the following pattern for the Yukawa couplings:

$$(y_u)_{ij} \approx \epsilon_{Q_i}\epsilon_{U_j}\epsilon_{H_u}, \qquad (y_d)_{ij} \approx \epsilon_{Q_i}\epsilon_{D_j}\epsilon_{H_d}, \tag{20}$$

$$(y_e)_{ij} \approx \epsilon_{L_i}\epsilon_{E_j}\epsilon_{H_d}, \qquad ((y_\nu)_{ij} \approx \epsilon_{L_i}\epsilon_{N_j}\epsilon_{H_u}), \tag{21}$$

where $\epsilon_r \equiv \mathcal{Z}_r^{-1/2}$, $i, j, = 1, 2, 3$ represent the generation indices, and y_u, y_d, y_e and y_ν are defined by

$$W = (y_u)_{ij} Q_i U_j H_u + (y_d)_{ij} Q_i D_j H_d + (y_e)_{ij} L_i E_j H_d \ \ (+(y_\nu)_{ij} L_i N_j H_u). \tag{22}$$

The soft supersymmetry breaking parameters and the supersymmetric Higgs mass, μ, take the following form. For the non-holomorphic scalar squared masses, we have

$$m_{\tilde{f}}^2 \approx m_0^2 + \epsilon_f^2 \epsilon_X^2 m^2 + \epsilon_f^4 \epsilon_X^2 m^2, \tag{23}$$

[3] A similar "separation" phenomenon may also be obtained through direct couplings of matter fields to strong conformal gauge dynamics [41].

$$m_{H_u}^2 \approx m_0^2 + \epsilon_{H_u}^2 \epsilon_X^2 m^2 + \epsilon_{H_u}^4 \epsilon_X^2 m^2, \qquad m_{H_d}^2 \approx m_0^2 + \epsilon_{H_d}^2 \epsilon_X^2 m^2 + \epsilon_{H_d}^4 \epsilon_X^2 m^2, \tag{24}$$

where $f = Q_i, U_i, D_i, L_i, E_i$ (and N_i), and m represents a generic mass of order F_X/M_* with F_X being the auxiliary field VEV in the canonically normalized basis. Here, we have added a universal scalar squared mass term m_0^2. This term does not arise from Eqs. (17 – 19), but it may appear in general through flavor universal mediations across the "space." For example, the gravity force is proportional to the wavefunction factors $\mathcal{Z}_r$ and $\mathcal{Z}_X$, so there may be a term $\mathcal{Z}_r \mathcal{Z}_X X^\dagger X \Phi_r^\dagger \Phi_r$ whose flavor structure is aligned to the kinetic terms $\mathcal{Z}_r$. It may also appear through exchange of flavor universal bulk states in the extra dimensional setup.

The limit of a large $\mathcal{Z}_X$ factor (without the m_0^2 term) corresponds to the standard "sequestering" case, and this class of scenarios is widely studied as a solution to the supersymmetric flavor problem [38, 46]. Here we take instead a similar but different approach to solving the flavor problem, by sequestering light generations from the supersymmetry breaking sector by making $\mathcal{Z}_r$ large. This setup has a virtue that the fermion mass hierarchy is simultaneously explained (see Eqs. (20, 21)). Given that the top quark has an $O(1)$ Yukawa coupling, $\mathcal{Z}_r$ for the (up-type) Higgs field should not be large, which is a desired situation from naturalness of the electroweak symmetry breaking.

For the A parameters, we have

$$(A_u)_{ij} \approx \epsilon_X m + (\epsilon_{Q_i}^2 + \epsilon_{U_j}^2 + \epsilon_{H_u}^2)\epsilon_X m, \qquad (A_d)_{ij} \approx \epsilon_X m + (\epsilon_{Q_i}^2 + \epsilon_{D_j}^2 + \epsilon_{H_d}^2)\epsilon_X m, \tag{25}$$

$$(A_e)_{ij} \approx \epsilon_X m + (\epsilon_{L_i}^2 + \epsilon_{E_j}^2 + \epsilon_{H_d}^2)\epsilon_X m, \qquad ((A_\nu)_{ij} \approx \epsilon_X m + (\epsilon_{L_i}^2 + \epsilon_{N_j}^2 + \epsilon_{H_u}^2)\epsilon_X m). \tag{26}$$

Here, we have assumed that the superpotential terms containing X (the second line in Eq. (19)) are present. If these terms are absent for some reason, the first term in each expression disappears. (Our definition for the A parameters is such that a scalar trilinear coupling is given by the product of the Yukawa coupling and the A parameter, e.g., $\mathcal{L} = -\sum_{i,j} (y_u)_{ij} (A_u)_{ij} \tilde{q}_i \tilde{u}_j H_u + \text{h.c.}$.) In addition, the flavor universal contribution, A_0, may be present for the same reason as m_0^2 in the scalar masses, which we have omitted in the above formulae.

The gaugino masses are given by

$$M_1 \approx \epsilon_X m, \qquad M_2 \approx \epsilon_X m, \qquad M_3 \approx \epsilon_X m. \tag{27}$$

Note that m represents a generic mass of order F_X/M_*, and we are not necessarily limiting ourselves to the case of universal gaugino masses.

The μ and μB terms are generated by the terms $H_u H_d$, $(X + X^\dagger) H_u H_d$ and $X^\dagger X H_u H_d$ in $\mathcal{F}$, which are not explicitly denoted in Eq. (17). They are given by

$$\mu \approx \epsilon_{H_u} \epsilon_{H_d} m_{3/2} + \epsilon_{H_u} \epsilon_{H_d} \epsilon_X m, \tag{28}$$

$$\mu B \approx \epsilon_{H_u} \epsilon_{H_d} m_{3/2}^2 + \epsilon_{H_u} \epsilon_{H_d} \epsilon_X m \, m_{3/2} + \epsilon_{H_u} \epsilon_{H_d} \epsilon_X^2 m^2, \tag{29}$$

where $m_{3/2}$ represents a generic mass of order the gravitino mass, and the first, second (and third) terms in Eq. (28) (Eq. (29)) arise from $H_u H_d$, $(X + X^\dagger) H_u H_d$ and $X^\dagger X H_u H_d$ in $\mathcal{F}$, respectively. Here, we have assumed $\langle X \rangle = 0$ and a generic value (phase) of F_X, for simplicity.

The structures given in Eqs. (20, 21, 23 – 29) represent the results of our particular assumption of Eqs. (17 – 19). In fact, this provides a parameterization for very large classes of theories, larger than the naive picture described above (i.e. the Higgs and supersymmetry breaking reside in the same "location"). This allows us to consider various interesting scenarios for supersymmetry breaking. For example, we can take $\epsilon_X \approx 10^{-2}$ and set $m \approx m_{3/2} \approx (10 \sim 100)$ TeV. In this case, anomaly mediation [38, 47] can give comparable contributions to the direct contributions given above. (The standard problem associated with the μ and μB terms must be solved, for example by replacing μ by a singlet field VEV.) While it is interesting to enumerate all these possibilities, here we instead concentrate on the simplest case arising from the naive picture that supersymmetry and electroweak symmetry breaking are in the same "location." In particular, we take $\epsilon_X \sim \epsilon_{H_u} \sim \epsilon_{H_d} = O(1)$ and $m \sim m_{3/2}$. We also assume that all the other ϵ's are smaller than ~ 1. Under these assumptions, the Yukawa couplings take the form

$$(y_u)_{ij} \approx \epsilon_{Q_i} \epsilon_{U_j}, \qquad (y_d)_{ij} \approx \epsilon_{Q_i} \epsilon_{D_j}, \qquad (y_e)_{ij} \approx \epsilon_{L_i} \epsilon_{E_j}, \qquad ((y_\nu)_{ij} \approx \epsilon_{L_i} \epsilon_{N_j}), \tag{30}$$

and the soft supersymmetry breaking and μ parameters

$$M_1 \approx \epsilon_X m, \qquad M_2 \approx \epsilon_X m, \qquad M_3 \approx \epsilon_X m, \tag{31}$$

$$(m^2_{\tilde{f}})_{ij} \approx m_0^2 + \epsilon_{f_i} \epsilon_{f_j} m^2, \qquad m^2_{H_u} \approx m_0^2 + m^2, \qquad m^2_{H_d} \approx m_0^2 + m^2, \tag{32}$$

$$(A_u)_{ij} \approx m + (\epsilon^2_{Q_i} + \epsilon^2_{U_j} + \delta_{ij}) m, \qquad (A_d)_{ij} \approx m + (\epsilon^2_{Q_i} + \epsilon^2_{D_j} + \delta_{ij}) m, \tag{33}$$

$$(A_e)_{ij} \approx m + (\epsilon^2_{L_i} + \epsilon^2_{E_j} + \delta_{ij}) m, \qquad ((A_\nu)_{ij} \approx m + (\epsilon^2_{L_i} + \epsilon^2_{N_j} + \delta_{ij}) m), \tag{34}$$

$$\mu \approx m, \qquad \mu B \approx m^2, \tag{35}$$

where $f = Q_i, U_i, D_i, L_i, E_i$ (and N_i), and m represents a generic mass parameter of order the weak scale. Note that the first terms in the A-term formulae (Eqs. (33, 34)) arises from the superpotential terms containing X (the second line in Eq. (19)), which can have nontrivial flavor dependences. The flavor universal contributions $\delta_{ij} m$ in the A terms originate from the $\epsilon^2_{H_u} \epsilon_X m$ or $\epsilon^2_{H_d} \epsilon_X m$ term, as well as from the A_0 term. These equations determine the correlations between the structures of the Yukawa couplings and the soft supersymmetry breaking parameters.

Let us now study consequences of Eqs. (30 – 35). We first consider the case where the $SU(5)$ relations are satisfied at the unification scale $M_U \approx 10^{16}$ GeV:

$$\epsilon_{Q_i} = \epsilon_{U_i} = \epsilon_{E_i}, \qquad \epsilon_{D_i} = \epsilon_{L_i}, \tag{36}$$

$$M_1 = M_2 = M_3, \tag{37}$$

$$m^2_{\tilde{Q}_i} = m^2_{\tilde{U}_i} = m^2_{\tilde{E}_i}, \qquad m^2_{\tilde{D}_i} = m^2_{\tilde{L}_i}, \tag{38}$$

$$(A_d)_{ij} = (A_e)_{ij}, \qquad ((A_u)_{ij} = (A_\nu)_{ij}), \tag{39}$$

although this need not be the case. (Unwanted fermion mass relations for the first two generations must be corrected somehow.)

With this assumption, we can determine the order of magnitude for the ϵ parameters from the fermion masses. From the Yukawa coupling of the up-type quarks, $\epsilon_i^{(10)}$ ($\equiv \epsilon_{Q_i} = \epsilon_{U_i} = \epsilon_{E_i}$) are obtained as

$$\epsilon_i^{(10)} \simeq \sqrt{(y_u)_{ii}} \simeq (3 \times 10^{-3},\ 4 \times 10^{-2},\ 8 \times 10^{-1}). \tag{40}$$

The $\epsilon_i^{(\bar{5})}$ ($\equiv \epsilon_{D_i} = \epsilon_{L_i}$) factors can be estimated by two ways; from down-type quarks or charged leptons. Those are given by

$$\epsilon_i^{(\bar{5})} \simeq \frac{(y_d)_{ii}}{\sqrt{(y_u)_{ii}}} \simeq \tan\beta \times (4 \times 10^{-3},\ 5 \times 10^{-3},\ 9 \times 10^{-3}), \tag{41}$$

and

$$\epsilon_i^{(\bar{5})} \simeq \frac{(y_e)_{ii}}{\sqrt{(y_u)_{ii}}} \simeq \tan\beta \times (1 \times 10^{-3},\ 1 \times 10^{-2},\ 1 \times 10^{-2}). \tag{42}$$

It is interesting that the above two are very similar. Moreover, the less hierarchical structure of $\epsilon_i^{(\bar{5})}$ is also consistent with large mixing angles and small mass hierarchies in the neutrino sector, provided that the neutrino masses are of the Majorana type $(LH_u)^2$ [49]. Equations (40 – 42) imply that, unless $\tan\beta$ is very large, only the third generation **10** multiplet, Q_3, U_3 and E_3, has an $O(1)$ ϵ factor.

3.1 Impact on electroweak symmetry breaking

Supersymmetry breaking parameters in our setup are essentially a modification of the mSUGRA ones in the Higgs and third generation sfermion sectors, due to $O(1)$ ϵ factors. Because the coefficients of the operators in Eqs. (17 – 19) are free parameters, the relevant parameters for electroweak symmetry breaking, $m^2_{H_u}$, $m^2_{H_d}$, μ, μB, $m^2_{\tilde{Q}_3}$, $m^2_{\tilde{U}_3}$, and M_3 at the unification scale, M_U, can all be taken as independent free parameters. Equivalently, we can treat m_A, μ, $\tan\beta$, and v ($\equiv (\langle H_u\rangle^2 + \langle H_d\rangle^2)^{1/2} = 174$ GeV) at a low energy as input parameters, instead of $m^2_{H_u}$, $m^2_{H_d}$, μ, and μB. The free parameters of our electroweak symmetry breaking analysis can thus be taken as m_A, μ and $\tan\beta$ at the weak scale, as well as $m^2_{\tilde{Q}_3}$, $m^2_{\tilde{U}_3}$ and M_3 at M_U.

It was shown in Ref. [24] that fine-tuning in electroweak symmetry breaking can be improved from the mSUGRA case by relaxing (one of) the relations among those parameters. The least fine-tuned region requires a large A_t parameter to avoid the constraint from the Higgs boson

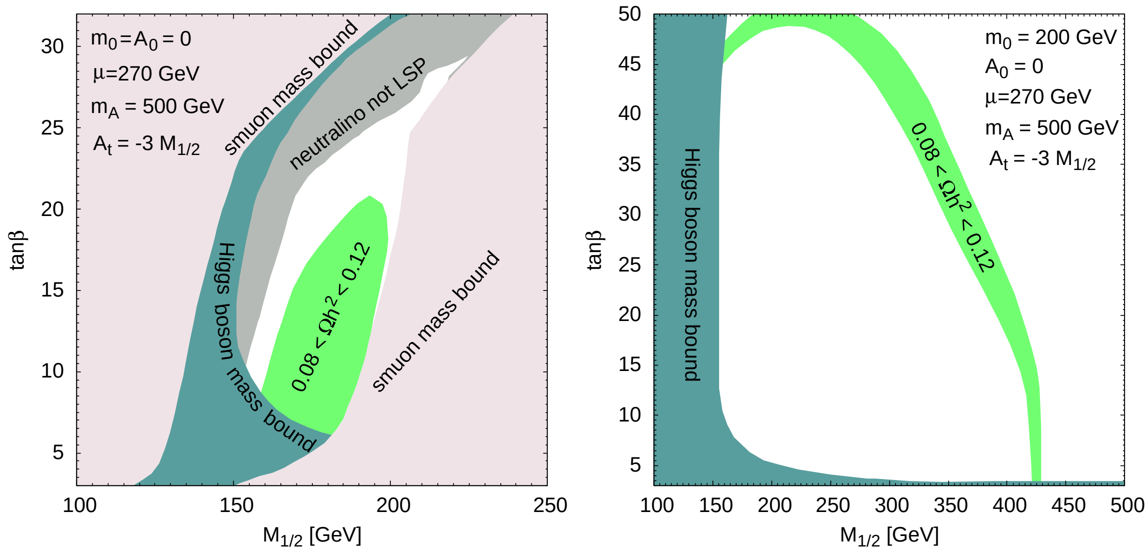

Figure 5: Viable parameter regions for the model with $m_0 = 0$ (left) and 200 GeV (right). We have fixed the values of $\mu = 270$ GeV and $m_A = 500$ GeV at low energies. Gaugino masses at the unification scale $M_{1/2}$ are taken to be universal and we take $A_t = -3M_{1/2}$ for the top squarks at the unification scale. The regions with correct dark matter abundance are also indicated.

mass bound. A large A_t term is naturally obtained in our setup, even if the contribution from the superpotential term $XQUH_u$ and the universal contribution A_0 are absent, due to the terms from the Kähler potential. (In fact, the absence of the superpotential term is somewhat favored by the constraints from flavor violating processes, as we discuss later.) With a large A_t term, the gluino and stop masses, $M_{\tilde{g}}$, $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$, can be as low as ≈ 440 GeV, ≈ 120 GeV and ≈ 430 GeV, respectively, without contradicting with the experimental constraints, including the Higgs boson mass bound. With these relatively small supersymmetry breaking parameters, significant fine-tuning among fundamental parameters is not needed to reproduce the correct scale for electroweak symmetry breaking.

In Fig. 5, we show viable parameter regions for the cases of $m_0 = 0$ and 200 GeV. The gaugino masses are taken to be universal $M_{1/2} \equiv M_1 = M_2 = M_3$ at M_U, and we have fixed the sign of μ to be $\mu M_{1/2} > 0$, motivated by the constraints from $b \to s\gamma$ and the muon anomalous magnetic moment. The values of third generation sfermion squared masses are chosen to be $m^2_{\tilde{Q}_3} = m^2_{\tilde{U}_3} = m^2_{\tilde{E}_3} = (\epsilon_3^{(10)} M_{1/2})^2$ and $m^2_{\tilde{D}_3} = m^2_{\tilde{L}_3} = (\epsilon_3^{(10)} \epsilon_3^{(\bar{5})}) M^2_{1/2}$ at M_U.

We find that there is a parameter region for $m_0 = 0$, which may be interesting from a theoretical point of view, since it arises from a simple form of Eqs. (17 – 19). In fact, the existence of the region is nontrivial. On one hand, $M_{1/2}$ should be larger than (120$\sim$200) GeV, since the first two generation sleptons obtain their masses only from one-loop running effects. On the other hand, the one-loop induced D-term for $U(1)_Y$ gives negative contributions to the right-handed slepton masses if $m_{H_d}^2$ is smaller than $m_{H_u}^2$ at M_U. With fixed values of μ and m_A, this happens when the gaugino mass is large, putting an upper bound on the gaugino mass. The Higgs boson mass bound is not severe in this model, since we can have a large A_t term, which is fixed to be $A_t = -3M_{1/2}$ at M_U in the figure. The excluded region is indicated, where we have imposed a conservative bound of $M_{\rm Higgs} > 113$ GeV to take into account theoretical uncertainties in the Higgs boson mass calculation. A relatively large value of $m_A = 500$ GeV also helps to obtain large values of $\tan\beta \approx (10 \sim 20)$ naturally. It is quite interesting that the region actually exists at a relatively low supersymmetry breaking scale, which is desired from naturalness. Solving the electroweak VEV, v, as a function of high energy parameter, and finding the severest cancellation among different contributions, we find that fine-tuning of the viable parameter region in Fig. 5 (left) is or order $(5\sim10)\%$, which is better than the simplest mSUGRA case. We note that a nonvanishing (positive) value of $m_{\tilde{Q}_3}^2 = m_{\tilde{U}_3}^2 = m_{\tilde{E}_3}^2$ at M_U helps to have values of $M_{1/2}$ as small as $\simeq 170$ GeV, by providing a positive contribution to the right-handed stau mass. Without them, the value of $M_{1/2}$ is pushed up to above $\simeq 200$ GeV making fine-tuning somewhat worse, although we can still obtain a consistent parameter region in this case.[4]

For $m_0 = 200$ GeV, there is no constraint from the slepton masses. The bound from the Higgs boson mass is $M_{1/2} \gtrsim 150$ GeV for large A_t. Reproducing the dark matter abundance, however, requires somewhat larger values of $M_{1/2}$ or $\tan\beta$, so electroweak fine-tuning is not as good as the case of $m_0 = 0$.

As explained before, a small μ term is an inevitable consequence of naturalness in electroweak symmetry breaking, which also provides a natural way of explaining the observed dark matter abundance. The regions with a correct dark matter abundance are superimposed in Fig. 5. It is clear that we do not have to live in special regions of the parameter space, such as $m_\chi \sim m_A/2$. Moreover, since the sign of μM_1 is positive, the direct detection of the neutralino dark matter is promising in this scenario.

[4] In fact, we do not need the entire scaling of soft supersymmetry breaking parameters in Eqs. (31 – 35) to obtain a desired parameter region in Fig. 5 (left). All we need are nonvanishing squared masses for the third generation **10** scalars at M_U (in addition to the gaugino masses and nonvanishing A terms). This arises in any setup where the first two generation (and the third generation in $\mathbf{5}^*$) fields are separated from supersymmetry breaking while the third generation (in **10**) and Higgs fields are not. A detailed analysis on this and related possibilities will be presented elsewhere.

3.2 Impact on flavor physics

With the flavor structure of supersymmetry breaking terms deduced from the fermion masses through Eqs. (31 – 35), we can make predictions on the magnitude of flavor violation in the low energy observables. Although we cannot precisely calculate the rates of the processes due to $O(1)$ ambiguities, simple relations between the rates and the Yukawa structure can be obtained. For the analysis of flavor violating processes, we follow the method of Ref. [50], where various constraints on flavor mixing parameters are listed.

The most important source of flavor violation comes from off-diagonal components of the A terms that arise from the couplings between the supersymmetry breaking field X and the MSSM fields in the superpotential (the second line in Eq. (19)). This is because, although the flavor mixings are suppressed by ϵ factors, these suppressions are compensated by the fact that an A-term insertion flips the chirality of the sfermion, and thus eliminates one factor of the Yukawa coupling from the amplitude. We thus first consider the effects of these terms, and later consider the case where these terms are somehow absent.

For $\mu \rightarrow e$ transition processes, the ratio of the off-diagonal to the diagonal components of the mass matrix, $(\delta^l_{12})_{LR}$ and $(\delta^l_{12})_{RL}$, are given by

$$(\delta^l_{12})_{LR} \simeq \epsilon_1^{(\bar{5})} \epsilon_2^{(10)} \left(\frac{\langle H_d \rangle}{m_{\rm SUSY}} \right) \simeq 4 \times 10^{-5} \left(\frac{v}{m_{\rm SUSY}} \right), \tag{43}$$

$$(\delta^l_{12})_{RL} \simeq \epsilon_2^{(\bar{5})} \epsilon_1^{(10)} \left(\frac{\langle H_d \rangle}{m_{\rm SUSY}} \right) \simeq 3 \times 10^{-5} \left(\frac{v}{m_{\rm SUSY}} \right), \tag{44}$$

where we have used $\epsilon_1^{(\bar{5})}$ obtained from the charged lepton masses in Eq. (42), and v and $m_{\rm SUSY}$ represent the Higgs VEV, $v \simeq 174$ GeV, and the mass scale of supersymmetry breaking parameters, respectively. Note that these mass insertion parameters do not depend on $\tan\beta$, in contrast to many supersymmetric models where the $\mu \rightarrow e$ transition amplitude is proportional to $\tan\beta$. Since the upper bounds on these variables from the branching ratio of the $\mu \rightarrow e\gamma$ decay are of order 10^{-6} (corresponding to $B(\mu \rightarrow e\gamma) \lesssim 10^{-11}$), we need somewhat small coefficients for the superpotential couplings between X and the MSSM fields, such as $O(0.1)$. Large supersymmetry breaking masses would help to suppress the amplitude, but it is disfavored from naturalness in electroweak symmetry breaking. This may imply that the superpotential couplings between X and the MSSM fields (at least for light generations) are somehow absent, although small couplings of $O(0.1)$ may still be regarded as "acceptable." In any case, if the superpotential couplings are present, we expect positive signals in future searches of $\mu \rightarrow e\gamma$ decay and the $\mu \rightarrow e$ conversion process in nuclei, which have sensitivities to the level of $O(10^{-8} \sim 10^{-7})$ in these parameters [51, 52].

For flavor violating τ decays, the following simple relations can be obtained:

$$\frac{B(\tau \to e\gamma)}{B(\mu \to e\gamma)} \sim 0.2 \left[\frac{\epsilon_1^{(\bar{5})} \epsilon_3^{(10)}}{\epsilon_1^{(\bar{5})} \epsilon_2^{(10)}} \frac{m_\mu}{m_\tau} \right]^2 \sim 0.2, \quad \frac{B(\tau \to \mu\gamma)}{B(\mu \to e\gamma)} \sim 0.2 \left[\frac{\epsilon_2^{(\bar{5})} \epsilon_3^{(10)}}{\epsilon_1^{(\bar{5})} \epsilon_2^{(10)}} \frac{m_\mu}{m_\tau} \right]^2 \sim 20. \tag{45}$$

Comparing with the current experimental sensitivities to the branching ratios of $O(10^{-7})$ for τ decays [53, 54] and $O(10^{-11})$ for $\mu \to e\gamma$ decay, we conclude that flavor violating τ decays are not likely to be observed in near future in this model.

Similar analyses can be performed for hadronic processes. For the K^0-$\bar{K}^0$ mixing, the contribution from the A term is simply

$$(\delta_{12}^d)_{LR} \simeq (\delta_{12}^l)_{RL}, \quad (\delta_{12}^d)_{RL} \simeq (\delta_{12}^l)_{LR}, \tag{46}$$

from the similarity of the down-type and charged-lepton Yukawa couplings. We find that the constraint from Δm_K, of $O(10^{-3})$, is much weaker than that from $\mu \to e\gamma$ decay. Other meson mixings such as Δm_{B_d}, Δm_{B_s} and Δm_D are also predicted to be much smaller than the experimental constraints. For the gluino mediated $b \to s\gamma$ decay, the most important mass insertion factor is:

$$(\delta_{23}^d)_{RL} \simeq \epsilon_2^{(\bar{5})} \epsilon_3^{(10)} \left(\frac{\langle H_d \rangle}{m_{\rm SUSY}} \right) \simeq 8 \times 10^{-3} \left(\frac{v}{m_{\rm SUSY}} \right). \tag{47}$$

The experimental constraint of $O(10^{-2})$ is marginally satisfied.

Let us now consider the case where the superpotential couplings between X and the MSSM fields are somehow suppressed. This setup is technically natural and does not require fine-tuning between parameters. In this case, contributions from the scalar mass terms in Eq. (32) become important sources of flavor violation, and we can repeat the same analysis as before to see the predictions. For $\mu \to e\gamma$ decay and the μ–e conversion in nuclei, the most significant bound comes from diagrams with $\tan\beta$ enhanced chirality flipping. The predictions of the model from such diagrams are parametrized as

$$(\delta_{12}^l)_{RL}^{\rm eff} \simeq \epsilon_1^{(10)} \epsilon_2^{(10)} \frac{m_\mu \tan\beta}{m_{\rm SUSY}} \simeq 4 \times 10^{-8} \tan\beta \left(\frac{v}{m_{\rm SUSY}} \right), \tag{48}$$

$$(\delta_{12}^l)_{LR}^{\rm eff} \simeq \epsilon_1^{(\bar{5})} \epsilon_2^{(\bar{5})} \frac{m_\mu \tan\beta}{m_{\rm SUSY}} \simeq 5 \times 10^{-9} \tan^3\!\beta \left(\frac{v}{m_{\rm SUSY}} \right). \tag{49}$$

Interestingly, the predictions are small enough to evade the current experimental constraints of $O(10^{-6})$ but large enough to be tested at future experiments. For $\tau \to e$ and $\tau \to \mu$ transitions, we obtain

$$\frac{B(\tau \to e\gamma)}{B(\mu \to e\gamma)} \sim 0.2 \left[\frac{\epsilon_1^{(\bar{5})} \epsilon_3^{(\bar{5})}}{\epsilon_1^{(\bar{5})} \epsilon_2^{(\bar{5})}} \right]^2 \sim 0.2, \quad \frac{B(\tau \to \mu\gamma)}{B(\mu \to e\gamma)} \sim 0.2 \left[\frac{\epsilon_2^{(\bar{5})} \epsilon_3^{(\bar{5})}}{\epsilon_1^{(\bar{5})} \epsilon_2^{(\bar{5})}} \right]^2 \sim 20, \tag{50}$$

for $\tan\beta \gtrsim 10$, finding the same relations as Eq. (45).

The largest contribution to the K^0-$\bar{K}^0$ mixing, Δm_K, comes from the mass insertion $(\delta^d_{12})_{RR}$:

$$(\delta^d_{12})_{RR} \simeq \epsilon_1^{(\bar{5})}\epsilon_2^{(\bar{5})} \simeq 1 \times 10^{-5}\tan^2\beta. \qquad (51)$$

The experimental constraint of order $10^{-2}\sim 10^{-1}$ can be easily satisfied unless $\tan\beta$ is extremely large. Relations among various meson mixings are predicted to be:

$$\frac{\Delta m_{B_d}}{\Delta m_K} \sim 1, \qquad \frac{\Delta m_{B_s}}{\Delta m_K} \sim 10^2, \qquad \frac{\Delta m_D}{\Delta m_K} \sim \frac{10^2}{\tan^4\beta}. \qquad (52)$$

With the experimental bound on Δm_K, it will be difficult to see deviations from the standard model predictions in B and D meson systems.

Finally, the gluino mediated $b \to s\gamma$ decay may occur through

$$(\delta^d_{23})^{\rm eff}_{RL} \simeq \epsilon_2^{(\bar{5})}\epsilon_3^{(\bar{5})}\frac{m_b\tan\beta}{m_{\rm SUSY}} \simeq 7 \times 10^{-7}\tan^3\beta\left(\frac{v}{m_{\rm SUSY}}\right). \qquad (53)$$

The experimental bound of order 10^{-2} is also satisfied.

3.3 Impact on inflation

In this subsection we make a comment on our assumption of large "wavefunction factors." Instead of introducing these factors in the function $\mathcal{F}$, as was done in Eqs. (17 – 19), we could introduce similar factors in the Kähler potential K (i.e. large factors only in front of the non-holomorphic quadratic terms in K). These two assumptions are physically distinct. For example, the "$\mathcal{F}$-based" case leads to (flavor universal) higher dimension operators in K, suppressed only by powers of the fundamental scale M_*. In the "K-based" case, on the other hand, these operators receive additional suppressions due to ϵ's (in the basis where fields are canonically normalized). A nonvanishing m_0^2 of order $m_{3/2}^2$ is also automatically obtained in this case. While the naive extra dimensional picture leads to the $\mathcal{F}$-based form, we do not find anything particularly wrong for the K-based form from a purely phenomenological point of view. (In particular, such an assumption is radiatively stable.)

It is interesting to point out that if we adopt the K-based assumption and introduce a large $\mathcal{Z}$ factor for a field, then the field has an almost minimal Kähler potential. This can provide a solution to the "η problem" of inflationary models, when applied to the inflaton field. This is because, assuming that the superpotential is (effectively) linear in the inflaton field, as in the case of hybrid inflation models, the minimal Kähler potential can avoid a generation of unwanted supergravity-induced mass term, of order the Hubble parameter, for the inflaton field [48]. It is interesting that another fine-tuning problem of supersymmetric models – the η problem – might be connected to the naturalness problems we are addressing, i.e. those of electroweak symmetry breaking, dark matter, and supersymmetric flavor.

3.4 Generalization of the model

The analyses so far have assumed unified relations on supersymmetry breaking parameters, such as universal gaugino masses and common wavefunction factors $\epsilon_{Q_i} \simeq \epsilon_{U_i} \simeq \epsilon_{E_i}$ and $\epsilon_{D_i} \simeq \epsilon_{L_i}$ for unified multiplets. In this subsection we discuss possible deviations from these assumptions.

We first consider the case where the universality of the gaugino masses is relaxed. We do not expect significant changes in Fig. 5 in this case. The allowed region for $m_0 = 0$ is mainly controlled by the right-handed slepton masses and the bino mass, which are determined only by M_1 and the one-loop induced $U(1)_Y$ D-term, and thus there is no significant effect by the non-universality. The Higgs boson mass, which is important for both cases of vanishing and nonvanishing m_0, is mainly determined by the A_t term and the top squark masses at low energies. These quantities can be significantly modified by changing the M_3 parameter, but its effects can be compensated by changing A_t, $m_{\tilde{Q}_3}^2$ and $m_{\tilde{U}_3}^2$ at the unification scale. The dark matter abundance is again mainly determined by M_1, μ, and m_A. Therefore, as far as the allowed region is concerned, the non-universality is not so important. However, it may be important for fine-tuning in electroweak symmetry breaking. Lowering M_3 leads to a suppression of the mass scale for all the colored superparticles, especially for the top squarks, reducing the one-loop correction to the Higgs mass squared parameter, $m_{H_u}^2$.

Modifications of the relations among different $\mathcal{Z}_r$ factors may cause quite dramatic changes in the predictions of flavor changing processes. However, the large branching ratio of $\mu \to e\gamma$ decay is quite generic, since

$$\max\left[(\delta^l_{12})_{LR}, (\delta^l_{12})_{RL}\right] \gtrsim 3 \times 10^{-5} \left(\frac{v}{m_{\rm SUSY}}\right), \tag{54}$$

$$\max\left[(\delta^l_{12})^{\rm eff}_{LR}, (\delta^l_{12})^{\rm eff}_{RL}\right] \gtrsim 1 \times 10^{-8} \tan^2\beta \left(\frac{v}{m_{\rm SUSY}}\right), \tag{55}$$

both of which result only from $\epsilon_{L_1}\epsilon_{E_1} \simeq (y_e)_{11} \simeq m_e/(v\cos\beta)$ and $\epsilon_{L_2}\epsilon_{E_2} \simeq (y_e)_{22} \simeq m_\mu/(v\cos\beta)$. Here, Eq. (54) is for the case where the superpotential couplings $XLHE$ are present with $O(1)$ coefficients, while Eq. (55) for the case where these couplings are suppressed.

4 Conclusions

For phenomenological analyses of supersymmetric models, the μ term is often taken not to be a fundamental input parameter but rather provided as a solution to the constraint equation for electroweak symmetry breaking. This conventional approach could hide phenomenologically important parameter regions. For example, in well-studied models such as minimal supergravity or gauge mediation models, large values are generally predicted for the μ parameter. However, this is by no means a general prediction of supersymmetric models. In fact, a large μ parameter is rather disfavored from a purely phenomenological point of view.

The large μ term in conventional models is caused by a large negative $m_{H_u}^2$ parameter at low energies, which requires a precise cancellation between $m_{H_u}^2$ and μ^2 in reproducing the correct scale for electroweak symmetry breaking, $v \simeq 174$ GeV. This cancellation is the source of the supersymmetric fine-tuning problem. Turning the argument around, once we assume that there is no such fine-tuning for some reason, the μ parameter should not be so large compared with the electroweak scale. In fact, this is even true in models with extended Higgs sectors. The effective μ parameter, which parametrizes the supersymmetric contribution to the Higgs potential (or the Higgsino mass), should not be large — no matter what its origin is.

After realizing that a large μ parameter is obtained only as a consequence of fine-tuning, it is sensible to take μ as an input parameter and study phenomenology of weak scale supersymmetry with a small μ parameter. The most striking effect is that the small μ term enhances the mixing between the Higgsino and the bino and significantly reduces the thermal relic abundance of the bino dark matter. We have shown that, with a small μ term, it is indeed quite easy to realize the neutralino dark matter without living in special parameter regions, such as near the A-pole or coannihilation regions. Furthermore, in such a situation, the detection rate for the neutralino dark matter in direct detection experiments is significantly enhanced. In the case where the gaugino masses are universal at the unification scale, we have obtained an absolute lower bound on the spin-independent cross section, $\sigma_{\rm SI} \gtrsim 10^{-46}$ cm^2, for electroweak fine-tuning no worse than $\approx 5\%$.

A possible realization of a small μ term is obtained by deviating from minimal supergravity by changing the $m_{H_u}^2$ parameter at the unification scale. We have presented a simple model to realize this situation, which is achieved by placing the supersymmetry breaking sector at the same "location" as the electroweak symmetry breaking sector (the Higgs fields). The hierarchy of the Yukawa couplings then implies that the third generation fields live "close" to the location of the Higgs fields, while the first two generations "away" from it. With this setup, the Higgs fields and the third generation sfermions feel supersymmetry breaking directly, while the first two generations only through renormalization group effects, which is desired for satisfying constraints from flavor changing processes. We have found that such a pattern of supersymmetry breaking masses indeed leads to viable parameter regions, where all the experimental constraints are satisfied and the dark matter of the universe is explained by the thermal relic abundance of the lightest neutralino. Making A_t large at the unification scale facilitates to evade the Higgs boson mass bound with a low overall scale of supersymmetry breaking masses, reducing fine-tuning (equivalent to reducing the μ parameter). Low-energy flavor violating processes are tightly related to the structure of the Yukawa couplings, so that their rates can be estimated. We have found that they are consistent with the current experimental bounds, but some of them are close. In particular, the $\mu \to e$ transition rates are predicted to be large, so that these processes should be discovered in near future experiments.

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