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Measurement of γ and $2\beta + \gamma$

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We report on the initial measurements of the angle γ and the sum of angles $2\beta + \gamma$ of the Unitarity Triangle. When compared with indirect information on the value of γ from other measurements of CKM parameters, the measurement of these angles will provide a precise test of Standard Model predictions, as statistics increase. There are several methods for directly measuring γ and $2\beta+\gamma$. We report on the status of each of these techniques, and the resulting constraints on the values of these angles.

1 Introduction

The comparison of measurements of the angles and sides of the Unitarity Triangle provides a test of the Standard Model, in which CP-violation is solely due to a single complex phase in V, the Cabbibo-Kobayashi-Maskawa (CKM) quark mixing matrix. The angle $\gamma \equiv \arg[-V_{\rm cd}V^*_{\rm cb}/V_{\rm ud}V^*_{\rm ub}]$ is considered to be the most difficult to measure of the three Unitarity Triangle angles. The difficulty is due to the fact that the interference terms that provide the sensitivity to γ tend to be small, due to small branching fractions, lower reconstruction efficiencies than with typical charmonium or charmless B decays, and relevant magnitudes of interfering amplitudes that are far from equal.

Nevertheless, there exist several techniques for directly measuring γ and $2\beta + \gamma$. These techniques can be divided into three classes: those that use a time-independent CP asymmetry between color-allowed $B \to D^0K$ and color-suppressed $B \to \overline{D}{}^0K$ amplitudes to directly measure γ, which is the relative weak phase between these amplitudes; those that use a time-dependent asymmetry between favored and suppressed $B \to D\pi$ or $B \to D^0K^0$ amplitudes; and a third type of technique, which uses a combination of time-dependent and time-independent asymmetries and branching fractions in $B \to D^{(*)}_{(s)}D^{(*)}$ to solve for the value of γ.

2 Time-Independent Techniques

The time-independent techniques each use an interference between color-allowed $B \to D^0K$ and color-suppressed $B \to \overline{D}{}^0K$ amplitudes to constrain γ through a CP-violating asymmetry in time-integrated decay rates. The D^0K and $\overline{D}{}^0K$ amplitudes have a relative weak phase of γ, but one always needs two more pieces of experimental information to form a constraint: the

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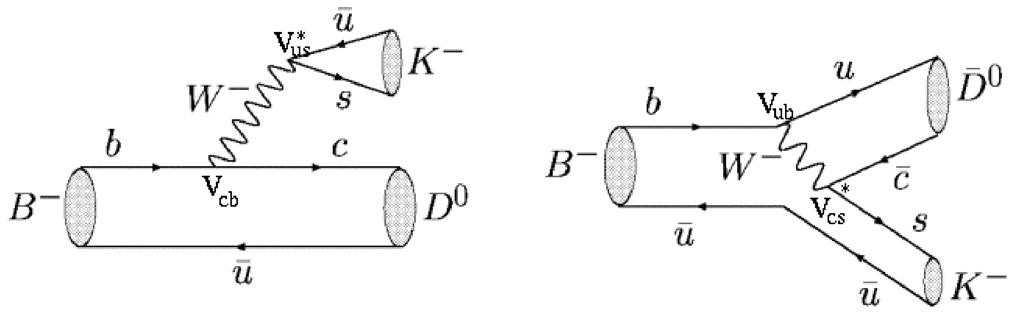

Figure 1: The $B \to D^0 K$ and $B \to \overline{D}^0 K$ amplitudes have a relative weak phase of γ.

relative magnitude of the amplitudes $r_B \equiv |\frac{A(b\to u)}{A(b\to c)}|$ and the strong phase difference between the two amplitudes δ_B. Naturally, the larger the interference, *i.e.* the closer r_B is to 1, the better the constraint will be on the value of γ for a given dataset.

The original technique for measuring γ, the Gronau-London-Wyler (GLW) method, uses an asymmetry between $B^- \to D^0 K^-$ and $B^+ \to \overline{D}^0 K^+$, where the D^0 ($\overline{D}^0$) decays to a *CP* eigenstate mode ($\pi^+\pi^-$, K^+K^-, $K^0_S\pi^0$, $K^0_S\phi$, or $K^0_S\omega$) [1]. There are 4 observables:

$$
\begin{aligned}
R_{CP\pm} &\equiv \frac{\Gamma(B^- \to D^0_{CP\pm}K^-) + \Gamma(B^+ \to D^0_{CP\pm}K^+)}{2\Gamma(B^- \to D^0_{\rm flav}K^-)} = 1 \pm 2r_B \cos\gamma\cos\delta_B + r_B^2 \\
A_{CP\pm} &\equiv \frac{\Gamma(B^- \to D^0_{CP\pm}K^-) - \Gamma(B^+ \to D^0_{CP\pm}K^+)}{\Gamma(B^- \to D^0_{CP\pm}K^-) + \Gamma(B^+ \to D^0_{CP\pm}K^+)} = \pm 2r_B \sin\gamma \sin\delta_B / R_{CP\pm}
\end{aligned}
$$

(where CP^+ refers to the *CP*-even final states $\pi^+\pi^-$ and K^+K^- and CP^- refers to the *CP*-odd final states $K^0_S\pi^0$, $K^0_S\phi$, and $K^0_S\omega$) thus allowing a solution of the 3 unknowns (r_B, δ_B, and γ), up to an 8-fold ambiguity in γ (when no external prior is taken for the value of δ_B).

The GLW method is theoretically clean, with nearly no hadronic uncertainty. However it is experimentally challenging, as the branching fractions to the relevant final states are small: the $B \to DK$ branching fractions are at the 10^{-4} level, the branching fractions of the D into *CP* eigenstate modes are of order 10^{-2}, and the overall detection efficiencies are around 25%. Thus, using a sample of 214×10^6 $B\overline{B}$ events, *BABAR*'s yield for the GLW final states is 93 ± 15 for $B \to D^0 K$ where the D^0 decays to the two *CP*-even final states and 76 ± 13 for D^0 decays to the *CP*-odd final state. *BABAR* also reconstructs $B \to D^0 K^*$ (with $K^* \to K^0_S\pi^-$), and obtains yields of 34.4 ± 6.9 events in the *CP*-even modes and 15.1 ± 5.8 events in the *CP*-odd modes of the D^0 in a sample of 227×10^6 $B\overline{B}$ events [2, 3].

Using the above event yields, the 4 GLW experimental observables are measured at *BABAR* to be $R_{CP^+} = 0.87 \pm 0.14 \pm 0.06$, $A_{CP^+} = 0.40 \pm 0.15 \pm 0.08$, $R_{CP^-} = 0.80 \pm 0.14 \pm 0.08$, $A_{CP^-} = 0.21 \pm 0.17 \pm 0.07$ for the $B \to D^0 K$ modes, and $R_{CP^+} = 1.77 \pm 0.37 \pm 0.12$, $A_{CP^+} = -0.09 \pm 0.20 \pm 0.06$, $R_{CP^-} = 0.76 \pm 0.29 \pm 0.06^{-0.04}_{-0.14}$, $A_{CP^-} = -0.33 \pm 0.34 \pm 0.10(+0.15 \pm 0.10) \cdot (A_{CP^-} - A_{CP^+})$ for the $B \to D^0 K^*$ modes, where the third uncertainty in the last two measurements reflects possible interference effects in final states with ϕ and ω [2, 3]. *BABAR*

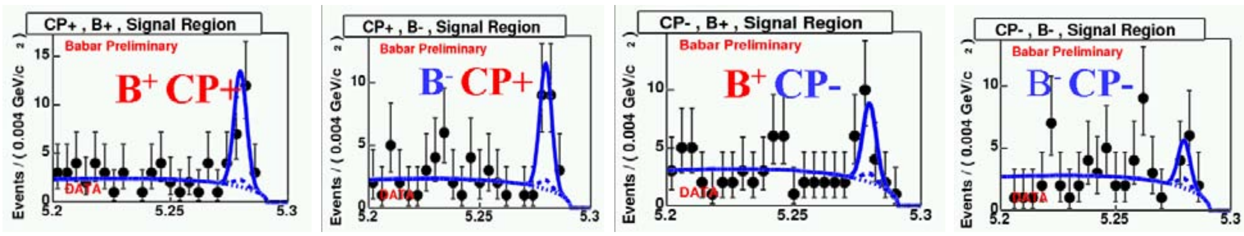

Figure 2: Yields at *BABAR* in a sample of 227×10^6 $B\overline{B}$ events for $B \to D^0 K^*$ (with $K^* \to K^0_S \pi^-$). The left two plots show B^+ and B^- yields to the *CP*-even D^0 decay modes $\pi^+\pi^-$ and K^+K^- (a total of 34.4 ± 6.9 events) and the right two show the yield to the *CP*-odd modes $K^0_S\pi^0$, $K^0_S\phi$, and $K^0_S\omega$ (15.1 ± 5.8 events) [3].

also measures $R_{CP^+} = 0.88 \pm 0.26^{+0.10}_{-0.08}$ and $A_{CP^+} = -0.02 \pm 0.24 \pm 0.05$ in $B \to D^{*0}K$ events, with $D^{*0} \to D^0\pi^0$, in a sample of 123×10^6 $B\overline{B}$ events [4].

Belle measures $R_{CP^+} = 0.98 \pm 0.18 \pm 0.10$, $A_{CP^+} = 0.07 \pm 0.14 \pm 0.06$, $R_{CP^-} = 1.29 \pm 0.16 \pm 0.08$, $A_{CP^-} = -0.11 \pm 0.14 \pm 0.05$ for the $B \to D^0 K$ modes, and $R_{CP^+} = 1.43 \pm 0.28 \pm 0.06$, $A_{CP^+} = -0.27 \pm 0.25 \pm 0.04$, $R_{CP^-} = 0.94 \pm 0.28 \pm 0.06$, $A_{CP^-} = 0.26 \pm 0.26 \pm 0.03$ for $B \to D^{*0}K$ ($D^{*0} \to D^0\pi^0$) modes, each in a sample of 250 fb^{-1} of data [5].

Using the $B \to D^0 K^*$ results, *BABAR* constrains the value of the theoretical parameter r_B^2 to be 0.23 ± 0.24. However, more statistics are needed to constrain γ from this method.

The Atwood-Dunietz-Soni (ADS) method also uses interference between color-allowed $B \to D^0 K$ and color-suppressed $B \to \overline{D}^0 K$ amplitudes, but instead of using *CP* eigenstate decays of the D^0, the decays $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^+\pi^-$ are used [6]. There are two observables:

$$
\begin{aligned}
R_{\text{ADS}} &\equiv \frac{\Gamma(B^- \to D^0(\to K^+\pi^-)K^-) + \Gamma(B^+ \to D^0(\to K^-\pi^+)K^+)}{\Gamma(B^- \to D^0(\to K^-\pi^+)K^-) + \Gamma(B^+ \to D^0(\to K^+\pi^-)K^+)} \\
&= r_D^2 + 2 r_B r_D \cos\gamma \cos(\delta_B + \delta_D) + r_B^2 \\
A_{\text{ADS}} &\equiv \frac{\Gamma(B^- \to D^0(\to K^+\pi^-)K^-) - \Gamma(B^+ \to D^0(\to K^-\pi^+)K^+)}{\Gamma(B^- \to D^0(\to K^+\pi^-)K^-) + \Gamma(B^+ \to D^0(\to K^-\pi^+)K^+)} \\
&= 2 r_B r_D \sin\gamma \sin(\delta_B + \delta_D)/R_{\text{ADS}}
\end{aligned}
$$

The value of $r_D \equiv \frac{|A(D^0 \to K^+\pi^-)|}{|A(D^0 \to K^-\pi^+)|}$ is constrained to the experimental value of 0.060 ± 0.003 [7]. The values of r_B and δ_B are equal to those from the GLW analysis described above. One is left with the two theoretical unknowns δ_D and γ, which can in principle be determined from the two experimental observables.

Similar to the GLW analysis, the ADS analysis is theoretically clean but suffers from highly suppressed decay rates into the relevant final states. Using a sample of 227×10^6 $B\overline{B}$ events, *BABAR* reconstructs $4.7^{+4.0}_{-3.2}$ events in the $B^- \to D^0 K^-$ ($D^0 \to K^+\pi^-$) channel, $-0.2^{+1.3}_{-0.8}$ events in the $B^- \to D^{*0}K^-$ ($D^{*0} \to D^0\pi^0, D^0 \to K^+\pi^-$) channel, and $1.2^{+2.1}_{-1.4}$ events in the $B^- \to D^{*0}K^-$ ($D^{*0} \to D^0\gamma, D^0 \to K^+\pi^-$) channel [8]. No significant signal is seen for any of these channels. Belle reconstructs 14.7 ± 7.6 events in the first of these channels, also not significant [9]. Using these values, *BABAR* constrains R_{ADS} from $B^- \to D^0 K^-$ channel to be less than 0.030 at 90% confidence level (c.l.). From this result, and allowing any value of δ_D and γ, one can

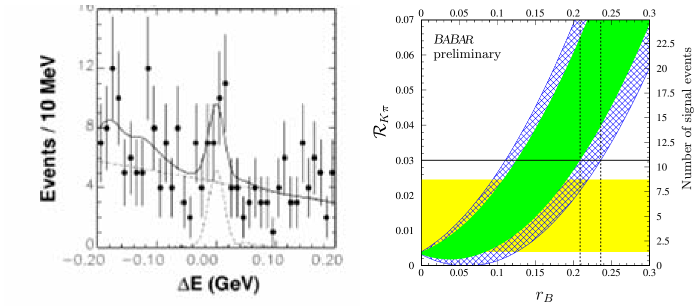

Figure 3: (Left) Yield at Belle for the ADS decay mode $B \to D^0(\to K\pi)K$ with opposite-sign kaons. Belle establishes a limit on this branching fraction of 7.6×10^{-7} at 90% c.l. [9] (Right) Constraints in the r_B-$R_{\rm ADS}$ plane from *BABAR*. The green region allows any value of δ_D and $48° < \gamma < 73°$, and the hatched region allows any value of γ. *BABAR* constrains $r_B < 0.23$ at 90% c.l. [8]

constrain r_B to be less than 0.23 at 90% c.l. as shown in Fig. 3 (right). Belle similarly constrains $R_{\rm ADS} < 0.047$ and $r_B < 0.28$, both at 90% c.l. However, similar to the GLW analysis, more statistics are needed to constrain γ from the ADS method.

A third method to constrain γ using interference between $B \to D^0K$ and $B \to \overline{D}^0K$ amplitudes is the Dalitz technique, which uses D^0 and $\overline{D}^0$ decays to the common final state $K_S\pi^+\pi^-$. By simultaneously fitting the Dalitz distributions of the $D^0 \to K_S\pi^+\pi^-$ decays from B^+ and B^- data, one can constrain the values of the three theoretical unknowns (r_B, δ_B, γ) [10].

One must first constrain the value of the strong phase δ_D and the ratio of magnitudes of interfering amplitudes r_D of the D^0 decay, parameters which are a function of position in the Dalitz plot. The sample of $B \to D^0K$ decays alone does not contain nearly enough statistics to constrain these functions. However, a sample of inclusive $D^0 \to K_S\pi^+\pi^-$ decays is of order 10^4 times larger, and provides sufficient statistics to constrain the δ_D and r_D variation. *BABAR* perfoms a fit to 16 different resonances, plus a non-resonant component, to the inclusive $D^0 \to K_S\pi^+\pi^-$ Dalitz distribution, and determines the amplitudes and relative phases of each, allowing the calculation of the δ_D and r_D variation. Belle performs a similar Dalitz fit. The distributions from *BABAR* and Belle are shown in Fig. 4 [11, 12].

Once δ_D and r_D are known, one can then observe the exclusive channels to constrain r_B, δ_B, and γ. Using a sample of 211×10^6 $B\overline{B}$ events, *BABAR* reconstructs 261 ± 19 events in the $B \to D^0(\to K_S\pi^+\pi^-)K$ channel. *BABAR* also reconstructs the decays $B \to D^{*0}(\to D^0x, D^0 \to K_S\pi^+\pi^-)K$, with $x = \pi^0$ or γ, and obtains 83 ± 11 and 40 ± 8 events in those channels respectively. Belle reconstructs 209 ± 16 and 58 ± 8 events in 250 fb^{-1} for the first two of these three channels (Belle does not presently reconstruct the $D^{*0} \to D^0\gamma$ mode).

Using a simultaneous fit to the B^+ and B^- Dalitz distributions, *BABAR* measures $\delta_B = (114 \pm 41 \pm 8 \pm 10)°$ and $\delta^*_B = (123 \pm 34 \pm 14 \pm 10)°$, where in both cases there is a $+(0°, 180°)$ ambiguity and the first error is statistical, the second is systematic, and the third is from uncertainty on the phase variation model. *BABAR* also finds $r^*_B = 0.155^{+0.070}_{-0.077} \pm 0.040 \pm 0.020$ and constrains r_B to be less than 0.19 at 90% c.l. The value of γ is constrained by *BABAR*, from a combined fit to the the D^0K and $D^{*0}K$ modes, to be $(70 \pm 26 \pm 10 \pm 10)°$.

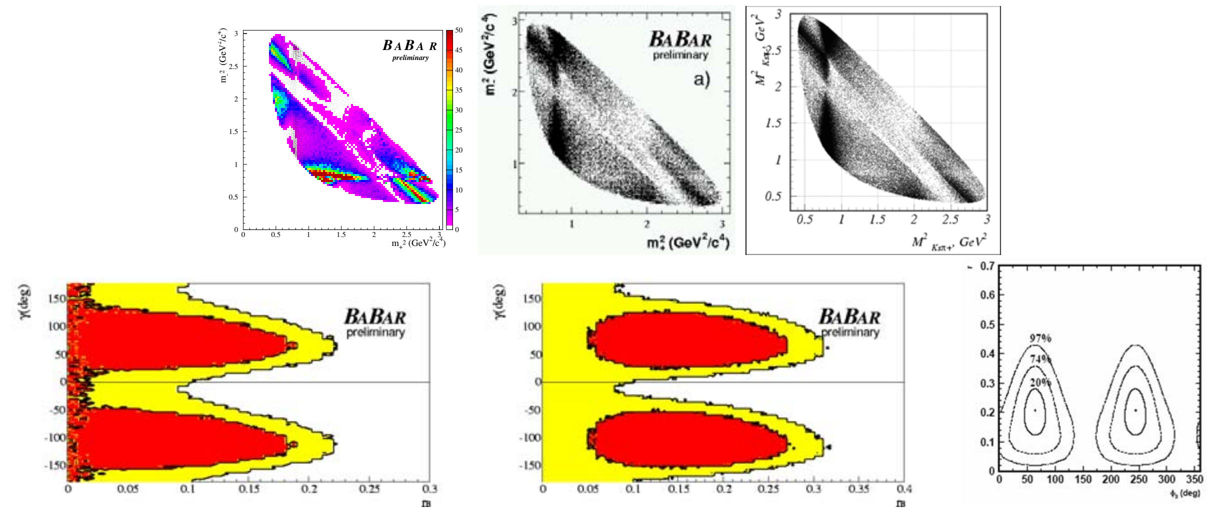

Figure 4: (Upper left) Sensitivity to γ for $B \to D^0 K$ events with $D^0 \to K_S \pi^+ \pi^-$ as a function of position within the Dalitz plot. (Upper middle and right) Dalitz distributions obtained using an inclusive sample of $D^0 \to K_S \pi^+ \pi^-$ events from *BABAR* and Belle respectively [11, 12]. Resulting constraints on the r_B-γ plane at 68% (red) and 90% (yellow) confidence levels from (lower left) $B \to D^0 K$ and (lower middle) $B \to D^{*0} K$. (Lower right) Constraints on the γ-r_B plane from Belle at 20%, 74%, and 90% confidence levels.

As noted above, the uncertainty on the value of γ for each of the time-independent techniques strongly depends on the value of r_B; a larger value of this parameter implies a larger $D^0 K$–$\overline{D}^0 K$ interference term, thus a smaller uncertainty on the measured value of γ. In each of the three analyses above, Belle reports a larger central value of r_B than *BABAR*. In the case of the GLW and Dalitz analyses, Belle's central values both are greater than the 90% c.l. upper limits on r_B placed by *BABAR*. While none of the inconsistencies are, by themselves, statistically significant, it is unclear why this trend has so far occured in each of the above analyses.

For the Dalitz analysis, Belle reports $r_B = 0.21 \pm 0.08 \pm 0.03 \pm 0.04$, $\delta_B = (64 \pm 19 \pm 13 \pm 11)^\circ$, and $\gamma = (64 \pm 19 \pm 13 \pm 11)^\circ$. The smaller uncertainty on γ, as compared with the *BABAR* analysis, is due to the apparent larger central value of r_B that Belle reconstructs. (This value has in fact declined from Belle's previous measurement of $r_B = 0.26^{+0.10}_{-0.14} \pm 0.03 \pm 0.04$, using an earlier sample of 140 fb^{-1} of data [13]. The declining value of r_B appears to explain why, after increasing their data sample by over a factor of two, Belle's uncertainty on γ has actually increased slightly.)

3 Measuring $\sin(2\beta + \gamma)$ Using Time-Dependent Asymmetries

Time-dependent asymmetries in $B^0 \to D^{(*)}\pi$, $D^{(*)}\rho$, and $D^{(*)0}(\overline{D}^{(*)0})K^{*0}$ can provide information on the value of $\sin(2\beta + \gamma)$ — and thus the value of γ, since β is so well-constrained.

The $B^0 \to D^{(*)}\pi$ and $D^{(*)}\rho$ methods use an interference between the usual Cabibbo-favored $b \to c$ channel and the doubly-Cabibbo-suppressed $b \to u$ channel [14]. These two amplitudes have a relative weak phase of γ, and a weak phase of 2β is provided by the $B^0\overline{B}^0$ mixing. As the amplitude for the $b \to u$ channel is very small compared with $b \to c$, the time-dependent asymmetry is a small effect, of order λ^2.

There are two experimental methods for reconstructing B^0 ($\overline{B}^0$) $\to D^{(*)}\pi$ and $D^{(*)}\rho$ decays to determine γ. One can perform exclusive reconstruction, where one fully reconstructs all the final state particles for the low-multiplicity hadronic $D^{(*)}$ decay modes. This method has a very high signal purity (typically near 90%), but one cannot reconstruct the majority of the $D^{(*)}$ decays, *i.e.* those into semi-leptonic or high-multiplicity hadronic decay modes. In order to obtain a higher efficiency, one can perform partial reconstruction of $D^*\pi$ and $D^*\rho$, by reconstructing only the slow pion, and not the D^0, from the $D^* \to D^0\pi$ decay. Using only the slow pion provides sufficient kinematic constraints to reconstruct this decay. While this technique provides an efiiciency approximately 5 times higher, it does suffer from far higher backgrounds.

The experimental observables are the coefficients of the sin and $\cos(\Delta M t)$ asymmetry terms in the time-dependent asymmetries of B^0 ($\overline{B}^0$) $\to D^{(*)}\pi$ and $D^{(*)}\rho$. The coefficient for the sin term is equal to $2r\sin(2\beta+\gamma)\cos\delta$, where r is the ratio of Cabibbo-favord and doubly-Cabibbo-suppressed amplitudes to the final state, and δ is the strong phase between those amplitudes. The coefficient for the cos term is equal to $2r\cos(2\beta+\gamma)\sin\delta$. *BABAR* obtains the results:

$$\begin{aligned}
2r\sin(2\beta+\gamma)\cos\delta &= -0.032 \pm 0.031 \pm 0.020 \\
2r\cos(2\beta+\gamma)\sin\delta &= -0.059 \pm 0.033 \pm 0.033 \\
2r_*\sin(2\beta+\gamma)\cos\delta_* &= -0.049 \pm 0.031 \pm 0.020 \\
2r_*\cos(2\beta+\gamma)\sin\delta_* &= 0.044 \pm 0.054 \pm 0.033
\end{aligned}$$

using exclusive reconstruction on a sample of 110×10^6 $B\overline{B}$ events [15], where the first two values are from the $D\pi$ channel and the last two are from $D^*\pi$, and

$$\begin{aligned}
2r_*\sin(2\beta+\gamma)\cos\delta_* &= -0.041 \pm 0.016 \pm 0.010 \\
2r_*\cos(2\beta+\gamma)\sin\delta_* &= -0.015 \pm 0.036 \pm 0.019
\end{aligned}$$

using partial reconstruction on a sample of 178×10^6 $B\overline{B}$ events [16]. Belle obtains

$$\begin{aligned}
2r\sin(2\beta+\gamma)\cos\delta &= -0.062 \pm 0.037 \pm 0.018 \\
2r\cos(2\beta+\gamma)\sin\delta &= -0.025 \pm 0.037 \pm 0.018 \\
2r_*\sin(2\beta+\gamma)\cos\delta_* &= 0.060 \pm 0.040 \pm 0.017 \\
2r_*\cos(2\beta+\gamma)\sin\delta_* &= 0.049 \pm 0.040 \pm 0.019
\end{aligned}$$

using exclusive reconstruction [17] and

$$\begin{aligned}
2r_*\sin(2\beta+\gamma)\cos\delta_* &= -0.031 \pm 0.028 \pm 0.018 \\
2r_*\cos(2\beta+\gamma)\sin\delta_* &= -0.004 \pm 0.028 \pm 0.018
\end{aligned}$$

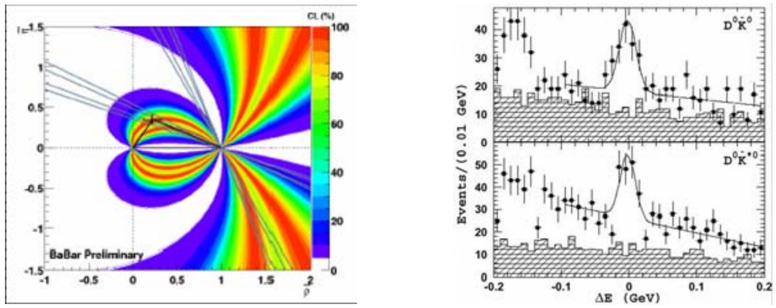

Figure 5: (Left) Constraints on the $\overline{\rho}-\overline{\eta}$ plane from *BABAR* $B^0 \to D^{(*)}\pi$ partial reconstruction results [16]. (Right) Belle yields for the decay modes $\overline{B}^0 \to D^0\overline{K}^0$ and $\overline{B}^0 \to D^0\overline{K}^{*0}$, where they obtain branching fractions of $(3.72 \pm 0.65 \pm 0.37) \times 10^{-5}$ and $(3.08 \pm 0.56 \pm 0.31) \times 10^{-5}$ respectively [21].

using partial reconstruction [18], both on a sample of 152×10^6 $B\overline{B}$ events. *BABAR* obtains constraints on the value of $|\sin(2\beta+\gamma)|$ from the partial reconstruction method: $|\sin(2\beta+\gamma)| > 0.75$ at 68% c.l. and > 0.58 at 90% c.l., resulting in constraints on the $\overline{\rho}-\overline{\eta}$ plane as shown in Fig. 5 left. Belle does not claim any constraints on the value of $|\sin(2\beta+\gamma)|$ however, assuming the strong phase $\delta = 0$ or π, Belle obtains $2r_* \sin(2\beta+\gamma) = 0.031 \pm 0.028 \pm 0.013$.

The relative weak phase of the $B^0 \to D^{(*)0}K^{*0}$ and $\overline{B}^0 \to D^{(*)0}\overline{K}^{*0}$ is also γ. As these final states are non *CP*-eigenstates, there is also a strong phase between the two decays, that can be solved for by additionally measuring the *CP* asymmetry in the decays $B^0 \to \overline{D}^{(*)0}K^{*0}$ and $\overline{B}^0 \to \overline{D}^{(*)0}\overline{K}^{*0}$ [19]. The sensitivity to $\sin(2\beta+\gamma)$ from these decays is given by the value of $r \equiv \frac{|A(\overline{B}^0 \to \overline{D}^{(*)0}\overline{K}^{*0})|}{|A(\overline{B}^0 \to D^{(*)0}\overline{K}^{*0})|}$. *BABAR* obtains the following branching fractions [20]:

$$\begin{aligned}
\mathcal{B}(\overline{B}^0 \to D^0\overline{K}^0) &= (6.2 \pm 1.2 \pm 0.4) \times 10^{-5} \\
\mathcal{B}(\overline{B}^0 \to D^{*0}\overline{K}^0) &= (4.5 \pm 1.9 \pm 0.5) \times 10^{-5} \\
\mathcal{B}(\overline{B}^0 \to D^0\overline{K}^{*0}) &= (6.2 \pm 1.4 \pm 0.6) \times 10^{-5}
\end{aligned}$$

but obtains just a limit on the numerator of r:

$$\mathcal{B}(\overline{B}^0 \to \overline{D}^0\overline{K}^{*0}) < 4.1 \times 10^{-5} \text{ at 90\% c.l.}$$

Similarly, Belle obtains [21]:

$$\begin{aligned}
\mathcal{B}(\overline{B}^0 \to D^0\overline{K}^0) &= (3.72 \pm 0.65 \pm 0.37) \times 10^{-5} \\
\mathcal{B}(\overline{B}^0 \to D^{*0}\overline{K}^0) &= (3.18^{+1.25}_{-1.12} \pm 0.32) \times 10^{-5} \\
\mathcal{B}(\overline{B}^0 \to D^0\overline{K}^{*0}) &= (3.08 \pm 0.56 \pm 0.31) \times 10^{-5} \\
\mathcal{B}(\overline{B}^0 \to D^{*0}\overline{K}^{*0}) &< 4.8 \times 10^{-5}; \text{ at 90\% c.l.} \\
\mathcal{B}(\overline{B}^0 \to \overline{D}^0\overline{K}^{*0}) &< 0.4 \times 10^{-5} \text{ at 90\% c.l.} \\
\mathcal{B}(\overline{B}^0 \to \overline{D}^{*0}\overline{K}^{*0}) &< 1.9 \times 10^{-5} \text{ at 90\% c.l.}
\end{aligned}$$

No constraints on $\sin(2\beta+\gamma)$ can be obtained so far from $B^0 \to D^{(*)0}(\overline{D}^{(*)0})K^{*0}$.

4 Using $B \to D_{(s)}^{(*)} D^{(*)}$ Decays to Measure γ

One can combine information from $B \to D^{(*)}\overline{D}^{(*)}$ and $B \to D_S^{(*)}\overline{D}^{(*)}$ branching fractions, along with *CP* asymmetry measurements in $B \to D^{(*)}\overline{D}^{(*)}$, to obtain a measurement of the Unitarity Triangle angle γ [22, 23]. The weak phase γ of the $u-$ and $c-$penguin amplitudes of the $B \to D^{(*)}\overline{D}^{(*)}$ decays may be extracted by using an SU(3) relation between the $B \to D^{(*)}\overline{D}^{(*)}$ and $B \to D_S^{(*)}\overline{D}^{(*)}$ decays. In this technique, the breaking of SU(3) is parametrized via the ratios of decay constants $f_{D_s^{(*)}}/f_{D^{(*)}}$, which are quantities measured in lattice QCD [24].

One obtains the relation (for $B^0 \to D^+D^-$ and individual helicity states of $B^0 \to D^{*+}D^{*-}$):

$$\mathcal{A}_{ct}^2 = \frac{a_R \cos(2\beta + 2\gamma) - a_{\text{indir}} \sin(2\beta + 2\gamma) - B}{\cos 2\gamma - 1} \tag{1}$$

where

$$\begin{aligned} B &\equiv \frac{1}{2}(|A^D|^2 + |\overline{A}^D|^2) = \mathcal{A}_{ct}^2 + \mathcal{A}_{ut}^2 + 2\mathcal{A}_{ct}\mathcal{A}_{ut}\cos\delta\cos\gamma, \\ a_{\text{dir}} &\equiv \frac{1}{2}(|A^D|^2 - |\overline{A}^D|^2) = -2\mathcal{A}_{ct}\mathcal{A}_{ut}\sin\delta\sin\gamma, \\ a_{\text{indir}} &\equiv \Im(e^{-2i\beta}A^{D*}\overline{A}^D) = -\mathcal{A}_{ct}^2\sin 2\beta - \\ & \quad 2\mathcal{A}_{ct}\mathcal{A}_{ut}\cos\delta\sin(2\beta+\gamma) - \mathcal{A}_{ut}^2\sin(2\beta+2\gamma), \end{aligned} \tag{2}$$

and

$$a_R^2 \equiv B^2 - a_{\text{dir}}^2 - a_{\text{indir}}^2. \tag{3}$$

B represents the branching fraction to a given $B \to D^{(*)}\overline{D}^{(*)}$ decay and a_{dir} and a_{indir} represent the corresponding direct and indirect *CP* asymmetries respectively. The phases β and γ are the angles of the Unitarity Triangle and δ is a strong phase. $\mathcal{A}_{ct} \equiv |(T+E+P_c-P_t-P_{EW}^C)V_{cb}^*V_{cd}|$ and $\mathcal{A}_{ut} \equiv |(P_u - P_t - P_{EW}^C)V_{ub}^*V_{ud}|$ are the norms of the combined $B \to D^{(*)}\overline{D}^{(*)}$ decay amplitudes containing $V_{cb}^*V_{cd}$ and $V_{ub}^*V_{ud}$ terms respectively, and the T, P, and E terms are the tree, penguin, and exchange amplitude components respectively [23]. As the modes $B^0 \to D^{*\pm}D^{\mp}$ are not *CP* eigenstates, a slightly more complicated formalism is needed for these modes [22].

Using these relations, there are 5 variables for each $B \to D^{(*)}\overline{D}^{(*)}$ decay for which to solve: $\mathcal{A}_{ct}$, $\mathcal{A}_{ut}$, δ, β, and γ. The branching fraction and the direct and indirect *CP* asymmetries of the $B \to D^{(*)}\overline{D}^{(*)}$ decay provide three measred quantities. Two further quantities are needed. The angle β can be obtained from the measurements of $\sin 2\beta$ using charmonium "golden modes" at the B-factories [25]. The other measurement that can be used is the branching fraction of the corresponding $B \to D_S^{(*)}\overline{D}^{(*)}$ decay.

If SU(3) were a perfect symmetry between $B \to D^{(*)}\overline{D}^{(*)}$ and $B \to D_S^{(*)}\overline{D}^{(*)}$ decays, then the norm of the $B \to D_S^{(*)}\overline{D}^{(*)}$ amplitude, denoted $\mathcal{A}'_{ct}$, would equal $\mathcal{A}_{ct}/\sin\theta_c$, where θ_c is the Cabibbo angle. However, SU(3)-breaking effects can spoil this relation. The SU(3)-breaking can be parametrized by the ratio of decay constants $f_{D_s^{(*)}}/f_{D^{(*)}}$, such that $\mathcal{A}'_{ct} = f_{D_s^{(*)}}/f_{D^{(*)}} \times \mathcal{A}_{ct}/\sin\theta_c$ (where the parentheses around the asterisks correspond to the $B \to D^{(*)}\overline{D}^{(*)}$ and $B \to D_S^{(*)}\overline{D}^{(*)}$ decays that are used). The theoretical uncertainty of this relation is determined to be 10% [23].

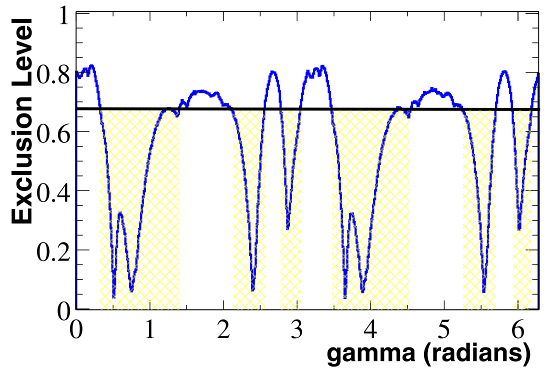

Figure 6: Constraints on the value of γ from $B \to D_{(s)}^{(*)} D^{(*)}$ decays.

Using these relations, together with measurements of $B \to D^{(*)}\overline{D}^{(*)}$ and $B \to D_S^{(*)}\overline{D}^{(*)}$ branching fractions and CP asymmetries from *BABAR* and Belle, constrains γ to lie in one of the ranges $[19.4^\circ - 80.6^\circ]$, $[120^\circ - 147^\circ]$, or $[160^\circ - 174^\circ]$ at 68% confidence level. There is an additional ($+0^\circ$ or 180°) phase ambiguity for each range. The constraints disappear for larger confidence levels, however the *BABAR* measurements used for these constraints were obtained on on a sample of only 88×10^6 $B\overline{B}$ events and thus can be significantly improved.

5 Conclusions

Although the angle γ is the most difficult to measure of the Unitarity Triangle angles at the B-Factories, surprising progress has been made in constraining it over the past few years. We now have initial measurements of the values of both γ and $\sin(2\beta+\gamma)$ from multiple channels, and have progressed toward precision measurements of this angle, which appear poised to have errors below $\pm 10^\circ$ prior to physics at the LHC. These precision measurements of γ are a critical test for the consistency of the Standard Model mixing sector.

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References

[1] M. Gronau and D. London, Phys. Lett. **B 253**, 483 (1991); M. Gronau and D. Wyler, Phys. Lett. **B 265**, 172 (1991).

[2] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408082, submitted to ICHEP '04.

[3] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408069, submitted to ICHEP '04.

[4] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408060, submitted to ICHEP '04.

[5] Belle Collaboration, K. Abe *et al.*, Belle-CONF-0443, submitted to ICHEP '04.

[6] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997); Phys. Rev. **D 63**, 036005.

[7] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **91**, 171801 (2003).

[8] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408028, submitted to ICHEP '04.

[9] Belle Collaboration, K. Abe *et al.*, hep-ex/0408129, submitted to ICHEP '04.

[10] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, Phys. Rev. **D 68**, 054018 (2003).

[11] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408088, submitted to ICHEP '04.

[12] Belle Collaboration, K. Abe *et al.*, hep-ex/0411049, submitted to FPCP '04.

[13] Belle Collaboration, A. Poluektov *et al.*, hep-ex/0406067.

[14] I. Dunietz, Phys. Lett. **B 427**, 179 (1998).

[15] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408059, submitted to ICHEP '04.

[16] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408038, submitted to ICHEP '04.

[17] Belle Collaboration, T.R. Sarangi *et al.*, Phys. Rev. Lett. **93**, 031802 (2004); erratum: Ibid **93**, 059901 (2004).

[18] Belle Collaboration, K. Abe *et al.*, hep-ex/0408106, submitted to ICHEP '04.

[19] B. Kayser and D. London, Phys. Rev. **D 61**, 116013 (2000); A.I. Sanda, hep-ph/0108031.

[20] *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/0408052, submitted to ICHEP '04.

[21] Belle Collaboration, P. Krokovny *et al.*, Phys. Rev. Lett. **90**, 141802 (2003).

[22] J. Albert, A. Datta, and D. London, hep-ph/0410015 (2004), accepted to Phys. Lett. **B**.

[23] A. Datta and D. London, Phys. Lett. **B 584**, 81 (2004).

[24] J. Simone, invited talk at *Lattice 2004*, Fermilab, June 2004: http://lqcd.fnal.gov/lattice04/presentations/paper252.pdf

[25] Heavy Flavor Averaging Group, world charmonium avg., http://www.slac.stanford.edu/xorg/hfag/triangle/summer2004/index.shtml.