

# Finite Temperature Corrections to Relic Density Calculations

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## Abstract

In this paper we evaluate finite temperature corrections to the dark matter relic density within the context of minimal supersymmetry with a neutralino LSP. We identify several regions of parameter space where the WIMP annihilation cross section is especially sensitive to small corrections to the underlying parameters. In these regions, finite temperature effects have the potential to be important. However, we shall show by explicit calculation that these effects are small. In the regions we investigated, the maximal corrections are on the order of  $10^{-4}$  and are therefore negligible compared with theoretical and experimental uncertainties.

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# 1 Introduction

It is now well accepted that approximately 20% of the matter in the universe is in the form of a new non-baryonic neutral and weakly interacting particle whose microscopic properties have yet to be determined. This material has been termed dark matter. Experimentally, the ratio of the dark matter density, on cosmological scales, to the critical density has been measured within 10% to be  $\Omega_{DM} \sim 0.2$  [1]. Given a theory of dark matter, the relic density can be predicted by calculating the particle's annihilation cross sections and integrating the Boltzmann equation through decoupling to the present day. It is interesting to note that if we assume that the dark matter particle annihilates through weak interactions and has a mass on the order of the electroweak scale, we obtain the correct value for the relic density (see, for example, [2]). This fact has led many theorists to concentrate on weakly interacting massive particle (WIMP) models of dark matter.

Assuming that dark matter is composed of WIMPs that are in thermal equilibrium in the early universe, their density today is given approximately by the expression [3]

$$\Omega = \frac{s_0}{\rho_c} \left( \frac{45}{\pi g_*} \right)^{1/2} \frac{x_f}{m_{pl}} \frac{1}{\langle \sigma v \rangle} \quad (1)$$

where  $s_0$  is the current entropy density of the universe,  $\rho_c$  is the critical density,  $g_* \approx 80$  is the number of relativistic degrees of freedom at the time of freeze-out,  $\langle \sigma v \rangle$  is the thermally averaged annihilation cross section and  $x_f = m/T$  is the ratio of the WIMP mass to the freeze-out temperature. For weak scale interactions  $x_f \sim 25$ . From the above value of  $x_f$  it is clear that the WIMP is non relativistic at the time of freeze-out and for this reason it has long been supposed that finite temperature effects can be neglected when calculating the annihilation cross section. Indeed, these corrections tend to be suppressed either exponentially, or by several powers of  $T/m$  (see, for example, [4]). However, currently favored models of dark matter can often lie in regions of parameter space where the annihilation cross section is extremely sensitive to small variations in the underlying parameters. For this reason we consider the issue of finite temperature corrections to the relic density worth revisiting.

For definiteness we consider models of dark matter within the context of the minimal supersymmetric standard model (MSSM). Dark matter scenarios in the MSSM have been discussed by many authors. In particular, the parameter space of a subclass of models - minimal supergravity (mSUGRA) - has been extensively mapped out [5, 7, 8]. Viable mSUGRA models fall roughly into four parameter regions: the bulk region, the focus point region, the coannihilation region and the A-funnel. In the bulk region the superpartners are all relatively light and the annihilation is dominated by lepton production through P-wave t-channel and s-channel slepton exchange. In

the focus point region the scalar superpartners are very heavy. In this case the LSP obtains a significant higgsino and wino component and the annihilation is dominated by s-channel W and Z production. The coannihilation region is characterized by a nearly degenerate next-to-lightest super partner (NLSP). In this scenario the NLSP has a finite density during freeze-out and coannihilates with the LSP. Finally, the A-funnel region is characterized by  $m_A \sim 2m_{N_1}$  where  $m_A$  is the mass of the CP-odd heavy Higgs  $A^0$  and  $m_{N_1}$  is the mass of the LSP. In this region of parameter space the WIMP annihilations proceed mainly through resonant s-channel  $A^0$  exchange.

Three of the above regions – the bulk, the coannihilation region and the A funnel – exhibit sensitivity to one or more of the underlying MSSM parameters. In the bulk, the S-wave annihilation is helicity suppressed. The dominant channel is thus P-wave, which is itself suppressed by two powers of the WIMP velocity,  $v$ . Any enhancement to the helicity breaking masses of the standard model fermions would thus significantly enhance the overall annihilation cross section. In the A-funnel the WIMP annihilation cross section is naturally highly sensitive to the width of the  $A^0$  boson. Finally, in the coannihilation region the cross section is sensitive to the abundance of the NLSP, which is exponentially dependent on its mass. In all these cases small corrections due to finite temperature effects have the potential to be significant.

Similar issues, in which dark matter annihilation cross sections are tuned by the adjustment of mass differences or particle mixings occur in many other models, both within and outside of the context of supersymmetry.

In this paper we will present specific estimates of the finite temperature effects on the relic density in the scenarios that we have discussed. The outline of the paper is as follows: In Section 2 we review elements of finite temperature field theory relevant to our analysis. In Section 3 we calculate the thermal corrections to the abundance of the NLSP at freeze-out in the stau coannihilation region. In Section 4 we analyze the enhancements to the helicity breaking standard model fermion masses and their effect on the relic density. In Section 5 we evaluate the thermal corrections to the width of the  $A^0$  boson in the A-funnel region. Finally, in Section 6 we review our analysis and present conclusions.

Other corrections to relic density predictions stemming from a better understanding of the high temperature environment in the early universe have been suggested. Most recently, Hindmarsh and Philipsen [6] have analyzed the deviation of the QCD equation of state from that of an ideal gas. They found that this effect could lead to a measurable correction to the dark matter abundance. It is, however, beyond the scope of this paper to discuss these recent developments.

## 2 Thermal Field Theory

At finite temperatures, field theory calculations must reflect the fact that all correlation functions are thermally averaged. The expression

$$\langle 0|\mathcal{O}|0\rangle$$

for the zero temperature vacuum expectation value of an operator must be replaced by

$$\frac{\sum_n \langle n|\mathcal{O}e^{-\beta H}|n\rangle}{\sum_n \langle n|e^{-\beta H}|n\rangle}.$$

where the sum is over a complete set of states. Perturbation theory then leads to a set of corrected Feynman rules for S-matrix elements. To a good level of accuracy the corrections to relic density calculations can be understood through the thermal shift in physical parameters such as the masses and widths of interacting particles. For this reason we will concentrate on the corrections to the real and imaginary parts of propagators and we do not find it necessary to evaluate general finite temperature Green's functions.

Two parallel formalisms exist for perturbative calculations at finite temperatures; the imaginary time formalism (ITF) which was first introduced by Matsubara [9] incorporates a contour along the imaginary time axis to obtain the thermal average. This technique is best suited for the calculation of equilibrium properties of hot plasmas but can be extended through analytic continuation to real time correlation functions. The real time formalism (RTF) was developed later by Schwinger [10], Mills [11] and Keldysh [12] and incorporates dynamics more naturally. The latter also provides more calculational ease. It is this technique which we shall utilize in the following.

In the RTF the degrees of freedom in a theory are doubled. The new fields do not create or annihilate external particles but may appear in loops. The new fields do not couple to the physical fields through Feynman diagram vertices but rather mix with them through propagators. It follows that these new fields do not appear at one loop level, and we can ignore them in this analysis. Then, the propagators of scalars, chiral fermions, and gauge bosons, respectively are [13]

$$D(P) = \frac{i}{P^2 - m^2 + i\eta} + 2\pi\delta(P^2 - m^2)n(p_0) \quad (2)$$

$$S(P) = (\sigma \cdot P) \left[ \frac{i}{P^2 - m^2 + i\eta} - 2\pi\delta(P^2 - m^2)\tilde{n}(p_0) \right] \quad (3)$$

$$D_{\mu\nu}(P) = -g_{\mu\nu} \left[ \frac{i}{P^2 - m^2 + i\eta} + 2\pi\delta(P^2 - m^2)n(p_0) \right]. \quad (4)$$

We have used Feynman gauge and the fermionic propagator is given in two component notation. The above equations contain the Bose-Einstein and Fermi-Dirac distributions, respectively,

$$\begin{aligned} n(p_0) &= \frac{1}{e^{\beta|p_0|} - 1} \\ \tilde{n}(p_0) &= \frac{1}{e^{\beta|p_0|} + 1}. \end{aligned}$$

Throughout this paper, upper case letters will denote four momenta and lower case letters will be reserved for the spatial components. If the fermionic Lagrangian contains a helicity breaking mass term we will obtain a corresponding Feynman rule for a mass insertion, given by

$$\langle \psi_L \psi_R^\dagger \rangle = -m^* c \left[ \frac{i}{P^2 + i\eta} - 2\pi\delta(P^2 - m^2)\tilde{n}(p_0) \right] \quad (5)$$

where  $c$  is the antisymmetric  $2 \times 2$  matrix defined so that  $c_{12} = -1$ . To evaluate the imaginary part of self energy diagrams we will use a generalization of the zero temperature cutting rules, due to Kobes and Semenoff [14]. As in the zero temperature case, the imaginary part of a diagram is obtained by replacing full propagators by “cut” propagators. For fermions these are given by

$$S^>(P) = 2\pi\epsilon(p_0)(1 - \tilde{f}(p_0))\delta(P^2 - m^2) \quad (6)$$

$$S^<(P) = -2\pi\epsilon(p_0)\tilde{f}(p_0)\delta(P^2 - m^2) \quad (7)$$

where  $\epsilon$  is the sign function and

$$\tilde{f}(p_0) = \frac{1}{e^{\beta p_0} + 1}.$$

Note the absence of the absolute value in the argument of the exponent.

Throughout this paper, we will take the plasma to be comprised solely of light ( $m_f \leq 10$  GeV) standard model fermions and massless gauge bosons. We will consistently neglect terms representing interactions with superparticles, heavy gauge bosons and heavy Higgs bosons in the thermal plasma. This is a reasonable approximation since such terms will be exponentially suppressed by the ratio of their mass to the freeze-out temperature.

### 3 The LSP-NLSP Mass Splitting

An often considered scenario for coannihilation involves the stau as the NLSP. In this section we will analyze the finite temperature corrections to the model. The annihilation rate is exponentially dependent on the mass splitting between the stau and

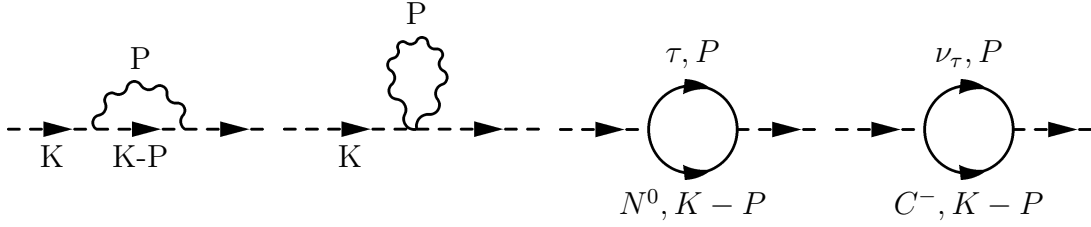


Figure 1: One loop diagrams contributing to the self energy of the stau.

the lightest superpartner. Thus it is interesting to compute the thermal corrections to this mass splitting.

The full sfermion propagator is given by

$$\frac{-i}{K^2 - m_0^2 - \Pi(K)^2}.$$

Where  $\Pi(K)$  is the one particle irreducible self energy. The particle's dispersion relation,  $k_0(k)$  is obtained by solving the equation

$$k_0^2 - k^2 - m_0^2 - \Pi(K)^2 = 0$$

and the mass shift is then given by the limit  $\delta m = k_0 - m_0 \Big|_{k=0}$ . At one loop level, the thermal corrections to the stau mass originate from the Feynman diagrams depicted in Fig. 1. Each propagator has two terms, shown in eqs. (2)-(4). The first represents a contribution from a virtual particle and the second from a thermal particle. The first term is the zero-temperature component of the propagator. To obtain the thermal contributions to the real part of a one loop diagram we need to consider terms involving one thermal and one virtual field. As mentioned, terms involving thermal heavy particles will be suppressed by small statistical factors and we can safely neglect them.

We begin with the photon loop. Using (2) and (4),

$$-i\Pi^{\gamma-loop} = (ie)^2 \int \frac{d^4 P}{(2\pi)^4} (K - P + K)^\mu D_{\mu\nu}(P) D(P - K) (K - P + K)^\nu \quad (8)$$

We write the real part of the self energy and ignore the  $T = 0$  terms as well as those representing interactions with sfermions in the plasma. This gives

$$Re\Pi_\beta^{\gamma-loop} = ie^2 Q^2 \int \frac{d^4 P}{(2\pi)^4} (4K^2 + P^2 - 4K \cdot P) \left[ \frac{i2\pi\delta(P^2)}{((P - K)^2 - m_0^2)} n(p_0) \right] \quad (9)$$

where  $m_0$  is the zero temperature mass of the stau and the subscript  $\beta$  denotes the thermal part of the diagram. This can be evaluated to give

$$Re\Pi_\beta^{\gamma-loop} = -\frac{\alpha}{2\pi} \int dp p \left[ 2 + (L_1^+(m_0) + L_1^-(m_0)) \frac{K^2 + m_0^2}{4kp} \right] n(p) \quad (10)$$

where, following [16], we have defined

$$L_1^\pm(K, p, m) \equiv \pm \ln \left( \frac{K^2 - m^2 \pm 2(k_0 + k)p}{K^2 - m^2 \pm 2(k_0 - k)p} \right). \quad (11)$$

Note that

$$\lim_{k \rightarrow 0} \frac{L_1^\pm}{k} = \frac{4p}{k_0^2 \pm 2k_0p - m_0^2}. \quad (12)$$

To first order in  $\alpha$  we can set  $k_0(0) = m_0$  inside the integral, which then considerably simplifies. We obtain for the mass shift

$$\delta(m^2)^{\gamma\text{-loop}} = -\frac{\alpha}{\pi} \int dp p n(p) = -\frac{\pi\alpha}{6} T^2. \quad (13)$$

The expression for the photon tadpole is

$$-i\Pi^{\text{tadpole}} = ig^2 \int \frac{d^4P}{(2\pi)^4} D_\mu^\mu(P). \quad (14)$$

Evaluating this and taking the limit as  $k \rightarrow 0$  we obtain

$$\delta(m^2)^{\text{tadpole}} = \frac{4\pi\alpha}{3} T^2. \quad (15)$$

Next, we compute the lepton-gaugino loops. Using (3) and following the same steps as those for the photon loops we obtain for a digram containing virtual neutralinos

$$\delta(m^2)^{\text{tau-N}} = \frac{8\alpha}{\pi} m_{\tilde{\tau}}^2 \sum_i |G_{\tau N_i}|^2 \int dx \frac{x^2 y}{(1 - \mu_i^2)^2 - 4y^2} \frac{1}{e^{y/t} + 1} \quad (16)$$

where  $G_{\tau N_i}$  are the couplings of the tau stau and neutralino,  $\mu_i = m_i/m_{\tilde{\tau}}$  is the mass of the  $i^{\text{th}}$  neutralino scaled by the stau mass,  $\mu_\tau = m_\tau/m_{\tilde{\tau}}$ ,  $y = \sqrt{x^2 + \mu_\tau^2}$ ,  $t = T/m_{\tilde{\tau}}$  and the sum runs over the four neutralino types. In the derivation of this expression we have consistently neglected terms that are proportional to the neutralino statistical factor. A similar expression gives the contribution from the diagrams with virtual charginos.

In the limit of the plasma being populated solely by light SM particles, the neutralino obtains a thermal mass shift from self energy diagrams of the type depicted in Fig. 2, with a virtual sfermion and a thermal standard model fermion. Note that we have neglected helicity flipping diagrams since these will be suppressed by light fermion masses.

$$-i\Sigma = -(ie)^2 \sum_f |G_{fN}|^2 \int \frac{d^4P}{(2\pi)^2} cS(P) cD(K+P) \quad (17)$$

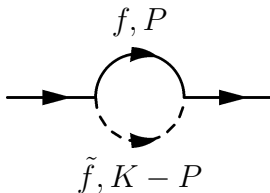


Figure 2: Feynman diagram contributing to the neutralino self energy at one loop order and in the limit of a plasma populated by light SM particles.

where the sum runs over all fermion species inhabiting the plasma and  $G_{fN}$  is the neutralino-fermion-sfermion coupling.

In order to extract the mass shift we should better understand the structure of this expression. The most general form of the 4-component self energy allowed by the Majorana nature of the neutralino is

$$\Sigma_4(K) = aK_\mu\gamma^\mu + bu_\mu\gamma^\mu\gamma^5 + c. \quad (18)$$

where  $u_\mu$  is the 4-velocity of the thermal bath. In the approximation that we ignore the helicity flip diagrams,  $c = 0$ . Then, the dispersion relation,

$$\det(K \cdot \gamma - \Sigma_4(K) - m_0) = 0$$

taken at zero momentum, leads to a thermal mass given by the equation

$$(1 - a)^2 m^2 = (m_0)^2 + b^2. \quad (19)$$

Examining (17) we see the  $\Sigma(K) = -\Sigma(-K)$  so we can conclude that  $b = 0$ . The identity

$$\delta m = am = \frac{1}{2} Tr \{ \text{Re} \Sigma \}.$$

then follows. Evaluating this, we obtain

$$\delta m = -\frac{2\alpha m_N}{\pi} |G_{fN}|^2 \int dx \frac{x^2 y}{(1 - \mu_f^2)^2 - 4y^2} \frac{1}{e^{y/t} + 1}. \quad (20)$$

where here  $y = \sqrt{x^2 + \mu_f^2}$ .

Using (13), (16) and (20) We can calculate the correction to the stau-neutralino mass splitting. A typical, well studied, point in the coannihilation region is given by benchmark point LCC3 which is described in detail in [17]. At this point the mass splitting between the stau and the lightest neutralino is approximately 10 GeV.



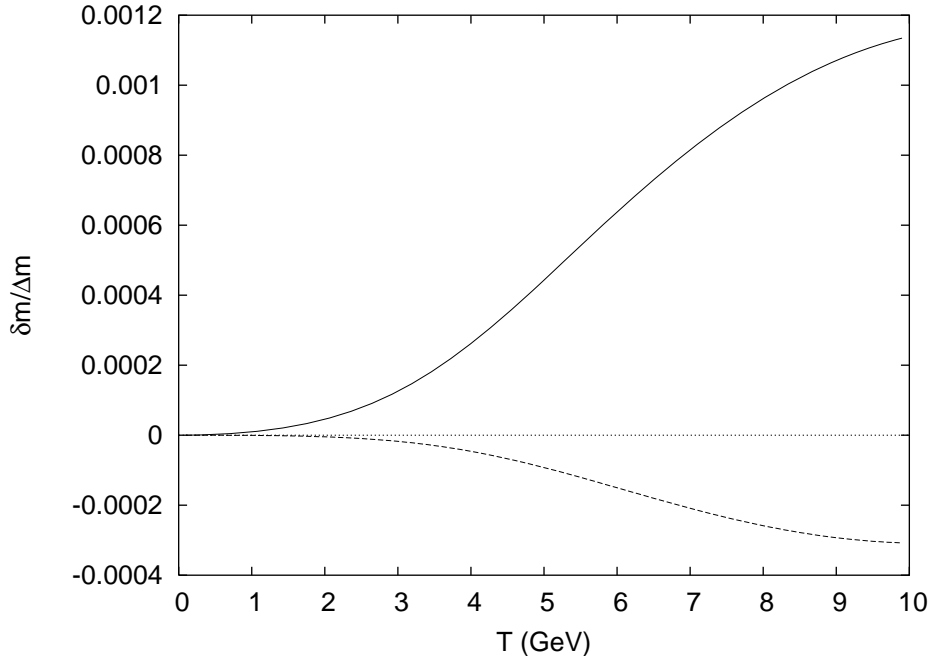


Figure 3: Thermal corrections to the masses of the stau (solid) and lightest neutralino (dashed) in the stau coannihilation region. The values are scaled by the zero-temperature mass splitting.

In Fig. 3 we plot the total shift in this mass splitting, at LCC3, as a function of temperature. At freeze-out, which occurs at approximately 5 GeV, thermal effects account for a fractional correction of about  $10^{-4}$ . The WIMP annihilation rate is determined by the relative Boltzman factor between the two particles, which is then corrected by

$$e^{-(m_{\tilde{\tau}}-m_N)/T} \rightarrow e^{-(m_{\tilde{\tau}}-m_N)/T} e^{-(m_{\tilde{\tau}}-m_N)10^{-4}/T} \approx e^{-(m_{\tilde{\tau}}-m_N)/T} (1 - 2 \times 10^{-4}). \quad (21)$$

We evaluated the effect of the shift in mass splitting on the relic density using the DarkSUSY [18] package. We found, as expected, that the change in the relic density prediction is roughly

$$\frac{\delta\Omega_{DM}}{\Omega_{DM}} \approx 1.0 \times \frac{\delta(\Delta m)}{\Delta m}. \quad (22)$$

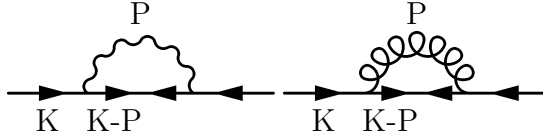


Figure 4: Feynman diagrams contributing, at one loop order, to the helicity breaking mass of the standard model fermions.

## 4 Helicity Breaking Fermion Masses

It would seem that a straightforward way for two WIMPs to annihilate would be by production of fermion pairs through t-channel and u-channel sfermion exchange. For gravity mediated supersymmetry breaking this is especially true since the LSP is typically almost pure bino and as such does not couple to the Higgs boson or to gauge bosons. But S-wave annihilation of neutralinos into fermion-antifermion pairs is helicity suppressed by the square of the ratio of the mass of the fermion to the mass of the LSP [19]. In the bulk region of the mSUGRA parameter space, this number is on the order of  $10^{-4}$  for tau or bottom quark production. P-wave annihilation is not helicity suppressed but is proportional to two powers of the WIMP velocity. This is typically about  $(T/m_N) \approx 10^{-2}$ . Thus, the helicity suppressed S-wave processes contribute to the annihilation at a percent level.

Thermal effects cannot on their own give fermions helicity breaking masses but they do have the potential to enhance (or suppress) existing mass operators. The thermal corrections to these operators must vanish as the zero temperature mass becomes small and will thus be proportional to it. It is possible that the coefficient of proportionality could be large since the SM fermions are light compared to the ambient temperature.

Assuming that only the light gauge bosons and SM fermions are thermal, the diagrams contributing to the fermion helicity breaking masses are those depicted in Fig. 4. For the photon loop

$$-i\Sigma = (ie)^2 \int \frac{d^4P}{(2\pi)^4} \bar{\sigma}^\mu S(K-P) D(P)_{\mu\nu} \bar{\sigma}^\nu. \quad (23)$$

We evaluate the integral and take the real part of  $\Sigma$ , neglecting the zero temperature terms, to obtain

$$Re\Sigma_\beta = \frac{\alpha}{\pi^3} m_f c \int d^4P \left[ \frac{2\pi\delta(P^2)}{(K+P)^2 - m_f^2} - \frac{2\pi\delta(P^2 - m_f^2)}{(K+P)^2} \right] n(p_0) \quad (24)$$

where  $m_f$  is the fermion mass. We can perform the  $p_0$  integration using the delta function. The first term in brackets yields

$$\begin{aligned} \text{Re}\Sigma_\beta^{(1)} &= \frac{\alpha}{\pi^2} m_f c \int \frac{d^3p}{p} \left[ \frac{1}{K^2 + 2(k_0 - k \cos \theta)p - m_f^2} + \frac{1}{K^2 - 2(k_0 + k \cos \theta)p - m_f^2} \right] n(p) \\ &= \frac{\alpha}{\pi k} m_f c \int dp (L_1^+(m_f) + L_1^-(m_f)) n(p) \end{aligned} \tag{25}$$

Since the fermions produced in the WIMP annihilation are on-shell, we may use  $K^2 = m_f^2$  inside the integrals. It is then clear from (11) that the integrand vanishes. So, we see that at one loop order massless gauge bosons cannot enhance the fermion mass operator. As we will see, all finite coorections will be due to massive particles populating the plasma. This brings us to the second term in (24) which yields a similar expression that is non zero only for fermions with a non-negligible mass. Thus, we consider only tau and bottom quark production. For the tau

$$\text{Re}\Sigma_\beta^{(2)} = \frac{\alpha}{2\pi k} m_f c \int dp \frac{p}{E} (L_2^+(m_f) + L_2^-(m_f)) \tilde{n}(E). \tag{26}$$

where

$$L_2^\pm(K, p, E, m_f) \equiv \pm \ln \left( \frac{K^2 + m_f^2 \pm 2(k_0 E + kp)}{K^2 + m_f^2 \pm 2(k_0 E - kp)} \right). \tag{27}$$

and  $E = \sqrt{p^2 + m_f^2}$ . This integral should be evaluated for on-shell fermions with momentum  $k$  on the order of the LSP mass. In the above expression, the coefficient of  $m_f c$  represents the correction to the mass operator. This term, too, would vanish identically but for the effect of the non-zero fermion masses. For the tau, this is the full result. The bottom quark receives another, stronger correction from thermal gluon loops. These will be equal to the above expression with an additional multiplicative factor of  $(4/9)(\alpha_s/\alpha)$ . For the purpose of this calculations we have considered the LCC1 benchmark point [20] as a canonical point in the bulk. We have evaluated  $\delta m/m$  numerically for the bottom and tau. The results are plotted in Fig. 5. We see that, at freeze-out, the corrections to the fermion mass operators are on the order of  $10^{-4}$  and are thus negligible compared to existing theoretical and experimental uncertainties.

## 5 The Width of the A Boson

In the so called A-funnel region of mSUGRA parameter space, the mass of the CP-odd heavy Higgs boson ( $A^0$ ) is roughly equal to twice the mass of the LSP and the

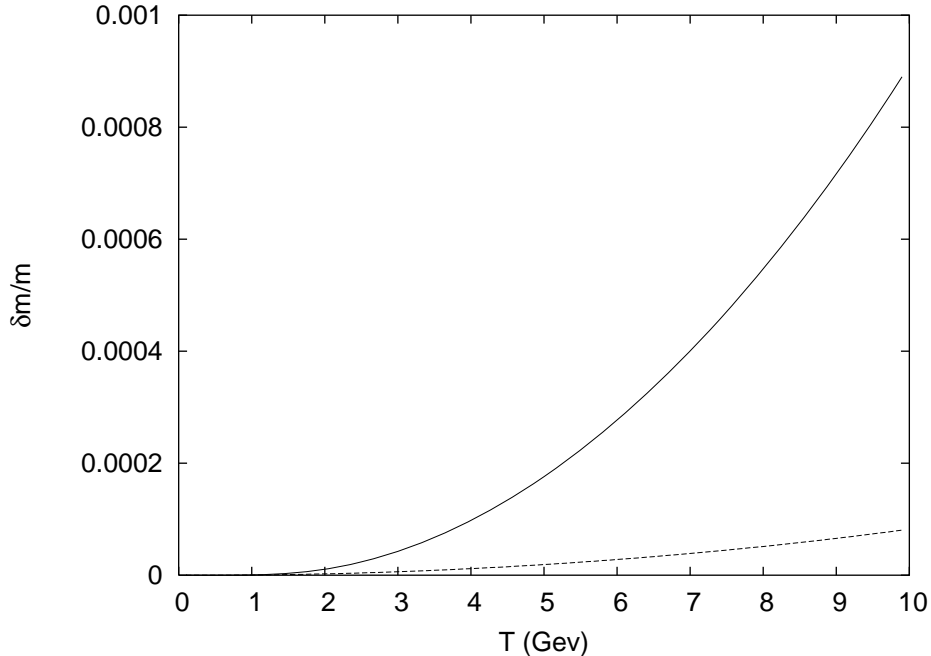


Figure 5: Thermal correction to the helicity breaking mass operator of the bottom quark (solid) and tau lepton (dashed).

WIMP annihilation cross section is dominated by resonant s-channel annihilation to  $b\bar{b}$ ,  $\tau^+\tau^-$ . The value of the cross section is, therefore, highly sensitive to the width of the  $A^0$ . We would like to investigate the thermal corrections to this quantity and their effect on the relic density predictions.

In the limit that the plasma is populated only by massless gauge bosons and SM fermions, the  $A^0$  self energy receives corrections from a single Feynman diagram - the fermion loop depicted in Fig. 6. We would like to evaluate the imaginary part of this diagram. As discussed in Sec. 2, we use the finite temperature cutting rules from [14] to do so. Following the notation in [21], we calculate

$$\Pi^> = -(ie)^2 \sum_f |G_{fA}|^2 \int \frac{d^4 P}{(2\pi)^4} Tr \{S^>(P)S^>(K-P)\} \quad (28)$$

where the sum is over all the fermion species in the plasma. As usual we consider only SM fermions to be thermal. Using (6) and (7) and some algebra this becomes

$$\Pi^> = -\frac{\alpha}{\pi} |G_{fA}|^2 \left( \frac{K^2}{2} - m_f^2 \right) \frac{1}{k} \int dp \frac{p}{E_1} [(1 - \tilde{n}_1)(\theta_- - \tilde{n}_2)\Theta_- + \tilde{n}_1(\theta_+ - \tilde{n}_2)\Theta_+]. \quad (29)$$

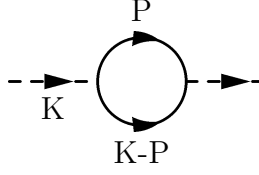


Figure 6: Feynman diagram contributing to the  $A^0$  boson self energy.

To avoid clutter we have defined

$$\begin{aligned}
 \tilde{n}_{1/2} &= \tilde{n}(E_{1/2}) \\
 \theta_{\pm} &= \theta(k_0 \pm E_1) \\
 E_1 &= \sqrt{p^2 + m_f^2} \\
 E_2 &= \sqrt{(\vec{k} - \vec{p})^2 + m_f^2}.
 \end{aligned}$$

and we have used the identity

$$\epsilon(p_0)(1 - \tilde{f}(p_0)) = \theta(p_0) - \tilde{n}(p_0) \quad (30)$$

The symbols

$$\Theta_{\pm} = \begin{cases} 1, & \text{if } -1 < (K^2 \pm 2k_0 E_1)/2kp < 1 \\ 0, & \text{otherwise} \end{cases}$$

impose kinematic constraints.

It appears that  $\Pi^>$  diverges as  $k \rightarrow 0$ . But, in this limit,  $\Theta_+$  vanishes completely and the region of integration determined by  $\Theta_-$  collapses to a point given by  $K^2 = 2k_0 E_1$ . This gives a finite result. To evaluate the limit we put the  $A^0$  on-shell and set  $K = (m, \vec{0})$ . This is a good approximation since  $\Gamma_A/m_A$  is typically on the order of 0.05. Writing the infinitesimal region of integration explicitly, we have

$$\lim_{k \rightarrow 0} \Pi^> = \frac{-4\alpha}{\pi} |G_{fA}|^2 \left( \frac{m_A^2}{2} - m_f^2 \right) \frac{1}{k} \int_{q-2k}^{q+2k} dp \frac{p}{E_1} (1 - \tilde{n}_1)(\theta_- - \tilde{n}_2). \quad (31)$$

Where  $q \equiv \sqrt{m_A^2/4 - m_f^2}$ . Finally, evaluating the integral, we obtain

$$\lim_{k \rightarrow 0} \Pi^> = -2\alpha m_A^2 \left( 1 - \frac{2m_f^2}{m_A^2} \right) \sqrt{1 - \frac{4m_f^2}{m_A^2}} \left( 1 - \tilde{n} \left( \frac{m_A}{2} \right) \right)^2 \theta \left( \frac{m_A}{2} - m_f \right). \quad (32)$$

The imaginary part of the real-time self energy at zero external momentum is then [21]

$$\begin{aligned}
\text{Im } \bar{\Pi} &= -\frac{1}{2} \left(1 - e^{-\beta k_0}\right) \Pi^> \\
&= \alpha m_A^2 |G_{fA}|^2 \left(1 - \frac{2m_f^2}{m_A^2}\right) \sqrt{1 - \frac{4m_f^2}{m_A^2}} \left(1 - 2\tilde{n}_1\left(\frac{m_A}{2}\right)\right) \theta(m_A - 2m_f).
\end{aligned}
\tag{33}$$

There are several points worth noting. First, the self energy has no imaginary part for  $m_A$  less than  $2m_f$ . This is expected for the following reason: at zero external momentum the processes that contribute to the boson's decay width are  $A^0 \rightarrow f\bar{f}$ . The additional processes  $A^0 f \rightarrow \bar{f}$  and  $A^0 \bar{f} \rightarrow f$ , corresponding to Landau damping, are kinematically disallowed. Thus, a minimum mass of  $m_A > 2m_f$  is required for a decay to take place. Instead of a power-law dependence on  $T$ , the thermal  $A^0$  width is suppressed by a fermionic statistical factor evaluated at half the  $A^0$  mass. We could have deduced this fact as well by noting, again, that the only relevant process is the one involving the production of a fermion-antifermion pair and its inverse. These two processes should be accompanied, respectively, by statistical factors  $(1 - \tilde{n}(m_A/2))^2$  and  $\tilde{n}(m_A/2)^2$  with a relative minus sign between them. The total statistical factor would then be

$$\left(1 - \tilde{n}\left(\frac{m_A}{2}\right)\right)^2 - \tilde{n}\left(\frac{m_A}{2}\right)^2 = 1 - 2\tilde{n}\left(\frac{m_A}{2}\right).$$

This is exactly the temperature dependence we obtained in (33). In Fig. 7 we plot the thermal corrections to the A width as a function of temperature. It is clear that at freeze-out ( $T \approx 5$  GeV), the thermal effects are completely negligible.

## 6 Conclusions

In this paper we have identified several mechanisms which lead to heightened sensitivity of the WIMP annihilation cross section, and thus the relic density, to the values of the underlying theory parameters. We then investigated the consequences of finite temperature effects in regions of parameter space where these mechanisms are important. We first considered the issue of thermal corrections to parameters to which the relic density is exponentially sensitive, specifically the LSP-NLSP mass splitting in the stau coannihilation region. We found a thermal correction of about  $10^{-4}$  to the mass splitting and a similar correction to the relative Boltzmann factor between the stau and the LSP at freeze-out. This correction is far too small to have any measurable consequences for current or upcoming relic density observations. In

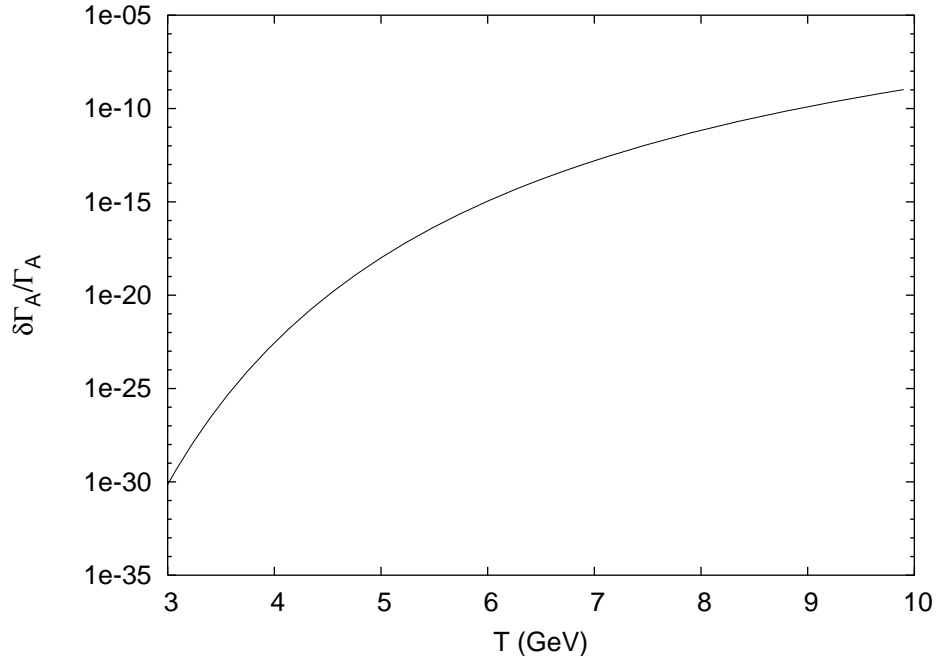


Figure 7: Thermal correction to the width of the  $A^0$  boson.

the bulk region we considered the issue of corrections to the standard model fermion masses and the corresponding shifts in helicity suppressed annihilation cross sections. While the branching ratio for these processes is small, the fermions are light and can thus receive significant mass corrections. It is the interplay of these two effects which we investigated. We found that, for a typical point in the mSUGRA bulk, the thermal effects correct the fermionic mass operators by at most a factor of  $10^{-4}$  at freeze-out. Finally, we considered thermal corrections to the width of the  $A^0$  boson in the A-funnel region. We found the value of this parameter to be exponentially suppressed by the ratio of the A mass to the freeze-out temperature and we provided a physical interpretation for this result.

Although in the specific family of models which we studied all thermal effects are considerably below the detection threshold of any current or planned observation of the relic density, we believe that generically these issues should be considered. For example, in any coannihilation scenario an exponential dependence on a small mass splitting is expected. Similarly, any resonant annihilation will exhibit heightened sensitivity to a small width associated with the virtual particle. Small corrections to these parameters cannot a priori be neglected. However, as we have seen, close analysis will often indicate that these effects are very small.

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