# Dalitz Plot Study of $B^{0} \rightarrow K^{+} K^{-} K_{s}^{0}$ Decays 

The BABAR Collaboration

July 25, 2005


#### Abstract

We present a study of the dynamics in $B^{0} \rightarrow K^{+} K^{-} K_{S}^{0}$ decays with approximately 230 million $B \bar{B}$ events collected by the BABAR detector at SLAC. We find that the Dalitz plot distribution is best parameterized with the $\phi K_{S}^{0}$ mode, an S-wave $K^{+} K^{-}$resonance near $1500 \mathrm{MeV} / c^{2}$, and a large non-resonant contribution. We set limits on resonances not included in our model, and study models for the non-resonant contribution.


Submitted at the International Europhysics Conference On High-Energy Physics (HEP 2005), 7/21-7/27/2005, Lisbon, Portugal

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Work supported in part by Department of Energy contract DE-AC02-76SF00515.

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## 1 INTRODUCTION

Charmless three-body decays of the $B$ meson are a rich laboratory for the physics of the Standard Model (SM), providing information on both the weak sector and the dynamics of the strong interaction. Decays to the final state $K^{+} K^{-} K_{S}^{0}$, which are dominated by $b \rightarrow s \bar{s} s$ amplitudes, are of particular interest due to their sensitivity to physics beyond the SM. However, little is known about the dynamical structure of this decay. Such an understanding is important because although the branching fraction is large [1, 2], the final state is a mixture of $C P$-even and $C P$-odd states [1, 3], hence any measurement of the $C P$ asymmetry is diluted due to the presence of both components. A key to more precise measurements of the $C P$ asymmetry is to identify and parameterize various contributions to the final state. An amplitude analysis, incorporating interference between decay submodes, allows for the development of a model of the decay dynamics and the extraction of the partial branching fractions and relative phases for the modeled resonances.

In this paper we present results from a full amplitude analysis of the decay $B^{0} \rightarrow K^{+} K^{-} K_{S}^{0}$. The decay kinematics of a spin-zero particle into three spin-zero daughters are completely determined by two degrees of freedom. In terms of the invariant masses of daughter pairs with four-momenta $p_{i}, m_{i j}^{2}=\left(p_{i}+p_{j}\right)^{2}$, the $B^{0}$ decay rate is

$$
\begin{equation*}
\frac{d \Gamma\left(q_{\mathrm{tag}}, m_{K^{+} K^{-}}, m_{K^{+} K_{S}^{0}}\right)}{d m_{K^{+} K^{-}}^{2} d m_{K^{+} K_{S}^{0}}^{2}}=\frac{1}{(2 \pi)^{3}} \frac{1}{32 M_{B^{0}}^{2}} \frac{1}{2} \times\left[|\mathcal{A}|^{2}+|\overline{\mathcal{A}}|^{2}-q_{\mathrm{tag}} \frac{|\mathcal{A}|^{2}-|\overline{\mathcal{A}}|^{2}}{\left(\Delta m_{d} \tau\right)^{2}+1}\right] \tag{1}
\end{equation*}
$$

where $M_{B^{0}}, \Delta m_{d}$, and $\tau$ are the mass, mixing frequency, and lifetime of the $B^{0}$, respectively. $\mathcal{A}$ $(\overline{\mathcal{A}})$ is the signal $B^{0}\left(\bar{B}^{0}\right)$ decay amplitude and $q_{\text {tag }}$ is $-1(1)$ when the other $B$ meson in the event is a $\bar{B}^{0}\left(B^{0}\right)$. Summing over $q_{\mathrm{tag}}$, we obtain

$$
\begin{equation*}
\frac{d \Gamma\left(m_{K^{+} K^{-}}, m_{K^{+} K_{S}^{0}}\right)}{d m_{K^{+} K^{-}}^{2} d m_{K^{+} K_{S}^{0}}^{2}}=\frac{1}{(2 \pi)^{3}} \frac{1}{32 M_{B^{0}}^{2}} \frac{1}{2} \times\left[|\mathcal{A}|^{2}+|\overline{\mathcal{A}}|^{2}\right] . \tag{2}
\end{equation*}
$$

Assuming $\mathcal{A} \approx \overline{\mathcal{A}}$, corresponding to no direct $C P$ asymmetry, this expression simplifies to

$$
\begin{equation*}
\frac{d \Gamma\left(m_{K^{+} K^{-}}, m_{K^{+} K_{S}^{0}}\right)}{d m_{K^{+} K^{-}}^{2} d m_{K^{+} K_{S}^{0}}^{2}} \approx \frac{1}{(2 \pi)^{3}} \frac{1}{32 M_{B^{0}}^{2}} \times|\mathcal{A}|^{2} . \tag{3}
\end{equation*}
$$

This assumption is consistent with the cosine term observed in time-dependent $C P$ asymmetry measurements [2, 4], and predicted by preliminary calculations based on the QCD-factorization model [5, 6].

To describe the decay dynamics, we parameterize the amplitude in the isobar model [7], as a sum of contributions $\mathcal{A}=\sum_{r} c_{r} \cdot \mathcal{A}_{r}$, where $c_{r}$ is a complex coefficient and the index $r$ runs over the resonances in the model plus a non-resonant component. For a resonance $r$ formed in the variable $m_{i j}^{2}$,

$$
\begin{equation*}
\mathcal{A}_{r}\left(m_{i j}^{2}, m_{i k}^{2}\right)=T_{r}\left(m_{i j}^{2}\right) \times F_{B}\left(\left|\overrightarrow{p_{k}}\right|\right) \times F_{L}\left(\left|\overrightarrow{p_{i}}\right|\right) \times Z_{L}\left(\overrightarrow{p_{i}}, \overrightarrow{p_{k}}\right), \tag{4}
\end{equation*}
$$

where the momenta $\vec{p}$ are measured in the resonance rest frame, $T_{r}$ is the dynamical function of the resonance, $F_{L}\left(F_{B}\right)$ is a Blatt-Weisskopf barrier factor for the resonance ( $B$ meson) decay, and $Z_{L}$ is a Zemach tensor [8, 9].

The Blatt-Weisskopf factors $F_{L}(z)$, with $z=\left|\overrightarrow{p_{i}}\right| R$ and the range $R=1.5 \mathrm{GeV}^{-1}$ [10], depend on the resonance angular momentum $L$ [8]:

$$
\begin{align*}
F_{L=0}(z) & =1, \\
F_{L=1}(z) & =\sqrt{\frac{1+z_{0}^{2}}{1+z^{2}}},  \tag{5}\\
F_{L=2}(z) & =\sqrt{\frac{9+3 z_{0}^{2}+z_{0}^{4}}{9+3 z^{2}+z^{4}}} .
\end{align*}
$$

Here $z_{0}=z\left(\left|\overrightarrow{p_{i}}\right|_{0}\right)$, where $\left|\overrightarrow{p_{i}}\right|_{0}=\left|\overrightarrow{p_{i}}\right|$ when $m_{i j}=m_{r}$, the resonance mass. For the factor $F_{B}$ for the parent $B$ meson, we use a range $R$ of zero. The angular distribution of the decay products for different $L$ is given in the Zemach tensor formalism by [9]:

$$
\begin{align*}
& Z_{L=0}\left(\overrightarrow{p_{i}}, \overrightarrow{p_{k}}\right)=1, \\
& Z_{L=1}\left(\overrightarrow{p_{i}}, \overrightarrow{p_{k}}\right)=-4 \overrightarrow{p_{i}} \cdot \overrightarrow{p_{k}},  \tag{6}\\
& Z_{L=2}\left(\overrightarrow{p_{i}}, \overrightarrow{p_{k}}\right)=\frac{16}{3}\left[3\left(\overrightarrow{p_{i}} \cdot \overrightarrow{p_{k}}\right)^{2}-\left(\left|\overrightarrow{p_{i}}\right|\left|\overrightarrow{p_{k}}\right|\right)^{2}\right] .
\end{align*}
$$

Most resonances are parameterized with the relativistic Breit-Wigner form

$$
\begin{equation*}
T_{r}\left(m_{i j}^{2}\right)=\frac{1}{m_{r}^{2}-m_{i j}^{2}-i m_{r} \Gamma\left(m_{i j}\right)} . \tag{7}
\end{equation*}
$$

$\Gamma\left(m_{i j}\right)$ is the mass-dependent width, given by

$$
\begin{equation*}
\Gamma\left(m_{i j}\right)=\Gamma_{r}\left(\frac{\left|\overrightarrow{p_{i}}\right|}{\left|\overrightarrow{p_{i}}\right|_{0}}\right)^{2 L+1}\left(\frac{m_{r}}{m_{i j}}\right)\left(F_{L}\left(\left|\overrightarrow{p_{i}}\right|\right)\right)^{2}, \tag{8}
\end{equation*}
$$

where $\Gamma_{r}$ is the nominal resonance width. For the $f_{0}(980)$, we use the coupled-channel parameterization of Flatté [11], with couplings taken from recent measurements [12, 13].

The inclusive charmless branching fraction $\mathcal{B}\left(B^{0} \rightarrow K^{+} K^{-} K^{0}\right)$ is measured to be $(24.7 \pm 2.3) \times$ $10^{-6}[1,2]$, where $\left(4.2_{-0.5}^{+0.6}\right) \times 10^{-6}$ is from the $\phi K^{0}$ channel $[14,15,16]$. We have previously reported on an angular moment analysis for this decay [3]. As a function of $m_{K^{+} K^{-}}$, we found the $\phi$ to be the only statistically significant P -wave component. The S -wave contribution is spread across the kinematic range, with a peaking structure at around $1500 \mathrm{MeV} / c^{2}$. Contributions of higher waves were consistent with zero. Our present analysis model is motivated by these results. In addition to the $\phi(1020) K_{S}^{0}$ channel, we include an S-wave resonance with a mass of about $1500 \mathrm{MeV} / c^{2}$, the $X(1500)$. An additional S-wave component, the "non-resonant" (NR) contribution, is added to account for the S -wave events spread across the phase space. We also try adding the $f_{0}(980)$ to the fit model.

The absence of higher waves in the $m_{K^{+} K^{-}}$angular analysis suggests the lack of resonant channels decaying to $K^{+} K_{S}^{0}$. The $B^{0} \rightarrow a_{0}^{-}(980) K^{+}$branching fraction has been measured as small [17]. Additionally, there are theoretical reasons to doubt the possibility of significant isovector contributions [18, 19].

Several decays involving $b \rightarrow c$ transitions can contribute to the $K^{+} K^{-} K_{S}^{0}$ final state. We coherently add an amplitude for the $\chi_{c 0} K_{S}^{0}$ mode to our isobar model, while the $D^{+} K^{-}$and $D_{s}^{+}$ $K^{-}$channels are added incoherently and are described with Gaussian shapes to account for the finite resolution in $m\left(K^{+} K^{-}\right)$mass.

Unfortunately, the present statistics do not allow for the parameterization of the P-wave outside of the $\phi K_{S}^{0}$ region and the subsequent extraction of the relative $S-P$ phase. Hence, we focus on the parameterization of the $S$-wave contribution which dominates the decay [3]. Theoretical models of the NR amplitude [5, 20] do not reproduce the distribution observed in data. Without reliable theoretical guidance, we test several ad-hoc parameterizations. The three most successful models are:

$$
\begin{align*}
& \sqrt{m_{K^{+} K^{-}}^{2}-4 m_{K^{+}}^{2}}\left[m_{K^{+} K^{-}}^{2} \log \left(\frac{m_{K^{+} K^{-}}^{2}}{\beta_{n r}^{2}}\right)\right]^{-1}  \tag{9}\\
& 1+\beta_{n r} e^{i \delta_{\beta}} m_{K^{+} K^{-}}^{2}, \text { and }  \tag{10}\\
& \exp \left(\beta_{n r} m_{K^{+} K^{-}}^{2}\right) \tag{11}
\end{align*}
$$

multiplied with isobar amplitudes and constant phases, and the parameters $\beta_{n r}$ are free in the fits. The first parameterization is inspired by a model from Ref. [5], where to improve agreement with data we have dropped terms involving $b \rightarrow u$ transitions and those that give a dependence on $m_{K^{+} K_{S}^{0}}$. The second parameterization is a linearization of theoretical models [20], and the third model was successfully used in an analysis of the decay $B^{+} \rightarrow K^{+} K^{+} K^{-}$[21]. We choose the model of Eq. (11) to parameterize the NR amplitude. This model gives good agreement with data and allows for direct comparison of the $\beta_{n r}$ parameter with that determined in $B^{+} \rightarrow K^{+} K^{+} K^{-}$ decays.

We also test for a possible NR amplitude dependence on the $K^{ \pm} K_{S}^{0}$ mass by adding terms with a linear dependence on $m_{K^{+} K_{S}^{0}}^{2}$.

Therefore, our main model includes four coherent contributions: the $\phi(1020)\left(m=1019.456 \mathrm{MeV} / c^{2}\right.$, $\left.\Gamma=4.26 \mathrm{MeV} / c^{2}\right), X(1500)\left(m=1507 \mathrm{MeV} / c^{2}, \Gamma=109 \mathrm{MeV} / c^{2}\right)$, and $\chi_{c 0}\left(m=3415.19 \mathrm{MeV} / c^{2}\right.$, $\Gamma=10.1 \mathrm{MeV} / c^{2}$ ) decaying to $K^{+} K^{-}$; and the NR S-wave [22].

## 2 ANALYSIS METHOD

This analysis uses a data sample of approximately 230 million $B \bar{B}$ events collected with the $B A B A R$ detector [23] at the SLAC PEP-II $e^{+} e^{-}$storage rings operating at the $\Upsilon(4 S)$ resonance. The basic method used for reconstruction and selection of $B^{0}$-candidates is described in Ref. [2]. We characterize events with standard topological variables which distinguish between the jet-like structure of the dominant continuum $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ background and relatively isotropic $B$ decays. These variables are combined into a Fisher discriminant $\mathcal{F}$ [2]. The shape of the $\mathcal{F}$ distribution for continuum events is seen to vary as a function of the Dalitz plot location, becoming more "signallike" away from the Dalitz plot edges. As a result, we do not include it in the fit, and instead impose a requirement on the value of $\mathcal{F}$ to select events for the fit.

Two kinematic variables are used to characterize $B^{0}$ candidates, the energy difference $\Delta E=$ $E_{B}-\sqrt{s} / 2$ and the beam-energy-substituted mass $m_{\mathrm{ES}}=\sqrt{\left(s / 2+\vec{p}_{i} \cdot \vec{p}_{B}\right)^{2} / E_{i}^{2}-\vec{p}_{B}^{2}} . \quad E_{B}$ is the reconstructed energy of the $B^{0}$ candidate in the CM frame and $\sqrt{s}$ is the total CM energy. ( $\vec{p}_{i}, E_{i}$ ) is the initial $e^{+} e^{-}$four-momentum and $\vec{p}_{B}$ is the reconstructed $B^{0}$-candidate momentum, both measured in the laboratory frame. For signal events, $\Delta E$ peaks at zero with a resolution of 18 MeV , while $m_{\mathrm{ES}}$ peaks at the $B^{0}$ mass with $2.6 \mathrm{MeV} / c^{2}$ resolution. We initially retain candidates with $|\Delta E|<200 \mathrm{MeV}$ and $m_{\mathrm{ES}}>5.2 \mathrm{GeV} / c^{2}$. For the Dalitz plot fit, we keep candidates in a signal region (SR) of $|\Delta E|<60 \mathrm{MeV}$ and $m_{\mathrm{ES}}>5.26 \mathrm{GeV} / c^{2}$. We also define a sideband (SB) region, with
$|\Delta E|<200 \mathrm{MeV}$ and $5.2<m_{\mathrm{ES}}<5.26 \mathrm{GeV} / c^{2}$, to study continuum backgrounds. For calculation of the Dalitz plot variables, candidates are refit with their mass fixed to the world average value [22], constraining the $B^{0}$ candidate to the kinematically-allowed Dalitz plot region.

Backgrounds due to $B$ decays are studied with samples of simulated events. The largest contribution arises from the random combination of tracks from both $B$ mesons. This combinatorial background is difficult to differentiate from continuum background, and is accounted for in the fits with the continuum background. $B$ decays with a missing pion in the final state combination fall outside both the SR and SB.

We use a high-statistics sample of simulated signal events to evaluate the signal efficiency in bins on the Dalitz plot. The efficiency varies primarily as a function of $m_{K^{+} K^{-}}$, falling from roughly $35 \%$ for low $m_{K^{+} K^{-}}$to about $10 \%$ at high $m_{K^{+} K^{-}}$. The average efficiency is $26 \%$. Signal events which are misreconstructed with a daughter from the other $B$ meson give a negligible contribution, making up less than $0.5 \%$ of the total events.

An unbinned extended maximum likelihood (ML) fit to the events in the SR is used to extract event yields and amplitude coefficients. Parameters of the probability density functions (PDF) for signal $\Delta E$ and $m_{\mathrm{ES}}$ are determined in simulated events, and those for the continuum background are fit to data. For the amplitude fit, we parameterize the event kinematics with the variables $m_{K^{+} K^{-}}$and $\cos \theta_{H}$, where $\theta_{H}$ is the angle between the flight direction of the $K^{+}$and $K_{S}^{0}$ in the $K^{+} K^{-}$center-of-mass (CM) frame. This change of variables introduces a Jacobian term $|J|=$ $\left(2 m_{K^{+} K^{-}}\right)\left(2\left|\vec{p}_{i}\right|\left|\vec{p}_{k}\right|\right)$ in the signal PDF, where $\left|\vec{p}_{i}\right|\left(\left|\vec{p}_{k}\right|\right)$ is the $K^{+}\left(K_{S}^{0}\right)$ momentum in the $K^{+} K^{-}$ CM frame. The total PDF is formed as $\mathcal{P}\left(m_{\mathrm{ES}}\right) \cdot \mathcal{P}(\Delta E) \cdot \mathcal{P}$ (Dalitz), where for signal events

$$
\begin{equation*}
\mathcal{P}(\text { Dalitz })=d \Gamma\left(m_{K^{+} K^{-}}, \cos \theta_{H}\right)|J| \varepsilon\left(m_{K^{+} K^{-}}, \cos \theta_{H}\right) \tag{12}
\end{equation*}
$$

before normalization, and $\varepsilon$ is the efficiency. The continuum $\mathcal{P}$ (Dalitz) PDF is modeled with a histogram of events from the SB region. Bin sizes are variable across the Dalitz plot to account for both narrow features and sparse statistics in different regions.

To explore the multiple solutions possible in the Dalitz fit, we performed several hundred fits with the initial amplitudes and phases randomized within reasonable values. In the fits, the amplitude and phase of the NR contribution are fixed. The relative contributions of each component are evaluated with the fit fraction $F F_{r}$, defined ignoring the interference between amplitudes, as

$$
\begin{equation*}
F F_{r}=\frac{\int\left|c_{r} \mathcal{A}_{r}\right|^{2}|J| d m_{K^{+} K^{-}} d \cos \theta_{H}}{\int\left|\sum_{s} c_{s} \mathcal{A}_{s}\right|^{2}|J| d m_{K^{+} K^{-}} d \cos \theta_{H}} . \tag{13}
\end{equation*}
$$

## 3 SYSTEMATIC STUDIES

We study several potential sources of systematic uncertainty. Fitting to events generated with the PDF and with a full MC sample, we test for any fit bias. No significant biases are found, and we conservatively include the largest deviation from the generated parameters as systematic errors. The parameters of the PDFs for the kinematic variables are fixed in the fit. We vary them by one standard deviation and take the change from the nominal fit as the systematic error. The masses and widths of all resonances except the $f_{0}(980)$ are taken to be the world averages [22], and we vary the values by their listed error and assign systematic errors based on the changes in the fit results. For the $f_{0}(980)$, the systematic error due to the parameterization is derived from the spread between the solutions given by the coupling values measured by BES and E791 [12, 13]. We derive an analogous uncertainty for the $X(1500)$ by fitting with the parameters given by Belle for
the $f_{X}(1500) \rightarrow K^{+} K^{-}$observed in $B^{+} \rightarrow K^{+} K^{+} K^{-}$decays [21] and comparing the results with the nominal fit that uses the world average values [22]. We estimate the systematic uncertainty due to the parameterization of the NR component from the difference in results for the three models.

An additional contribution to the systematic error arises from the spread of values at each of the quoted solutions. We assign the RMS of this spread as the uncertainty. We observe that several of the fit parameters are the same for multiple clusters of solutions, with only one or two parameters differing. In these cases, we quote the average as our result for the parameters that are nearly the same and take the difference from the average as a systematic error on the result.

## 4 RESULTS

There are eleven free parameters in the fit: signal and background yields, the NR parameter $\beta_{n r}$, and five strengths and three relative phases. The phases for the three interfering components are defined with respect to the NR component for which the phase of the coupling to the $B$ meson is set to zero. In the extended ML fit with a total of 1842 events, we find $530 \pm 28$ signal events. Figure 1 shows the data overlayed on $m_{\mathrm{ES}}$ and $\Delta E$ projections of the fit function. Signal and background yields are correlated with the Dalitz plot parameters at the $1 \%$ level.


Figure 1: Projections of the fit function for $m_{\mathrm{ES}}$ (left) and $\Delta E$ (right) shown with data (points). The solid curve is the total PDF, and the continuum background PDF is shown as a dashed curve.

In several hundred fits with random initial values for parameters varied in the fit, we find two two-fold ambiguities resulting in four solutions with similar likelihood values. We list these solutions in Table 1, where the fit fraction $F F_{r}$ for each component $r$ was defined in Eq. (13). Note that interference terms are neglected, so the sum of fit fractions does not necessarily add to $100 \%$. Plots showing projections of the fit function with data are shown in Figure 2.

We observe ambiguity in the fraction and the phase of $X(1500) K_{S}^{0}$ decays. The solution with the relative phase close to zero has a small value for the $X(1500) K_{S}^{0}$ fraction, but the fraction is 7 times larger when the phase is close to $\pi / 2$. In Tab. 1 and Fig. 2, we denote the former solution " A " and the latter " B ". Another ambiguity occurs in the $\phi K_{S}^{0}$ component where there are two solutions approximately 3.5 radians apart in phase, but consistent in fraction. Here we use " 1 " and " 2 " to label the solutions. Both ambiguities are reproduced in Monte Carlo studies where a fit to
a sample of events generated according to one of the solutions leads to the same ambiguities in the fitted parameters as observed in the fit to data.

All NR models given by Eqs. (9)-(11) result in a good fit, with consistent fractions and phases. Using Eq. (11), we get $\beta_{n r}=-0.14 \pm 0.02$, which is consistent with the parameter from the linearized model of Eq. (10). In the alternative model described with Eq. (9) we fix $\beta_{n r}=0.3 \mathrm{GeV} / c^{2}$ as suggested by [5], and we get $\beta_{n r}=0.5 \pm 0.1 \mathrm{GeV} / c^{2}$ if varied in the fit. All fit solutions show the NR parameter to be inconsistent with a "flat" (phase-space) model. When we attempt to fit with the flat model, the fit does not converge in most cases ( $99 \%$ ), and we do not observe distinct clusters of solutions. We also probe for a NR dependence on the $K^{ \pm} K_{S}^{0}$ mass by introducing an explicit dependence on $K^{ \pm} K_{S}^{0}$ mass into our linear model, but the corresponding coefficients are consistent with zero.

As a goodness-of-fit measure, we divide the phase space into bins and compute the $\chi^{2}$ difference between signal-weighted events [24] and the expectation based on the fitted signal model. All bins have at least 10 events. We find for a two-dimensional binning the value of $\chi^{2} /$ dof $=77.1 / 70$, and in projections in $m_{K^{+} K^{-}}$and $\cos \theta_{H}$, respectively, we get $\chi^{2} /$ dof $=48.9 / 40$ and $\chi^{2} /$ dof $=11.7 / 16$. We find negligible differences in $\chi^{2}$ /dof among solutions.

Table 1: Dalitz plot fit results. Labels used in the final column are described in the text. The upper limit for the $f_{0}(980) K_{S}^{0}$ mode is at $90 \%$ confidence.

| Mode | $\left\|c_{r}\right\|$ | Fit Fraction $F F_{r}(\%)$ | Phase | Solutions |
| :--- | ---: | ---: | ---: | :--- |
| $\phi(1020) K_{S}^{0}$ | $0.085 \pm 0.009$ | $15.4 \pm 3.4 \pm 0.6$ | $-1.47 \pm 0.16 \pm 0.2$ | $1 \mathrm{~A}, 1 \mathrm{~B}$ |
|  |  |  | $2.06 \pm 0.18 \pm 0.2$ | $2 \mathrm{~A}, 2 \mathrm{~B}$ |
| $X(1500) K_{S}^{0}$ | $0.71 \pm 0.15$ | $5.2 \pm 2.2 \pm 0.9$ | $-0.17 \pm 0.21 \pm 0.1$ | $1 \mathrm{~A}, 2 \mathrm{~A}$ |
|  | $1.93 \pm 0.18$ | $38.9 \pm 7.3 \pm 0.9$ | $1.19 \pm 0.08 \pm 0.1$ | $1 \mathrm{~B}, 2 \mathrm{~B}$ |
| Non-resonant | 7.07 (fixed) | $70.7 \pm 3.8 \pm 1.7$ | 0 (fixed) | All |
| $\chi_{c 0} K_{S}^{0}$ | $0.32 \pm 0.07$ | $3.1 \pm 1.6 \pm 0.8$ | $0.2 \pm 0.4 \pm 0.6$ | All |
| $f_{0}(980) K_{S}^{0}$ | $1.57 \pm 0.41$ | $5.7 \pm 3.2 \pm 1.0(<9.7)$ | - |  |

## 5 DISCUSSION

We find that the majority of the $B^{0}$ decays to $K^{+} K^{-} K_{S}^{0}$ belong to the $K^{+} K^{-} S$-wave that we identify as the non-resonant contribution. We find three equally good parameterizations that describe this contribution as an $S$-wave with a dependence on $K^{+} K^{-}$mass. When we apply the same model as in $B^{+} \rightarrow K^{+} K^{+} K^{-}$decays [21], we find consistent values for the shape parameter. We do not observe any variation of this component with $K^{ \pm} K_{S}^{0}$ mass. This is consistent with our previous angular moment analysis [3] that did not find higher angular moments that would arise from such a variation. We find good agreement with a theoretical model [5] only when ignoring amplitude terms with features that are not found in the data.

In addition to the non-resonant $S$-wave, we identify a contribution with a $K^{+} K^{-}$mass around $1500 \mathrm{MeV} / c^{2}$. A two-fold ambiguity is observed in the fraction, depending on the relative phase with respect to the large non-resonant background. The nature of this contribution is unclear. Identification of this state as the $f_{0}(1500)$ is inconsistent with the measurement of the $B^{0} \rightarrow$ $f_{0}(1500) K_{S}^{0}, f_{0} \rightarrow \pi^{+} \pi^{-}$decay [25]. Since the ratio $\Gamma\left(f_{0}(1500) \rightarrow K^{+} K^{-}\right) / \Gamma\left(f_{0}(1500) \rightarrow \pi^{+} \pi^{-}\right)$


Figure 2: Top row: $m_{K^{+} K^{-}}$(left) and $m_{K^{+} K_{S}^{0}}$ (right) projections of the nominal solutions (lines) with data (points). For the $m_{K^{+} K_{S}^{0}}$ plot, there are two entries per event. The peak at $m_{K^{+} K_{S}^{0}} \approx$ $1.8 \mathrm{GeV} / c^{2}$ is due to $D^{+} K^{-}$and $D_{s}^{+} K^{-}$decays, while the peaks at higher values of $m_{K^{+} K_{S}^{0}}$ are reflections of forward and backward $\phi$ decays. Bottom row: Left, $\cos \theta_{H}$ projection of the nominal solutions (lines) with data (points); right, Dalitz plot showing distribution of signal events. Lines show solutions 1A (black solid), 1B (red dashed), 2A (green dotted), 2B (blue dash-dot) (See Tab.1). All plots show background-subtracted signal events [24].
is $0.1845 \pm 0.0195$ [22], we would expect only about two events in $B^{0} \rightarrow X(1500) K_{S}^{0}$. Our dataset has insufficient statistics for an independent determination of the mass and width of the state, so we use the world average values for the $f_{0}(1500)$ [22].

In the low $K^{+} K^{-}$mass region, we find the $P$-wave contribution from $\phi K_{S}^{0}$ decays with a fraction consistent with previous measurements [22]. With the available statistics, we do not observe a significant $S$-wave under the $\phi$ resonance. We perform a fit with the $f_{0}(980)$ resonance included in the model and parameterized with the Flatté lineshape, and set an upper limit on the fraction for this decay of $9.7 \%$ at the $90 \%$ confidence level.

## 6 SUMMARY

We present the first study of the decay dynamics in $B^{0} \rightarrow K^{+} K^{-} K_{S}^{0}$ decays and report preliminary results on the fractions and relative phases of intermediate states that contribute to the decay. This is of particular importance for the interpretation of measurements of $C P$ asymmetries in these decays, and searches for physics beyond the Standard Model. We assume no direct CP asymmetry, allowing us to use all events in this statistics-challenged analysis. This assumption is supported by a cosine term in time-dependent $C P$ asymmetry measurements consistent with zero, and preliminary calculations based on the QCD-factorization model [5, 6].

One of the main goals of this paper has been to provide input to theoretical studies that can result in better models. We extended our previous analysis [3] by providing a parameterization for the dominant $S$-wave contribution, and reported on features that are not seen in our dataset.

## 7 ACKNOWLEDGMENTS

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), Institute of High Energy Physics (China), the Commissariat à l'Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Science and Technology of the Russian Federation, and the Particle Physics and Astronomy Research Council (United Kingdom). Individuals have received support from CONACyT (Mexico), the A. P. Sloan Foundation, the Research Corporation, and the Alexander von Humboldt Foundation.

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