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Luminosity Loss due to Beam Distortion and the Beam-Beam Instability*

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Abstract

In a linear collider, sources of emittance dilution such as transverse wakefields or dispersive errors will couple the vertical phase space to the longitudinal position within the beam (the so-called 'banana effect'). When the Intersection Point (IP) disruption parameter is large, these beam distortions will be amplified by a single bunch kink instability which will lead to luminosity loss. We study this phenomena both analytically using linear theory and via numerical simulation. In particular, we examine the dependence of the luminosity loss on the wavelength of the beam distortions and the disruption parameter. This analysis may prove useful when optimizing the vertical disruption parameter for luminosity operation with given beam distortions.

Introduction

To achieve the desired luminosity in a future linear collider, the beams are focused to small spot sizes and the resulting beam-beam forces can be very large. With oppositely charged beams, the beam-beam forces will lead to a mutual focusing or pinch which further increases the beam densities and the luminosity and is referred to as the luminosity enhancement. In addition, if the beams are offset from each other, the attractive beam-beam force can bring the beams closer together possibly recovering some of the lost luminosity. Unfortunately, if the beam-beam force is too large, this attraction can lead to an instability much like a plasma two-stream instability which is referred to as a single bunch kink instability [1].

The beam-beam force and resulting kink instability can be parameterized with the disruption parameter: $D_{x(,y)} \equiv \sigma_z/[f_{x(,y)}] = 2N_b r_e \sigma_z/[\gamma\sigma_{x(,y)}(\sigma_x+\sigma_y)]$, where $f_{x(,y)}$ is the focal length due to the beam-beam force; $\sigma_{x(,y,z)}$ is the rms beam size, N_b the number of particle per beam, r_e the electron classical radius, and γ the Lorentz factor. Normally, for a flat beam, $D_x \ll 1$, and hence the beam acts as a thin lens in the horizontal plane. In contrast, $D_y \gg 1$ and the two beams begin to oscillate in the vertical plane during the collision with the number of oscillations given by $n \approx 0.15\sqrt{D_y}$. In simulations with $D_y \leq 10$, the beam-beam force is observed to reduce the luminosity loss due to centroid offsets, however for larger D_y the luminosity becomes increasingly sensitive to small offsets [2]. In this paper, we investigate the impact of internal distortions in the beam. The many sources of emittance dilution lead to vertical position and angle offsets $\{y_w(z), y'_w(z)\}$ of the slice centers as a function of their longitudinal position z in the beam. It can be expected that the two-stream instability will resonantly amplify distortions with wavelengths that are close to resonance.

Equations of Motion

Suppose that two beams move towards each other. For simplicity, their longitudinal distribution is assumed to be uniform, and their transverse distribution to be Gaussian. Assume that the beams collide with a relative vertical displacement and each beam is assumed to have the same number of particles per unit length λ but with opposite charge. Assuming no redistribution of charges occurs during the collision, the Lorentz force near the axis is [4]

$$F_{y,inc} \approx \frac{4\lambda e^2}{4\pi\epsilon_0}\frac{Y_d}{\sigma_y(\sigma_x+\sigma_y)}\,, \qquad (1)$$

where e is the elementary charge, ϵ_0 the vacuum permittivity, Y_d the particle vertical displacement relative to the centroid of the other beam. We are interested in the luminosity loss due to the variation of the beam centroid along the bunch; hence, we need to integrate over the transverse distribution. Assuming small offsets of the two beams, this reduces the effective incoherent force in Eq. (1) by a factor of 2 to get the so-called coherent force of $F_{y,coh} \approx F_{y,inc}/2$. We study cold beams, and the equations of motion for the two beams, moving with velocity v, read [1]

$$\left(\frac{\partial}{\partial t} \pm v\frac{\partial}{\partial s}\right)^2 y_{l(,r)} = -\frac{2\lambda r_e c^2\left[y_{l(,r)} - y_{r(,l)}\right]}{\sigma_y(\sigma_x+\sigma_y)\gamma}, \qquad (2)$$

where, $y_{l(,r)}$ is the centroid displacements of the electron (positron) beam from the reference axis. In the above model, the evolution of the beam envelope due to the pinch effect is not included. However, since the beam-beam instability dominates, this simplification is quite good, as evidenced by the comparison with simulation in the following.

Initial Value Problem

The internal coordinate z is introduced to label the slice at a distance z from the head of the beam, and $0 < z < l$, where l is the beam length. We define $t = 0$ when the heads of the two beams collide. We also define $s = 0$ as the IP where the two beams first collide. The positive s direction is to the right. Hence in the left-coming beam, the slice z will be located at $s = vt - z$ at time t when the beam head is at the location $s = vt$. In the right-coming beam, we also introduce z to describe the distance between a certain slice and the head of the beam, and again $0 < z < l$. Hence, when the head of the right-coming beam is at location $s = -vt$ at time t, the slice z is at location $s = -vt+z$. We now

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use (s,z) as the independent variable pair, and we define $k_0^2 \equiv 2\lambda r_e/\left[\sigma_y(\sigma_x+\sigma_y)\gamma\right]$. Please refer to Fig. 1 of Ref. [5] for definition of the coordinate system.

Now let us study the case where the right-coming beam has an initial sinusoidal modulation, and the left-coming beam is undistorted and on-axis. The initial conditions are then

$$y_l(0,z)=0 \ \text{ and } \ \left.\frac{\partial y_l(s,z)}{\partial s}\right|_{s=0}=0\,, \tag{3}$$

for the left-coming electron beam. Similarly,

$$y_r(0,z)=y_{r0}\sin(kz) \ \text{ and } \ \left.\frac{\partial y_r(s,z)}{\partial s}\right|_{s=0}=0\,, \tag{4}$$

for the right-coming positron beam. The equations of motion together with the initial conditions yield the following integral representation of the solution

$$\begin{aligned} y_l(s,z) &= y_l(z/2,z)\cos\left[k_0(s-z/2)\right] \\ &+ \left.\frac{\partial y_l(s,z)}{\partial s}\right|_{s=z/2}\frac{\sin\left[k_0(s-z/2)\right]}{k_0} \\ &+ k_0\int_{z/2}^{s} ds' y_r(-s',2s'-z)\sin[k_0(s-s')]\,. \end{aligned} \tag{5}$$

Similarly, for the right-coming beam, we have

$$\begin{aligned} y_r(s,z) &= y_r(-z/2,z)\cos\left[k_0(s+z/2)\right] \\ &+ \left.\frac{\partial y_r(s,z)}{\partial s}\right|_{s=-z/2}\frac{\sin\left[k_0(s+z/2)\right]}{k_0} \\ &+ k_0\int_{-z/2}^{s} ds' y_l(-s',-2s'-z)\sin[k_0(s-s')]. \end{aligned} \tag{6}$$

Series Solution

Let us solve the above set of Eqs. (5) and (6) via a series solution approach [6, 7]. We expand $y_{r(,l)}(s,z)$ in a series of powers in k_0

$$y_{r(,l)}(s,z)=\sum_{n=0}^{\infty} y_{r(,l)}^{(n)}(s,z)\,, \tag{7}$$

and obtain the nth-order term from the $(n-1)$th-order term. According to Eq. (5), for $n=1,2,3,\cdots$, $y_l(s,z)$ would be

$$y_l^{(n)}(s,z)=k_0\int_{z/2}^{s} ds' y_r^{(n-1)}(-s',2s'-z)\sin[k_0(s-s')]; \tag{8}$$

and similarly, according to Eq. (6), for $y_r(s,z)$, we have

$$y_r^{(n)}(s,z)=k_0\int_{-z/2}^{s} ds' y_l^{(n-1)}(-s',-2s'-z)\sin[k_0(s-s')]. \tag{9}$$

General Solution According to Eqs. (5) and (6) with the initial conditions in Eqs. (3) and (4), we get, $y_r^{(0)}(s,z) = y_{r0}\sin(kz)\cos\left[k_0\left(s+z/2\right)\right]$, and $y_l^{(0)}(s,z)=0$. Noticing that, in the limit of $k_0\to 0$, we have $y_r^{(0)}(s,z)=y_{r0}\sin(kz)$. This is just the initial modulation profile.

For a large D_y, which leads to many oscillations during the collision, we are eligible to consider the limit of $k_0 s \gg 1$ and also $ks\gg 1$. Therefore, we keep only the largest terms in $k_0 s$ and ks. The asymptotic solutions are:

$$\begin{aligned} y_r(s,z) &\approx iy_{r0}\frac{kk_0}{k_0^2-4k^2}\sqrt{\frac{|2s+z|}{z}} \\ &\times J_1\left(ik_0\sqrt{z|2s+z|}/2\right)\cos[k_0(s+z)], \end{aligned} \tag{10}$$

for $-z/2>s>-(l+z)/2$; and

$$\begin{aligned} y_l(s,z) &\approx -y_{r0}\frac{kk_0}{k_0^2-4k^2} \\ &\times J_0\left(ik_0\sqrt{z(2s-z)}/2\right)\sin[k_0(s-z)], \end{aligned} \tag{11}$$

for $(l+z)/2>s>z/2$. Notice that, the solutions given in Eqs. (10) and (11) have a singular point at $k_0=2k$, which indicates a resonance behavior studied in the following.

Resonance Case At resonance, $k_0=2k$, the recursion relation of Eq. (8) and (9) should be revised by replacing k_0 with $2k$. The asymptotic solutions are:

$$y_r(s,z)\approx\frac{y_{r0}}{2}J_0\left(ik\sqrt{z|2s+z|}\right)\sin[2k(s+z)], \tag{12}$$

for $-z/2>s>-(l+z)/2$; and

$$y_l(s,z)\approx\frac{iy_{r0}}{2}\sqrt{\frac{2s-z}{z}}J_1\left(ik\sqrt{z(2s-z)}\right)\cos[2k(s-z)], \tag{13}$$

for $(l+z)/2>s>z/2$.

Luminosity

The luminosity is defined as [8]

$$\mathcal{L}=2N_b^2 v\int dxdydsdt\; n_l(x,y,z_l,t)n_r(x,y,z_r,t), \tag{14}$$

where $z_l=vt-s$ and $z_r=vt+s$ and we have assumed the same number population N_b in each beam and head-on collisions. The distribution function is normalized to unit, *i.e.*, $\int dxdyds\; n_{l(,r)}(x,y,z_{l(,r)},t)=1$. Assuming Gaussian transverse distributions and a uniform longitudinal distribution, and ignoring the luminosity enhancement due to beam-beam pinch, the 'geometric' luminosity is $\mathcal{L}_{00}=N_b^2/\left[4\pi\sigma_x\sigma_y\right]$. Finally, the nominal luminosity $\mathcal{L}_0$, including the effect of the luminosity enhancement, is found by multiplying by the enhancement factor H_D which is typically between 1 and 2 for flat beam collisions, *i.e.*, $\mathcal{L}_0=\mathcal{L}_{00}H_D$. Now, let us study the luminosity loss due to the beam-beam disruption. Given the analytical general solutions in Eqs. (10) − (13), we can numerically compute the luminosity. On the other hand, we also simulate the luminosity loss via GuineaPig [9] for longitudinal uniform distribution and Gaussian distribution as well.

Table 1: Summary of the parameters for the US Cold [3].

E (GeV)	N_b (10^{10})	σ_x (μm)
250	2.0	0.543
$\sigma_{x'}$ (μrad)	σ_y (nm)	$\sigma_{y'}$ (μrad)
36	5.7	14
σ_z (m m)	σ_δ (%)	D_y
0.3	0.1	22.0

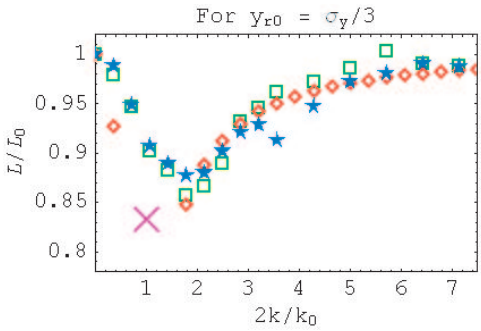

Figure 1: The $\mathcal{L}/\mathcal{L}_0$ as a function of $2k/k_0$. The "square" is the simulation for truncated (at $\pm\sqrt{3}\sigma_z$) Gaussian distribution; the "$\star$" is the simulation for a uniform distribution; the "$\diamond$" is the result with solutions in Eqs. (10) and (11); the "$\times$" is the result with solutions in Eqs. (12) and (13).

Results

Now let us illustrate how the beam-beam effect leads to a large luminosity loss, even if the emittance is not degraded much. We study the ILC US Cold [3] with the parameters in Table 1. Shown in Fig. 1, we find that for $y_{r0} = \sigma_y/3$, the beam-beam interaction leads to a resonance where 14 % luminosity loss is observed. Now, if one does not notice the instability studied in this paper, then he will find 3 % emittance growth, which leads to only 1 % "geometric" luminosity loss simply due to the initial distortion. Hence, the geometric emittance growth in this case is not a good measure for the luminosity loss [10]. Similar situation is found for different y_{r0}. In Figs. 2, we show the luminosity

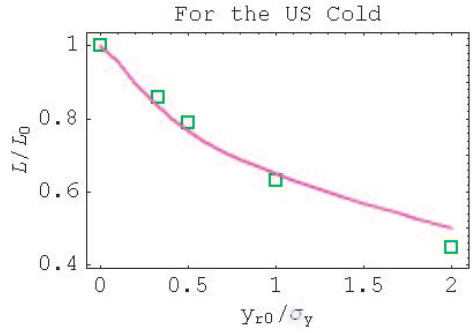

Figure 2: The $\mathcal{L}/\mathcal{L}_0$ as a function of y_{r0}/σ_y with "square" having same meaning as in Fig. 1 and the curve is the result with solutions in Eqs. (12) and (13).

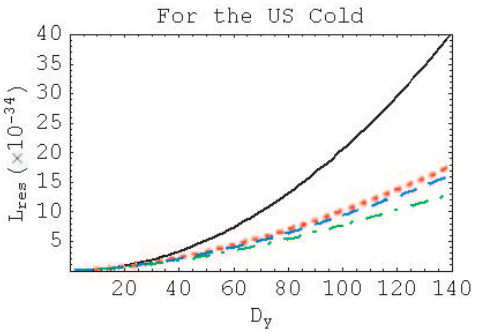

Figure 3: The resonance luminosity $\mathcal{L}_{\rm res}$ as a function of D_y. The dotted, dashed, and dotted-dashed curves stand for the cases where $y_{r0}/\sigma_y = 1/3, 1/2$, and 1, respectively. As a comparison, the solid curve stands for the case with no modulation, *i.e.*, $\mathcal{L}_0$, or $y_{r0} = 0$.

loss as a function of y_{r0}. Once again, very good agreement between the analytical result and the simulation is found.

The study shows that the luminosity loss computed analytically agrees with the simulation results quite well. The results for a longitudinal uniform distribution is very close to those for a Gaussian distribution. The study also indicates that this luminosity loss due to the beam-beam interaction is very sensitive to the disruption parameter D_y. To illustrate this dependency, we plot in Fig. 3, the luminosity at resonance, *i.e.*, with solution given in Eqs. (12) and (13), as a function of D_y which relates to $k_0(= 2k)$. In our study, we vary N_b alone to vary D_y. Without the beam-beam instability considered in this paper, increasing the disruption parameter D_y will increase the luminosity $\mathcal{L}_0$ quadratically. On the other hand, the beam-beam instability introduces a large luminosity loss when D_y increases. This leads to the phenomena that beyond certain value of $D_{y,c}$ ($D_{y,c} \sim 20$ in our paper), increasing D_y will no longer efficiently increase the luminosity. To conclude, the luminosity loss due to beam-beam instability is substantial, and needs serious consideration in future ILC design.

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