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# Applications of AdS/CFT Duality to QCD\*

Stanley J. Brodsky  
Stanford Linear Accelerator Center, Stanford University  
Stanford, California 94309  
e-mail: [sjbth@slac.stanford.edu](mailto:sjbth@slac.stanford.edu)

Guy F. de Téramond  
Universidad de Costa Rica  
San José, Costa Rica

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# Applications of AdS/CFT Duality to QCD

Stanley J. Brodsky

*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, USA*

Guy F. de Téramond

*Universidad de Costa Rica, San José, Costa Rica*

Even though quantum chromodynamics is a broken conformal theory, the AdS/CFT correspondence has led to important insights into the properties of QCD. For example, as shown by Polchinski and Strassler, dimensional counting rules for the power-law falloff of hadron scattering amplitudes follow from dual holographic models with conformal behavior at short distances and confinement at large distances. We find that one also obtains a remarkable representation of the entire light-quark meson and baryon spectrum, including all orbital excitations, based on only one mass parameter. We also show how hadron light-front wavefunctions and hadron form factors in both the space-like and time-like regions can be predicted.

## 1. Introduction

The central mathematical principle underlying AdS/CFT duality is the fact that the group  $SO(2,4)$  of Poincaré and conformal transformations of physical  $3+1$  space-time has an elegant mathematical representation on  $AdS_5$  space where the fifth dimension has the anti-de Sitter warped metric. The group of conformal transformations  $SO(2,4)$  in  $3+1$  space is isomorphic to the group of isometries of AdS space,  $x^\mu \rightarrow \lambda x^\mu$ ,  $r \rightarrow r/\lambda$ , where  $r$  represents the coordinate in the fifth dimension. The dynamics at  $x^2 \rightarrow 0$  in  $3+1$  space thus matches the behavior of the theory at the boundary  $r \rightarrow \infty$ . This allows one to map the physics of quantum field theories with conformal symmetry to an equivalent description in which scale transformations have an explicit representation in AdS space.

Even though quantum chromodynamics is a broken conformal theory, the AdS/CFT correspondence has led to important insights into the properties of QCD. For example, as shown by Polchinski and Strassler,<sup>1</sup> the AdS/CFT duality, modified to give a mass scale, provides a nonperturbative derivation of the empirically successful dimensional counting rules<sup>2,3</sup> for the leading power-law fall-off of the hard exclusive scattering amplitudes of the bound states of the gauge theory. The modified theory generates the hard behavior expected from QCD instead of the soft behavior characteristic of strings. Other important applications include the description of spacelike hadron form factors at large transverse momentum<sup>4</sup> and deep inelastic scattering structure functions at small  $x$ .<sup>5</sup> The power falloff of hadronic

light-front wave functions (LFWF) including states with nonzero orbital angular momentum is also predicted.<sup>6</sup>

In the original formulation by Maldacena,<sup>7</sup> a correspondence was established between a supergravity string theory on a curved background and a conformally invariant  $\mathcal{N} = 4$  super Yang-Mills theory in four-dimensional space-time. The higher dimensional theory is  $AdS_5 \times S^5$  where  $R = (4\pi g_s N_C)^{1/4} \alpha_s'^{1/2}$  is the radius of AdS and the radius of the five-sphere and  $\alpha_s'^{1/2}$  is the string scale. The extra dimensions of the five-dimensional sphere  $S^5$  correspond to the  $SU(4) \sim SO(6)$  global symmetry which rotates the particles present in the SYM supermultiplet in the adjoint representation of  $SU(N_C)$ . In our application to QCD, baryon number in QCD is represented as a Casimir constant on  $S^5$ .

The reason why AdS/CFT duality can have at least approximate applicability to physical QCD is based on the fact that the underlying classical QCD Lagrangian with massless quarks is scale-invariant.<sup>8</sup> One can thus take conformal symmetry as an initial approximation to QCD, and then systematically correct for its nonzero  $\beta$  function and quark masses.<sup>9</sup> This “conformal template” approach underlies the Banks-Zak method<sup>10</sup> for expansions of QCD expressions near the conformal limit and the BLM method<sup>11</sup> for setting the renormalization scale in perturbative QCD applications. In the BLM method the corrections to a perturbative series from the  $\beta$ -function are systematically absorbed into the scale of the QCD running coupling. An important example is the “Generalized Crewther Relation”<sup>12</sup> which relates the Bjorken and Gross-Llewellyn sum rules at the deep inelastic scale  $Q^2$  to the  $e^+e^-$  annihilation cross sections at specific commensurate scales  $s^*(Q^2) \simeq 0.52 Q^2$ . The Crewther relation<sup>13</sup> was originally derived in conformal theory; however, after BLM scale setting, it becomes a fundamental test of physical QCD, with no uncertainties from the choice of renormalization scale or scheme.

QCD is nearly conformal at large momentum transfers where asymptotic freedom is applicable. Nevertheless, it is remarkable that dimensional scaling for exclusive processes is observed even at relatively low momentum transfer where gluon exchanges involve relatively soft momenta.<sup>14</sup> The observed scaling of hadron scattering amplitudes at moderate momentum transfers can be understood if the QCD coupling has an infrared fixed point.<sup>15</sup> In this sense, QCD resembles a strongly-coupled conformal theory.

## 2. Hadron Spectra from AdS/CFT

The duality between a gravity theory on  $AdS_{d+1}$  space and a conformal gauge theory at its  $d$ -dimensional boundary requires one to match the partition functions at the  $AdS$  boundary,  $z = R^2/r \rightarrow 0$ . The physical string modes  $\Phi(x, z) \sim e^{-iP \cdot x} f(r)$ , are plane waves along the Poincaré coordinates with four-momentum  $P^\mu$  and hadronic invariant mass states  $P_\mu P^\mu = \mathcal{M}^2$ . For large- $r$  or small- $z$ ,  $f(r) \sim r^{-\Delta}$ , where the dimension  $\Delta$  of the string mode must be the same dimension as that of the interpolating operator  $\mathcal{O}$  which creates a specific hadron out of the vacuum:  $\langle P | \mathcal{O} | 0 \rangle \neq 0$ .

The physics of color confinement in QCD can be described in the AdS/CFT approach by truncating the AdS space to the domain  $r_0 < r < \infty$  where  $r_0 = \Lambda_{\text{QCD}} R^2$ . The cutoff at  $r_0$  is dual to the introduction of a mass gap  $\Lambda_{\text{QCD}}$ ; it breaks conformal invariance and is responsible for the generation of a spectrum of color-singlet hadronic states. The truncation of the AdS space insures that the distance between the colored quarks and gluons as they stream into the fifth dimension is limited to  $z < z_0 = 1/\Lambda_{\text{QCD}}$ . The resulting 3+1 theory has both color confinement at long distances and conformal behavior at short distances. The latter property allows one to derive dimensional counting rules for form factors and other hard exclusive processes at high momentum transfer. This approach, which can be described as a “bottom-up” approach, has been successful in obtaining general properties of the low-lying hadron spectra, chiral symmetry breaking, and hadron couplings in AdS/QCD,<sup>16</sup> in addition to the hard scattering predictions.<sup>1,5,6</sup>

In this “classical holographic model”, the quarks and gluons propagate into the truncated AdS interior according to the AdS metric without interactions. In effect, their Wilson lines, which are represented by open strings in the fifth dimension, are rigid. The resulting equations for spin 0,  $\frac{1}{2}$ , 1 and  $\frac{3}{2}$  hadrons on  $AdS_5 \times S^5$  lead to color-singlet states with dimension 3, 4 and  $\frac{9}{2}$ . Consequently, only the hadronic states (dimension-3)  $J^P = 0^-, 1^-$  pseudoscalar and vector mesons, the (dimension- $\frac{9}{2}$ )  $J^P = \frac{1}{2}^+, \frac{3}{2}^+$  baryons, and the (dimension-4)  $J^P = 0^+$  glueball states, can be derived in the classical holographic limit.<sup>17</sup> This description corresponds to the valence Fock state as represented by the light-front Fock expansion. Hadrons also fluctuate in particle number, in their color representations (such as the hidden-color states<sup>18</sup> of the deuteron), as well as in internal orbital angular momentum. The higher Fock components of the hadrons are manifestations of the quantum fluctuations of QCD; these correspond to the fluctuations of the bulk geometry about the fixed AdS metric. Similarly, the orbital excitations of hadronic states correspond to quantum fluctuations about the AdS metric.<sup>19</sup> We thus can consistently identify higher-spin hadrons with the fluctuations around the spin 0,  $\frac{1}{2}$ , 1 and  $\frac{3}{2}$  classical string solutions of the  $AdS_5$  sector.<sup>17</sup>

As a specific example, consider the twist-two (dimension minus spin) glueball interpolating operator  $\mathcal{O}_{4+L}^{\ell_1 \dots \ell_m} = FD_{\{\ell_1 \dots \ell_m\}} F$  with total internal space-time orbital momentum  $L = \sum_{i=1}^m \ell_i$  and conformal dimension  $\Delta_L = 4 + L$ . We match the large  $r$  asymptotic behavior of each string mode to the corresponding conformal dimension of the boundary operators of each hadronic state while maintaining conformal invariance. In the conformal limit, an  $L$  quantum, which is identified with a quantum fluctuation about the AdS geometry, corresponds to an effective five-dimensional mass  $\mu$  in the bulk side. The allowed values of  $\mu$  are uniquely determined by requiring that asymptotically the dimensions become spaced by integers, according to the spectral relation  $(\mu R)^2 = \Delta_L(\Delta_L - 4)$ .<sup>17</sup> The four-dimensional mass spectrum follows from the Dirichlet boundary condition  $\Phi(x, z_0) = 0$ ,  $z_0 = 1/\Lambda_{\text{QCD}}$ , on the AdS string amplitudes for each wave functions with spin  $< 2$ . The eigen-

spectrum is then determined from the zeros of Bessel functions,  $\beta_{\alpha,k}$ . The predicted spectra<sup>17</sup> of mesons and baryons with zero mass quarks is shown in Figs. 1 and 2. The only parameter is  $\Lambda_{\text{QCD}} = 0.263$  GeV, and 0.22 GeV for mesons and baryons, respectively.

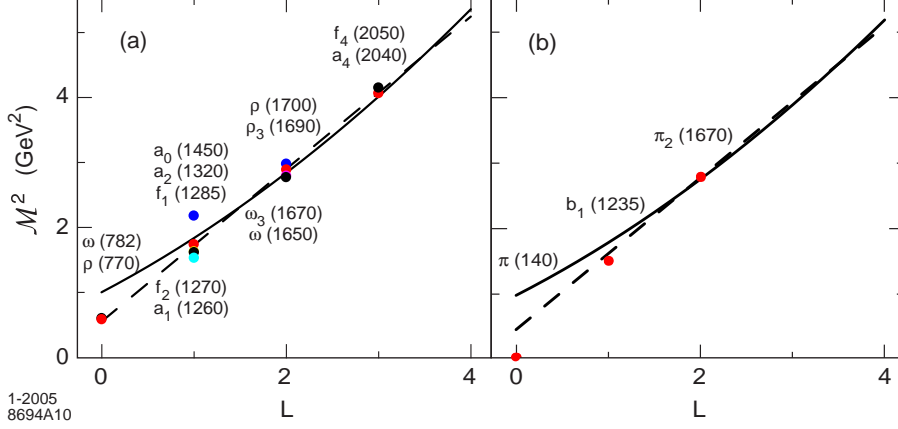


Fig. 1. Light meson orbital states for  $\Lambda_{\text{QCD}} = 0.263$  GeV: (a) vector mesons and (b) pseudoscalar mesons. The dashed line is a linear Regge trajectory with slope  $1.16 \text{ GeV}^2$ .

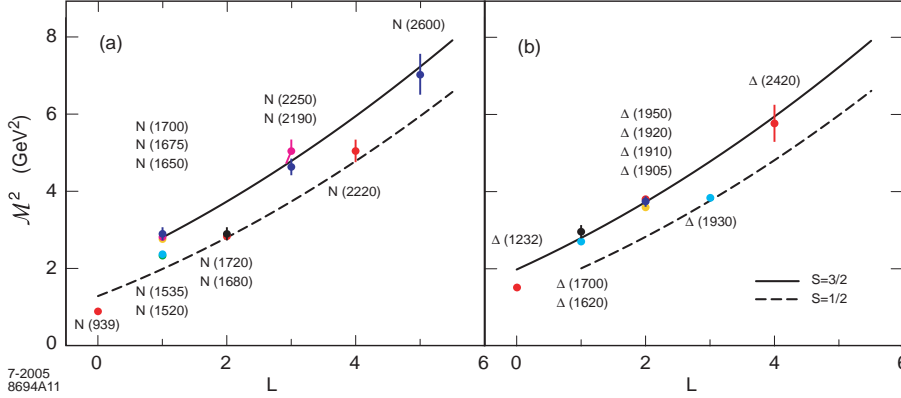


Fig. 2. Light baryon orbital spectrum for  $\Lambda_{\text{QCD}} = 0.22$  GeV: (a) nucleons and (b)  $\Delta$  states.

### 3. Dynamics from AdS/CFT

Current matrix elements in AdS/QCD are computed from the overlap of the normalizable modes dual to the incoming and outgoing hadron  $\Phi_I$  and  $\Phi_F$  and the

non-normalizable mode  $J(x, z)$ , dual to the external source

$$F(Q^2)_{I \rightarrow F} \simeq R^{3+2\sigma} \int_0^{z_o} \frac{dz}{z^{3+2\sigma}} \Phi_F(z) J(Q, z) \Phi_I(z), \quad (1)$$

where  $\sigma_n = \sum_{i=1}^n \sigma_i$  is the spin of the interpolating operator  $\mathcal{O}_n$ , which creates an  $n$ -Fock state  $|n\rangle$  at the AdS boundary.  $J(x, z)$  has the value 1 at zero momentum transfer, and as boundary limit the external current, thus  $A^\mu(x, z) = \epsilon^\mu e^{iQ \cdot x} J(Q, z)$ . The solution to the AdS wave equation subject to boundary conditions at  $Q = 0$  and  $z \rightarrow 0$  is<sup>5</sup>  $J(Q, z) = zQK_1(zQ)$ . At large enough  $Q \sim r/R^2$ , the important contribution to (1) is from the region near  $z \sim 1/Q$ . At small  $z$ , the  $n$ -mode  $\Phi^{(n)}$  scales as  $\Phi^{(n)} \sim z^{\Delta_n}$ , and we recover the power law scaling,<sup>2</sup>  $F(Q^2) \rightarrow [1/Q^2]^{\tau-1}$ , where the twist  $\tau = \Delta_n - \sigma_n$ , is equal to the number of partons,  $\tau_n = n$ . A numerical computation for the pion form factor gives the results shown in Fig. 3, where the resonant structure in the time-like region from the AdS cavity modes is apparent.

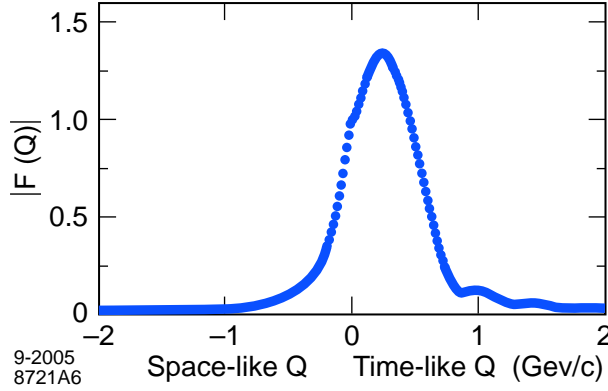


Fig. 3. Space-like and time-like structure for the pion form factor in AdS/QCD.

#### 4. AdS/CFT Predictions for Light-Front Wavefunctions

The AdS/QCD correspondence provides a simple description of hadrons at the amplitude level by mapping string modes to the impact space representation of LFWFs. It is useful to define the partonic variables  $x_i \vec{r}_{\perp i} = \vec{R}_{\perp} + \vec{b}_{\perp i}$ , where  $\vec{r}_{\perp i}$  are the physical position coordinates,  $\vec{b}_{\perp i}$  are frame-independent internal coordinates,  $\sum_i \vec{b}_{\perp i} = 0$ , and  $\vec{R}_{\perp}$  is the hadron transverse center of momentum  $\vec{R}_{\perp} = \sum_i x_i \vec{r}_{\perp i}$ ,  $\sum_i x_i = 1$ . We find for a two-parton LFWF the Lorentz-invariant form

$$\tilde{\psi}_L(x, \vec{b}_{\perp}) = C x(1-x) \frac{J_{1+L} \left( |\vec{b}_{\perp}| \sqrt{x(1-x)} \beta_{1+L,k} \Lambda_{\text{QCD}} \right)}{|\vec{b}_{\perp}| \sqrt{x(1-x)}}. \quad (2)$$

The  $\beta_{1+L,k}$  are the zeroes of the Bessel functions reflecting the Dirichlet boundary condition. The variable  $\zeta = |\vec{b}_\perp| \sqrt{x(1-x)}$ ,  $0 \leq \zeta \leq \Lambda_{QCD}^{-1}$ , represents the invariant separation between quarks. In the case of a two-parton state, it gives a direct relation between the scale of the invariant separation between quarks,  $\zeta$ , and the holographic coordinate in AdS space:  $\zeta = z = R^2/r$ . The ground state and first orbital eigenmode are depicted in the figure below.

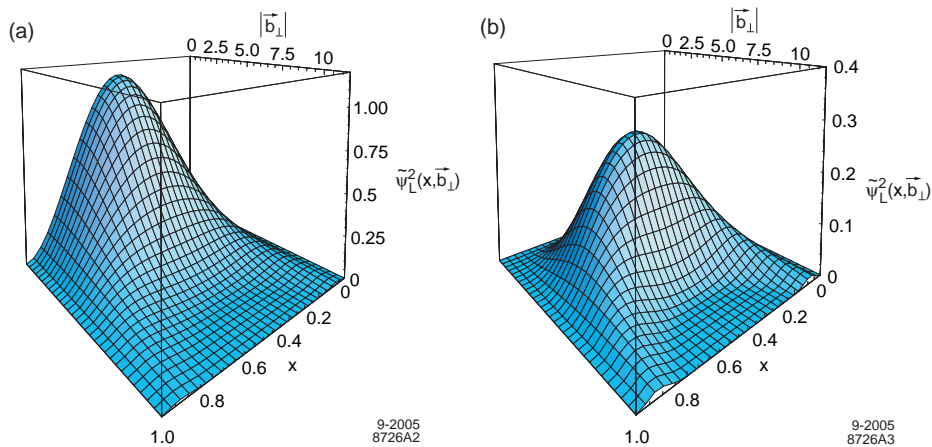


Fig. 4. Prediction for the square of the two-parton bound-state light-front wave function  $\tilde{\psi}_L(x, \vec{b}_\perp)$  as function of the constituents longitudinal momentum fraction  $x$  and  $1-x$  and the impact space relative coordinate  $\vec{b}_\perp$ : (a)  $L=0$  and (b)  $L=1$ .

The holographic model is quite successful in describing the known light hadron spectrum. The only mass scale is  $\Lambda_{QCD}$ . The model incorporates confinement and conformal symmetry. Only dimension-3,  $\frac{9}{2}$  and 4 states  $\bar{q}q$ ,  $qqq$ , and  $gg$  appear in the duality at the classical level. As we have described, non-zero orbital angular momentum and higher Fock-states require the introduction of quantum fluctuations. The model gives a simple description of the structure of hadronic form factors and LFWFs, which can be used as an initial approximation to the actual eigensolutions of the light-front Hamiltonian for QCD. It also explains the suppression of the odderon. The dominance of the quark-interchange mechanism in hard exclusive processes also emerges naturally from the classical duality of the holographic model.

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## References

1. J. Polchinski and M. J. Strassler, Phys. Rev. Lett. **88**, 031601 (2002) [arXiv:hep-th/0109174].
2. S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).
3. V. A. Matveev, R. M. Muradian and A. N. Tavkhelidze, Lett. Nuovo Cim. **7**, 719 (1973).
4. J. Polchinski and L. Susskind, arXiv:hep-th/0112204.
5. J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003) [arXiv:hep-th/0209211].
6. S. J. Brodsky and G. F. de Teramond, Phys. Lett. B **582**, 211 (2004) [arXiv:hep-th/0310227].
7. J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. G473 **38**, 1113 (1999)] [arXiv:hep-th/9711200].
8. G. Parisi, Phys. Lett. B **39**, 643 (1972).
9. S. J. Brodsky, Y. Frishman and G. P. Lepage, Phys. Lett. B **167**, 347 (1986).
10. T. Banks and A. Zaks, Nucl. Phys. B **196**, 189 (1982).
11. S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D **28**, 228 (1983).
12. S. J. Brodsky, G. T. Gabadadze, A. L. Kataev and H. J. Lu, Phys. Lett. B **372**, 133 (1996) [arXiv:hep-ph/9512367].
13. R. J. Crewther, Phys. Rev. Lett. **28**, 1421 (1972).
14. G. F. de Teramond and S. J. Brodsky, arXiv:hep-ph/0507273.
15. S. J. Brodsky, arXiv:hep-ph/0408069.
16. H. Boschi-Filho and N. R. F. Braga, Eur. Phys. J. C **32**, 529 (2004) [arXiv:hep-th/0209080]; G. F. de Teramond and S. J. Brodsky, arXiv:hep-th/0409074; J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, arXiv:hep-ph/0501128; S. Hong, S. Yoon and M. J. Strassler, arXiv:hep-ph/0501197; L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005) [arXiv:hep-ph/0501218]; H. Boschi-Filho, N. R. F. Braga and H. L. Carrion, arXiv:hep-th/0507063.
17. G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. **94**, 201601 (2005) [arXiv:hep-th/0501022].
18. S. J. Brodsky, C. R. Ji and G. P. Lepage, Phys. Rev. Lett. **51**, 83 (1983).
19. S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Nucl. Phys. B **636**, 99 (2002) [arXiv:hep-th/0204051].