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Moduli Decays and Gravitinos

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Abstract

One proposed solution of the moduli problem of string cosmology requires that the moduli are quite heavy, their decays reheating the universe to temperatures above the scale of nucleosynthesis. In many of these scenarios, the moduli are approximately supersymmetric; it is then crucial that the decays to gravitinos are helicity suppressed. In this paper, we discuss situations where these decays are, and are not, suppressed. We also comment on a possible gravitino problem from inflaton decay.

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1 Introduction

It is possible that nature exhibits an approximate $N = 1$ supersymmetry. In string theory, such an approximate supersymmetry is often accompanied by approximate moduli. For cosmology, these moduli are both intriguing and problematic. Intriguing because it is tempting to connect them with inflation; problematic because they tend to carry too much energy, spoiling the successes of big bang nucleosynthesis. One possible resolution of these problems is to suppose that the moduli have masses well above the masses of squarks and gauginos (assumed to be of order TeV or so) [1]. Then the moduli decay reheats the universe to temperatures above nucleosynthesis temperature. In such a scenario, the main issues are production of dark matter and gravitinos. Production of dark matter was addressed in an early paper of Moroi and Randall [2]. These authors argued that decays of moduli to gauginos were helicity suppressed, and used this in a series of estimates of the dark matter density. It has been argued that similar suppression ratio takes place for the branching ratio to gravitinos[1] [2, 3]. Subsequently, other authors have considered variations of this scenario, supposing similar helicity suppressions [4].

Moroi and Randall [2] considered situations where the moduli masses were comparable to $m_{3/2}$. But it is also possible that the moduli have masses which are large compared to $m_{3/2}$, and approximately supersymmetric. This is the situation in the model of Kachru, Kallosh, Linde and Trivedi (KKLT) [5], and it has been assumed by many authors that there is a similar suppression there. Recently, however, two groups have examined this question. In [6] and [7] it was argued, quite generally, that neither the gravitino nor the gaugino branching ratio are helicity suppressed. Both sets of authors argue specifically that in the models of [5], the decay rate is unsuppressed.

In this note, we look at both of these questions. We find, in agreement with both groups, that in models with supersymmetric moduli, there is, in general, no suppression of the gaugino decay rate. The situation of the gravitino is more complicated. We will exhibit examples where there is no suppression of the gravitino rate. But in models like that of KKLT, with a simple supersymmetry breaking sector, we show that there is a suppression. As guidance for our analysis, we rely heavily on the Goldstino equivalence theorem. This theorem is analogous to the equivalence theorem in spontaneously broken gauge theories, which asserts that at high energies the amplitudes for processes involving longitudinal gauge bosons are the same as those for the would-be Goldstone bosons of the symmetry breaking. In supergravity, one just replaces "gauge

[1] In reference [2], this question was moot since the moduli masses were assumed to be of order $m_{3/2}$.

bosons" by "gravitinos" and "Goldstone bosons" by "Goldstinos" to obtain the corresponding equivalence theorem. In the KKLT-type models, working directly with the Goldstinos, it is easy to see that all of the decay amplitudes are suppressed by $m_{3/2}$. With a bit more effort, one can find the suppression in terms of the longitudinal gravitinos of unitary gauge. For the models without suppression, it is easy to see the equivalent descriptions.

We will see, though, that whether or not there is suppression depends strongly on the assumptions about supersymmetry breaking. If the hidden sector contains approximate moduli with mass of order $m_{3/2}$, the decays of the heavy moduli to gravitinos are suppressed; if not, they are unsuppressed. This observation resuscitates the moduli problem, since one either suffers from a gravitino problem or another modulus. On the other hand, in models like KKLT, the gravitino mass is expected to be quite large, so the decays of this other modulus can themselves reheat the universe to nucleosynthesis temperatures. So, with a slight modification of the original scenario [2] of Moroi and Randall, a viable cosmology is possible.

Having established criteria for suppression, we briefly discuss the implications of these observations for various cosmological models in the literature, especially that of KKLT. We also comment on the inflaton decay into gravitinos, which is discussed recently in Ref. [8]. More detailed studies will appear in a subsequent publication.

2 Some Studies of the Goldstino Equivalence Theorem

The validity of the equivalence theorem follows from a simple physical argument. Consider a theory with supersymmetry broken at a low energy scale, $M^2_{susy} = F$, well below the *intermediate scale*, $M^2_{int} = \mathrm{TeV} \cdot M_p$. For the Goldstone particles of this theory, and their would-be superparners, gravity is completely irrelevant. For example, in a hidden-sector-type model, approximate moduli will interact with gravitinos with interaction strength $1/F \gg 1/M^2_p$.

As a simple model, consider a theory with a massive field, ϕ, and a Polonyi sector, with fields z, and superpotential

$$W = W_o + \mu^2 z + \frac{M}{2}\phi^2 \tag{1}$$

where W_o is adjusted so that the cosmological constant vanishes (this is helpful conceptually but will not be particularly important for our discussion). In this model, supersymmetry is broken. The Goldstino is the fermionic component of z, ψ_z. We take a Kähler potential including interaction between ϕ and z supermultiplets:

$$K = \phi^\dagger \phi + z^\dagger z + \frac{1}{\Lambda} \phi^\dagger z^2 + \text{c.c.} \; . \tag{2}$$

Λ might be the mass of some fields which have been integrated out. With this interaction term, the minimum of the potential lies at $z \sim \Lambda^2/M_p$. The modulus coupling to gravitinos becomes

$$\frac{1}{\Lambda} F_\phi^\dagger \psi_z \psi_z = \frac{M}{\Lambda} \phi \psi_z \psi_z \, . \tag{3}$$

This leads to an amplitude:

$$\mathcal{A}(\phi \to \psi_z \psi_z) = \frac{M}{\Lambda}. \tag{4}$$

We can find the same result in terms of the longitudinal gravitinos. The basic coupling, which will figure repeatedly in our discussions, is:

$$\mathcal{L}_{cv} = -e^{G/2} \left[\psi_\mu \sigma^{\mu\nu} \psi_\nu + \text{h.c.} \right] . \tag{5}$$

Here $G = K + \ln(W) + \ln(W^*)$ and we set $M_p = 1$ whenever this is unambiguous. The coupling to scalars, ϕ_i arises from Taylor series expanding the exponential:

$$\mathcal{L}_{cv} = -\frac{e^{G/2}}{2} \; G_i \phi_i \psi_\mu \sigma^{\mu\nu} \psi_\nu. \tag{6}$$

In our case, $G_\phi = 0$, $e^{G/2} G_z = \mu^2$. The massive field is an admixture of ϕ and z. Eliminating F_ϕ by its equation of motion, the mass matrix has the structure:

$$M^2 |\phi|^2 + \frac{2}{\Lambda} F_z M \phi z + \text{c.c.} \tag{7}$$

up to terms of $\mathcal{O}(m_{3/2}^2)$. As a result, the massive scalar is:

$$\Phi = \phi + \frac{2F_z}{M\Lambda} z^\dagger \; . \tag{8}$$

The longitudinal gravitino is:

$$\psi_\mu(k) = \sqrt{\frac{2}{3}} \frac{k_\mu}{m_{3/2}} u_\alpha(k). \tag{9}$$

This can be obtained from the Rarita-Schwinger action, or by writing the supersymmetry transformation to unitary gauge (the $\sqrt{3}$ factor results from the relation $|G_z|^2 = 3$ which holds for vanishing cosmological constant). Using the Dirac equation, and keeping only terms of order $k/m_{3/2}$ gives for the amplitude:

$$\mathcal{A}(\Phi \to \psi_\mu \psi_\mu) = \frac{2}{3} \left(\frac{k \cdot k'}{m_{3/2}^2} \right) \frac{e^{G/2}}{2} \frac{2F_z}{M\Lambda} \; G_z. \tag{10}$$

Using $F_z^2 = 3m_{3/2}^2$, $e^{G/2} = m_{3/2}$ and $2k \cdot k' = M^2$ gives precisely the amplitude of eqn. (4).

Note that this amplitude is not chirality suppressed. If we take $\Lambda = M_p$, this is of precisely the size found, in another context, in [6, 7].

3 The KKLT Model

In the simplest version of the KKLT model, the only moduli which are light relative to the fundamental scale are the Kähler moduli. We will assume that there is only one such modulus, which we will call ρ. Supersymmetry breaking can arise, as suggested by KKLT, due to the presence of anti-D3 branes. Alternatively, similar scaling for the moduli potential may result from the hidden sector fields. We will first consider the simplest hidden sector model, with a single field, z. We will take the Kähler potential and superpotential to be

$$K = -3\ln(\rho + \rho^\dagger) + z^\dagger z \ , \qquad W = e^{-\rho} + W_o + \mu^2 z. \tag{11}$$

Here we have taken various coefficients to be numbers of order unity. (For simplicity, we have taken the coefficient of ρ in the exponential to be -1; our analysis does not depend on this choice.) W_o is assumed to be very small. The supersymmetry breaking effect in the hidden sector (z sector) is mediated to the ρ field through gravitational ($1/M_p$ suppressed) interaction. To leading order in $1/\rho$, the ρ sector is supersymmetric and the vacuum is determined by

$$D_\rho W = 0 \ , \qquad V_z = 0. \tag{12}$$

Here D_ρ denotes the Kähler derivative,

$$D_\rho W = \frac{\partial W}{\partial \rho} + \frac{\partial K}{\partial \rho} W = W G_\rho. \tag{13}$$

In this case,

$$z \approx \sqrt{3} - 1 \ , \qquad W_o \approx (2 - \sqrt{3})\mu^2 \ , \qquad \rho \approx -\ln(W_o / \ln(W_o)) \gg 1 \ , \tag{14}$$

and $m_{3/2} = \mathcal{O}(\mu^2)$. The ρ field has a supersymmetric mass of order $\rho m_{3/2} \gg m_{3/2}$ in this vacuum, which justifies the approximation $G_\rho \approx 0$. Gravity-mediated supersymmetry breaking effects give corrections to the ρ potential characterized by $1/\rho$. In counting powers of ρ, it is useful to note that $W_\rho \sim W/\rho$, and $W_{\rho\rho} \sim W_\rho$.

One can now study the amplitude for the modulus decay in powers of $1/\rho$. In the unitary gauge the amplitude is proportional to G_ρ (see eqn. (6)). As argued in [6, 7], in a model of this type, G_ρ would receive corrections of order $1/\rho$. Recalling the structure of the unitary gauge amplitude, and that $m_\rho \sim \rho m_{3/2}$, it would seem that the amplitude is enhanced over naive expectations by a factor of $m_\rho/m_{3/2}$.

It is difficult, however, to find such an enhancement in the Goldstino picture. Here there are various types of terms. First, there are terms in the action involving derivatives. However, all of these *do have a chirality suppression.* Then there are helicity-flip terms, involving two matter fermions. The relevant terms in the supergravity action are:

$$\mathcal{L}_{\chi\chi} = -\frac{1}{2} e^{G/2} \left(G_{ij} + \frac{1}{3} G_i G_j - \Gamma^k_{ij} G_k \right) \psi_i \psi_j, \tag{15}$$

where the connection is defined by $\Gamma^k_{ij} = g^{k\bar{l}} G_{ij\bar{l}}$. To obtain the coupling of the heavy modulus to Goldstinos we need to Taylor expand the factors of G in powers of the modulus and project matter fermions onto Goldstinos. The Goldstino is a linear combination of ψ_ρ and ψ_z, of the form:

$$\tilde{G} = \cos\theta \psi_z + \sin\theta \psi_\rho \,, \tag{16}$$

where $\theta \sim 1/\rho$, as we will see shortly. For the calculation of the leading contribution in the $1/\rho$ expansion, we can take $\psi_i = \psi_j = \psi_z$ which gives a coupling of ρ to a pair of Goldstinos:

$$-\frac{1}{2} e^{G/2} \left(G_{zz\rho} + \frac{2}{3} G_{z\rho} G_z \right) \rho \psi_z \psi_z. \tag{17}$$

But both terms in parenthesis are of order $1/\rho$. When we rescale ρ to obtain a canonical kinetic term, we obtain an amplitude of order $m_{3/2}$. We obtain the same result if we take one of the fermions to be ψ_ρ; the projection onto the Goldstino again leads to a term of order $m_{3/2}$.

How does one reconcile the results of these two different gauges (descriptions)? The physical Goldstino is a linear combination of ψ_z and ψ_ρ; the orthogonal combination, Ψ, is massive. Similarly, the heavy scalar Φ is a linear combination of z and ρ. In fact, it is precisely this mixing that is responsible for the non-vanishing G_ρ found in Refs. [6, 7]. However, in the KKLT model under consideration the mixing is supersymmetric at order $1/\rho$! This means that at this order the only non-vanishing F-term is in the same supermultiplet as the true Goldstino. As we will now explain, for the heavy field

$$G_\Phi = \mathcal{O}\left(\frac{1}{\rho^2}\right). \tag{18}$$

It is very helpful to work with the supergravity action written in terms of the quantity G, rather than K and W separately. In terms of G, the potential is:

$$V = e^G [g^{i\bar{j}} G_i G_{\bar{j}} - 3]. \tag{19}$$

There are two things we need to calculate: the F term for ρ – which is now the problem of finding G_ρ, and the eigenstates of the mass matrix. From the condition that $\partial V/\partial\rho^\dagger = 0$, we find

$$G_\rho = -\left[\frac{\rho+\rho^\dagger}{\sqrt{3}}\right]^{-2} \frac{G_{\bar\rho\bar z}}{G_{\bar\rho\bar\rho}}\, G_z \ , \tag{20}$$

at the leading order in the $1/\rho$ expansion. Indeed this is of $\mathcal{O}(1/\rho)$ after the field rescaling such that ρ has the canonically normalized kinetic term, i.e., $\rho \to (\langle\rho+\rho^\dagger\rangle/\sqrt{3})\rho$ and thus $G_\rho \to (\langle\rho+\rho^\dagger\rangle/\sqrt{3})G_\rho$.

Now let's consider the mass matrix. We will keep the leading order contribution in the $1/\rho$ expansion for each component. After the rescaling of ρ we have

$$V_{\rho\bar z} = e^G \left[\frac{\langle\rho+\rho^\dagger\rangle}{\sqrt{3}}\right]^3 G_{\rho\rho}G_{\bar\rho\bar z} \ , \quad V_{\rho\bar\rho} = e^G \left[\frac{\langle\rho+\rho^\dagger\rangle}{\sqrt{3}}\right]^4 G_{\rho\rho}G_{\bar\rho\bar\rho} \ , \tag{21}$$

and $V_{z\bar z}$ is of the order of e^G. Note that these terms are supersymmetric; there are corresponding large terms in the fermion mass matrix. Because of this mixing, the mass eigenstate of the heavy scalar has a small component of the field, z, such that

$$\Phi = \hat\rho + \epsilon^* z \ , \tag{22}$$

where

$$\epsilon = \left[\frac{\langle\rho+\rho^\dagger\rangle}{\sqrt{3}}\right]^{-1} \frac{G_{\bar\rho\bar z}}{G_{\bar\rho\bar\rho}} \ , \tag{23}$$

and $\hat\rho$ represents the canonically normalized ρ field. The fermion has the same mixing factor, and thus the angle θ in eqn. (16) is given by $\theta = -\epsilon$. The G_Φ factor is calculated to be

$$G_\Phi = \epsilon G_z + \frac{\langle\rho+\rho^\dagger\rangle}{\sqrt{3}} G_\rho = 0 \ . \tag{24}$$

The $\mathcal{O}(1/\rho)$ contribution cancels out and the unitary gauge and Goldstino picture are reconciled. At next order, there is no reason for things to vanish. But this corresponds to amplitudes which are suppressed by two powers of ρ, i.e. which are proportional to $m_{3/2}$.

We should note, however, that the suppression we have found here does not result from general symmetry principles, but is a feature of this particular model. We can modify the KKLT Kähler potential in a way which yields a decay amplitude as large as that suggested in [6, 7]. If we take

$$K = -3\ln(\rho+\rho^\dagger - z^\dagger z + \kappa(zz + z^\dagger z^\dagger)), \tag{25}$$

where κ is a real parameter, then the Christoffel symbol Γ^{ρ}_{zz} has a non-vanishing value $\Gamma^{\rho}_{zz} = 2\kappa$. In this case, the terms in the supergravity Lagrangian:

$$\frac{1}{2} e^{G/2} \Gamma^k_{ij} G_k \psi^i \psi^j = \kappa m_{3/2} G_\rho \psi_z \psi_z + \cdots \tag{26}$$

give a coupling of order

$$\kappa m_{3/2} G_{\rho\rho} \rho \psi_z \psi_z \simeq \frac{-3}{\langle \rho + \rho^\dagger \rangle} \kappa m_{3/2} \rho \psi_z \psi_z. \tag{27}$$

Note that this term vanishes when $\kappa = 0$, as in the KKLT model in Ref. [6]. The other terms are $\mathcal{O}(1/\rho^2)$. Rescaling the ρ and z fields so that their kinetic terms are canonical, this gives a decay amplitude of order $\rho m_{3/2}$, i.e. of order m_ρ. So determining the presence or absence of chirality suppression requires detailed microphysical understanding of the model.

In general, for a particle ϕ which has a supersymmetric mass $m_\phi \gg m_{3/2}$, only the third term in the parenthesis in eqn. (15) has the potential of enhancing the decay amplitude. With the canonically normalized kinetic terms for ϕ and z, the Taylor expansion of the term gives

$$\begin{aligned} \mathcal{L} &\ni \frac{1}{2} e^{G/2} \Gamma^\phi_{zz} G_{\phi\phi} \phi \psi_z \psi_z + \text{h.c.} \\ &\sim m_\phi \Gamma^\phi_{zz} \phi \psi_z \psi_z + \text{h.c.} \end{aligned} \tag{28}$$

Here we have used $G_{\phi\phi} \sim W_{\phi\phi}/W \sim m_\phi/m_{3/2}$. Therefore there is a helicity suppression unless a direct $\phi^\dagger zz$ coupling is present in the Kähler potential such that $\Gamma^\phi_{zz} = \mathcal{O}(1)$.

4 Models Without Light Moduli in the Hidden Sector

It was crucial to the cancellation of the previous section that the mixing between ρ and z was supersymmetric, and in particular that all soft masses were small compared to the mass of the heavy modulus. One might suspect that if the soft mass of the field z was *large* compared to m_ρ, then this cancellation would no longer occur. Such a large mass might arise if supersymmetry is broken dynamically [9, 10, 11], or if the hidden sector involves non-trivial interactions, as in the O'Raifeartaigh model. To show that there is no cancellation, in general, we can modify the Kähler potential for the field z so that it obtains a large, non-supersymmetric mass:

$$\delta K = -\frac{1}{\Lambda^2} (z^\dagger z)^2. \tag{29}$$

Here $\Lambda \ll M_p$. This model effectively describes the situation without flat directions such as models with dynamical supersymmetry breaking, e.g., in Ref. [9, 10, 11]. In general, the above term can be generated by integrating out heavy fields with masses $\Lambda \gtrsim (m_{3/2}M_p)^{1/2}$. In this case, there is no longer a cancellation, in the unitary gauge picture. The above term gives a mass term for the z field

$$m^2_{\rm Hidden} \simeq 12 m^2_{3/2} \left(\frac{M_p}{\Lambda}\right)^2 \gg m^2_{3/2} \ . \tag{30}$$

When z is much heavier than ρ in the KKLT example, the lighter mass eigenstate Φ, which mainly consists of ρ, is obtained to be

$$\Phi = \hat{\rho} + \epsilon'^* z \ , \tag{31}$$

where ϵ' is[2]

$$\epsilon' = \mathcal{O}\left(\frac{1}{\rho}\frac{m^2_\rho}{m^2_{\rm Hidden}}\right) \ . \tag{32}$$

Therefore, for $m_\rho \ll m_{\rm Hidden}$,

$$G_\Phi = \frac{\langle \rho + \rho^\dagger \rangle}{\sqrt{3}} G_\rho + \epsilon' G_z \simeq \frac{\langle \rho + \rho^\dagger \rangle}{\sqrt{3}} G_\rho = \mathcal{O}\left(\frac{1}{\rho}\right) \ . \tag{33}$$

The cancellation does not take place in this case. The amplitude is $\mathcal{O}(m_\rho)$.

In the Goldstino picture, it is the Γ^z_{zz} coupling which leads to the unsuppressed decay. The term in the Lagrangian,

$$\frac{1}{2} e^{G/2} \Gamma^z_{zz} G_z \psi_z \psi_z \tag{34}$$

contains a coupling of Φ to Goldstinos through the mixing ϵ':

$$\frac{\epsilon'^*}{2} m_{3/2} G_{zz\bar{z}\bar{z}} G_z \Phi^\dagger \psi_z \psi_z \ . \tag{35}$$

With $G_{zz\bar{z}\bar{z}} = -4/\Lambda^2$ and $G_z = \sqrt{3}$, the amplitude is estimated to be $\mathcal{O}(m_\rho)$, consistent with the calculation in the unitary gauge.

This leaves us with a cosmological problem. Either z is very heavy, in which case ρ decays to gravitinos are problematic. Or z is light compared to ρ, in which case the modulus z is potentially problematic.

But in a scenario like that of KKLT, z might behave in precisely the manner envisioned by Moroi and Randall [2]. The gravitino in such a model is likely to be quite massive, easily 100

[2] Note that this expression is only valid for $m_{\rm Hidden} \gtrsim m_\rho$, otherwise the mixing factor is given by ϵ in eqn. (23) with $\mathcal{O}(m^2_{\rm Hidden}/\rho m^2_\rho)$ corrections.

TeV or so [12, 13, 14]. Similarly, then, the z field will be very massive. If its mass is less than twice the gravitino mass, it will not decay to gravitinos. In this circumstance, an acceptable cosmology is possible with a slight modification of the scenario of Moroi and Randall, even if z dominates the energy density of the universe. A large enough reheating temperature for nucleosynthesis can be obtained from z decay by introducing an explicit coupling of z to vector-like visible sector particles (X and $\bar{X}$) in the Kähler potential

$$\lambda \frac{1}{M_p} z^\dagger \bar{X} X + \text{h.c.}. \tag{36}$$

If the coupling constant, λ, is $\mathcal{O}(1)$, the reheating temperature is above nucleosynthesis temperatures. Moroi and Randall discussed this scenario with the role of X and $\bar{X}$ being played by the Higgs fields, but in our case the X and $\bar{X}$ fields cannot be particles in the minimal supersymmetric standard model (MSSM) since the above term gives a mass term of $\mathcal{O}(100 \text{ TeV})$. If, instead, we introduce X and $\bar{X}$ as new fields and assign them the same quantum numbers and R-parity as the Higgs fields, they can decay quickly through mixing with Higgs to ordinary quarks and leptons. The production of the R-parity odd particles is suppressed compared to the quark/leptons or radiation and, therefore, the overproduction of the dark matter can be avoided.

Alternatively, we can arrange m_{Hidden} such that $m_{\text{Hidden}} < m_\rho$ but z decays earlier than ρ. However, gravitino production in z decay is potentially problematic, even accounting for the dilution effects from the ρ decay; precisely how serious this problem is in this model depends on the details of the cosmological history, such as the initial amplitude of the z and ρ oscillations.

5 Gaugino Emission

In Refs. [6, 7], it is also pointed out that the decay of heavy moduli to gauginos is not necessarily helicity suppressed. This is easy to see. Consider a modulus S with a coupling to gauge fields through

$$\mathcal{L}_{gauge} = f(S) W_\alpha^2 \,, \tag{37}$$

and the Kähler potential

$$-3 \log(S + S^\dagger) \,. \tag{38}$$

Correspondingly, there is a coupling to a pair of canonically normalized gauginos,

$$\mathcal{L}_{\lambda\lambda} = \frac{f'(S)}{f(S)} g^{S\bar{S}} e^{G/2} G_{\bar{S}} \lambda\lambda. \tag{39}$$

Then writing $S = S_o + \hat{S}$, where $\hat{S}$ represents the fluctuating field, the coupling of $\hat{S}$ (after rescaling it to canonical normalization) to $\lambda\lambda$ is

$$g^{S\bar{S}} e^{G/2} G_{\bar{S}\bar{S}} \hat{S}\lambda\lambda \approx M_S \hat{S}\lambda\lambda \tag{40}$$

where M_S is the supersymmetric part of the S mass. In the above we assumed a generic scaling $f'(S)/f(S) \sim 1/S$. Although (40) is a consequence of the specific form of the Kähler potential, this result is quite general.

Provided that M_S is large compared to the gravitino or gaugino mass, there is clearly no suppression by powers of m_λ. The branching ratio of S to gauge bosons and gauginos will be of order one. We should note that, unlike the possible enhancement of the gravitino rate, which is a cosmological catastrophe, the enhancement of the gaugino rate, for a very heavy scalar decay, is not necessarily a problem. For a modulus with mass of order 1000 TeV or so, the annihilation rate for gauginos produced in decays is large enough that one can naturally obtain a density suitable for the dark matter density.

6 Gravitinos from Inflaton Decays

In Ref. [8], it was argued that there is a gravitino problem from inflaton decays. The authors assumed that there is no helicity suppression for inflaton decay into gravitinos, and obtained quite stringent constraints from nucleosynthesis or overproduction of the dark matter. As a result, they argued that it is critical to a successful cosmology that $G_\phi = 0$ at the minimum of the inflaton potential[3]. However, in light of our previous discussion, it is natural to ask whether inflaton decays *can* be helicity suppressed.

As we have already seen, the question of whether these helicity suppressed or not depends crucially on the structure of the hidden sector. The decay is helicity suppressed unless there is a direct coupling to the hidden sector of a specific type, or the hidden sector field with non-vanishing F-term is heavier than the inflaton.

We demonstrate this helicity suppression by analyzing a very simple model with

$$K = \phi^\dagger \phi + z^\dagger z - \frac{(z^\dagger z)^2}{\Lambda^2} \ , \tag{41}$$

$$W = \frac{m_\phi}{2}(\phi - \phi_0)^2 + \mu^2 z + W_o \ . \tag{42}$$

[3] Cosmology with a general heavy scalar field has also been studied very recently in Ref. [15] .

This is essentially the same situation as that of the models discussed in [8] and of the KKLT model analyzed in sections 3 and 4. The inflaton field has a large supersymmetric mass term m_ϕ compared to the gravitino and the minimum of the potential $\phi = \phi_0$ is displaced from the origin. We have again introduced the $-(z^\dagger z)^2/\Lambda^2$ term such that we can treat the z mass, $m_{\rm Hidden} = \mathcal{O}(m_{3/2}M_p/\Lambda)$, as a free parameter.

When $m_\phi > m_{\rm Hidden}$, the decay amplitude is suppressed by a factor of $m_{3/2}/m_\phi$ or $(m_{\rm Hidden}/m_\phi)^2$ as we see in the following. In the unitary gauge calculation, the mismatch between the direction of the F-term and the inflaton mass eigenstate appears to be small compared to the naive estimation of $\mathcal{O}(m_{3/2}/m_\phi)$ by a factor of $\mathcal{O}(m_{\rm Hidden}^2/m_\phi^2)$ or $\mathcal{O}(m_{3/2}/m_\phi)$. Explicitly, the G_ϕ and G_z factors are obtained to be

$$G_\phi = -\frac{G_{\bar{z}\bar{\phi}}}{G_{\bar{\phi}\bar{\phi}}} G_z = -\frac{\sqrt{3}\phi_0 m_{3/2}}{m_\phi} G_z \ , \quad G_z = \sqrt{3} \tag{43}$$

at the leading order in the $m_{3/2}/m_\phi$ expansion. The $\mathcal{O}(m_{3/2}/m_\phi)$ contribution appeared in G_ϕ with a factor ϕ_0 representing the displacement from the origin. On the other hand, the inflaton mass eigenstate Φ has a small z component

$$\Phi = \phi + \epsilon''^* z \ , \tag{44}$$

with

$$\epsilon'' = \frac{\sqrt{3}\phi_0 m_{3/2}}{m_\phi} + \mathcal{O}\left(\frac{m_{3/2}}{m_\phi}\frac{m_{\rm Hidden}^2}{m_\phi^2}, \frac{m_{3/2}^2}{m_\phi^2}\right) . \tag{45}$$

Therefore, the G_Φ factor is

$$G_\Phi = \epsilon'' G_z + G_\phi = \mathcal{O}\left(\frac{m_{3/2}}{m_\phi}\frac{m_{\rm Hidden}^2}{m_\phi^2}, \frac{m_{3/2}^2}{m_\phi^2}\right) . \tag{46}$$

The $\mathcal{O}(m_{3/2}/m_\phi)$ contribution cancels as anticipated. Therefore, the decay amplitude has either $(m_{\rm Hidden}/m_\phi)^2$ or $m_{3/2}/m_\phi$ suppression. The same result can be obtained in the Goldstino picture. The Γ^z_{zz} term and the mixing gives the coupling of the inflaton to Goldstinos:

$$\frac{\epsilon''^*}{2} m_{3/2} G_{zz\bar{z}\bar{z}} G_z \Phi^\dagger \psi_z \psi_z \simeq \frac{1}{2} m_\phi \phi_0 \left(\frac{m_{\rm Hidden}}{m_\phi}\right)^2 \Phi^\dagger \psi_z \psi_z \ . \tag{47}$$

On top of the factor ϕ_0, the amplitude has a suppression factor of $(m_{\rm Hidden}/m_\phi)^2$. Other terms in the Lagrangian give $\mathcal{O}(m_{3/2}/m_\phi)$ suppressed amplitudes.

With sufficiently small $(m_{\rm Hidden}/m_\phi)^2$ and $m_{3/2}/m_\phi$, we can easily satisfy the constraint from gravitino cosmology [16]. The model predictions are obtained by multiplying the larger of two factors $m^2_{\rm Hidden}/m^2_\phi$ or $m_{3/2}/m_\phi$ by the value of G_ϕ presented in Figure 1 in Ref. [8]. If we are to avoid the gravitino production from the z field, the decay of z should happen earlier than the inflaton decay. This gives a constraint on the reheating temperature of the universe T_R. The naive expectation for the decay width of z is

$$\Gamma_z \sim \frac{1}{4\pi}\frac{m^3_{\rm Hidden}}{\Lambda^2}\ , \tag{48}$$

and Λ^2 is related to $m_{\rm Hidden}$ by eqn. (30). Indeed the partial decay width into the gravitinos is this size. By comparing the inflaton decay width $\Gamma_\phi \sim T_R^2/M_p$, we obtain a condition

$$T_R \ll 10^9 \text{ GeV } \left(\frac{m_{3/2}}{100 \text{ GeV}}\right)^{-1}\left(\frac{m_{\rm Hidden}}{10^8 \text{ GeV}}\right)^{5/2}\ . \tag{49}$$

Consistent parameter regions can be easily found.

7 Conclusions: Cosmological Implications

We began this paper with studies of the equivalence theorem for gravitinos. In several exercises, we saw that it is usually easy to compute amplitudes for particle decays to gravitinos in a goldstino picture, but that in unitary gauge there are sometimes subtle cancellations, and it is easy to overestimate these amplitudes. From these examples, we learned that, while there is not necessarily a helicity suppression in the decays of massive scalars to Goldstinos, there often is. Perhaps of greatest current interest, we saw that there is a suppression by $m_{3/2}$ in the simplest version of the KKLT model. On the other hand, we saw that if we alter the form of the Kähler potential in a specific way, we can obtain an enhanced result.

Even when there is a suppression, however, the cosmology of the KKLT models is challenging. For in these cases, there is always another modulus in the hidden sector. For the realistic cosmological scenarios, we need to include those fields in the discussion. We will leave more detailed discussion of the cosmology for another publication, but we noted that it is likely that these moduli are very heavy, and their decays may reheat the universe to temperatures above those of nucleosynthesis. With the addition of vector-like fields to the MSSM, these decays can lead to an acceptable dark matter density. We conclude from these observations that the idea that decays of heavy moduli heat the universe above nucleosynthesis temperatures, producing the dark matter in their decays, remains a viable one; the reader can judge whether it is more plausible than scenarios without light moduli.

The possibility that inflaton decays to gravitinos are not helicity suppressed raises the prospect of significant constraints on inflationary cosmology. Here, however, we again found that there can be significant suppression of moduli decays to gravitinos. The results depend on the structure of the hidden sector. In the case where the inflaton is heavier than the hidden sector particle whose F-term breaks supersymmetry, the amplitude of the inflaton decay into two gravitinos is suppressed by $(m_{\rm Hidden}/m_\phi)^2$ or $m_{3/2}/m_\phi$. This is sufficient to avoid the constraints from nucleosynthesis or overproduction of the dark matter in many inflation models. Problems with the decay of the hidden sector particle can be avoided for low enough reheating temperature. So it appears that the constraints on inflation from inflaton decays to gravitinos are quite mild.

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