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CMB Fluctuation Amplitude from Dark Energy Partitions* †

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It is assumed that the dark energy observed today is frozen as a result of a phase transition involving the source of that energy. Postulating that the dark energy de-coherence which results from this phase transition drives statistical variations in the energy density specifies a class of cosmological models in which the cosmic microwave background (CMB) fluctuation amplitude at last scattering is approximately 10^{-5}.

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In a previous paper (reference [1], equation 14), it has been shown that the fluctuation amplitude of the Cosmic Microwave Background (CMB) can be theoretically estimated to be of the order 10^{-5}, in agreement with observational evidence, using a minimal set of assumptions. Although the scale parameter of the cosmological expansion at the time of dark energy de-coherence enters the calculation, the final result depends only on the normalized dark energy density Ω_Λ, the red shift at last scattering z_{LS}, and the red shift at the time when the energy density of the non-relativistic pressure-less matter observed today was equal to the radiation energy density z_{eq}. The observation that the final result was independent of any specifics at the time of de-coherence suggested to the authors of this paper that this result is more general than

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the mechanism suggested in reference [1]. This paper examines the generality of the previous result, and presents a plausible justification for the size of the fluctuations at the time of de-coherence. A tentative conclusion is that any *phenomenological* theory that accepts the "cosmological constant" Λ, or the dark energy density $\rho_\Lambda = \frac{\Lambda}{8\pi G_N}$, or the deSitter scale radius $R_\Lambda = \sqrt{\frac{3}{\Lambda}}$, as a reasonable way to fit the observational data over the relevant range of red shifts, and which also assumes some sort of phase transition that decouples the residual dark energy from the subsequent dynamics of the energy density, will fall within a class of theories all of which fit the magnitude of the observed fluctuations.

A likely candidate for the dark energy is some form of vacuum energy. Physical systems for which vacuum energy density directly manifests include those that exhibit the Casimir effect[2]. Lifshitz and his collaborators[3] demonstrated that the Casimir force can be thought of as the statistical superposition of the van der Waals attractions between individual molecules that make up the attracting media resulting from the zero-point motions of the sources (independent of the couplings involved). Since these motions are inherently a quantum effect, one expects these systems to exhibit the space-like correlations consistent with a quantum phenomenon.

A weakly interacting sea of the quantum fluctuations due to zero point motions should exhibit statistical variations in this "dark energy" density. At de-coherence, one should be able to use simple counting arguments to quantify these variations. If the zero-point motions of the sources have a statistical weight $\Omega(E_A)$ associated with a partition A having energy E_A while holding total energy fixed, then the probability of such a partitioning is given by

$$P(E_A) \;=\; \frac{\Omega(E_A)}{\Omega_{tot}} \;=\; \frac{\Omega_A(E_A)\,\Omega_{\bar{A}}(E_{tot}-E_A)}{\Omega_{tot}}, \tag{1}$$

where $\bar{A}$ represents everything external to the A partition. Requiring that the most likely configuration of energy partitions results when (the log of) this probability is maximized, this distribution gives a uniform dark energy distribution if

$$\frac{1}{E_\Lambda^A} \;\equiv\; \frac{d}{dE_A} log\Omega(E_A) \qquad , \qquad E_\Lambda^A = E_\Lambda^{\bar{A}} \equiv E_\Lambda. \tag{2}$$

Here E_Λ is an intensive energy (chemical potential) associated with the statistical reservoir and boundary conditions. This result is analogous to the zeroth law of thermodynamics. Similarly, using arguments analogous to those used to establish the second law of thermodynamics, $log\Omega(E)$ is expected to be a non-decreasing function for previously isolated systems placed in mutual contact.

If one next examines a "small" partition A for which the reservoir $\bar{A}$ has energy $E_{tot} - E_A$, one can examine the (log of the) lowest order fluctuations from uniformity for the reservoir to show that

$$\Omega_{\bar{A}}(E_{tot} - E_A) \;\cong\; \Omega_{\bar{A}}(E_{tot})\, e^{-E/E_\Lambda}, \tag{3}$$

thus defining a probability distribution of the form

$$P(E) \;=\; \frac{e^{-E/E_\Lambda}}{\sum_{E'} e^{-E'/E_\Lambda}}. \tag{4}$$

For such an ensemble, the fluctuations-dissipation theorem gives

$$< (\delta E)^2 > = \; E_\Lambda^2 \frac{d}{dE_\Lambda} < E > . \tag{5}$$

A typical equation of state will connect the extensive variable $< E >$ to another extensive variable that counts the available degrees of freedom N_{DoF}. On dimensional grounds, the terms in a typical equation of state which depend on E_Λ should take the general form $< E >= N_{DoF} \frac{E_\Lambda^a}{\epsilon^{a-1}}$, where ϵ is a constant with dimensions of energy. The expected fluctuations are then given by

$$\frac{< (\delta E)^2 >}{< E >^2} \;=\; \frac{a}{N_{DoF}} \left(\frac{\epsilon}{E_\Lambda} \right)^{a-1} \;=\; a \, \frac{E_\Lambda}{< E >} \,. \tag{6}$$

As seen from the second form in equation 6, the dimensionless fluctuations are of the order of $\frac{1}{N_{DoF}}$, i.e. inversely proportional to a dimensionless extensive parameter relative to the scale of the de-coherence. In terms of the densities, one can directly write $\frac{<(\delta E)^2>}{<E>^2} \;=\; \frac{<(\delta \rho)^2>}{\rho^2} \sim \frac{\rho_\Lambda}{\rho}$.

Therefore, the energy available for fluctuations in the two point correlation function is expected to be given by the cosmological dark energy, in a manner similar to the way that background thermal energy $k_B T$ drives the fluctuations of thermal systems. This means that the amplitude of relative fluctuations $\delta\rho/\rho$ is expected to be of the order

$$\Delta_{PT} \;\equiv\; \left(\frac{\rho_\Lambda}{\rho_{PT}} \right)^{1/2} \tag{7}$$

where ρ_{PT} is the cosmological energy density at the time of the phase transition that decouples the dark energy. The scale dependence of this form will be explored in the remainder of this letter.

Next, such a phase transition which occurs during the epoch of radiation domination will be considered. Using the densities at radiation-matter (dust) equality $\rho_M(z_{eq}) = \rho_{rad}(z_{eq})$, one can extrapolate back to the phase transition period to determine the redshift at that time. The (non-relativistic) plasma is expected to scale using $\rho_M(z) = \rho_{Mo}(1+z)^3$ until it is negligible, whereas, the radiation scales during the early expansion using $\rho_{rad}(z) = \rho_{PT}\left(\frac{1+z}{1+z_{PT}}\right)^4$. Ignoring threshold effects (which give small corrections near particle thresholds while they are non-relativistic), this gives

$$1 + z_{PT} = \left[\frac{\rho_{PT}}{\rho_{Mo}}(1+z_{eq})\right]^{\frac{1}{4}}. \tag{8}$$

Here, Ω_{Mo} is the present normalized mass density.

For adiabatic perturbations (those that fractionally perturb the number densities of photons and matter equally), the matter density fluctuations grow according to[4]

$$\Delta = \begin{cases} \Delta_{PT}\left(\frac{R(t)}{R_{PT}}\right)^2 & radiation - dominated \\ \Delta_{eq}\left(\frac{R(t)}{R_{eq}}\right) & matter - dominated \end{cases} \tag{9}$$

This allows an accurate estimation for the scale of fluctuations at last scattering in terms of those during de-coherence given by

$$\Delta_{LS} = \left(\frac{R_{LS}}{R_{eq}}\right)\left(\frac{R_{eq}}{R_{PT}}\right)^2 \Delta_{PT} = \frac{(1+z_{PT})^2}{(1+z_{eq})(1+z_{LS})}\Delta_{PT}. \tag{10}$$

Using equations 8, 10, and 7, this amplitude at last scattering is given by

$$\Delta_{LS} = \frac{(1+z_{PT})^2}{(1+z_{eq})(1+z_{LS})}\left(\frac{\rho_\Lambda}{\rho_{PT}}\right)^{1/2} \cong \frac{1}{1+z_{LS}}\sqrt{\frac{\Omega_{\Lambda o}}{(1-\Omega_{\Lambda o})(1+z_{eq})}} \cong 2.6\times 10^{-5}, \tag{11}$$

where a spatially flat cosmology and radiation domination has been assumed. The values taken for the phenomenological parameters are given by $\Omega_{\Lambda o} \cong 0.73$, $z_{eq} \cong 3400$, and $z_{LS} \cong 1100$. This estimate for a transition during the radiation dominated regime is independent of the density during the phase transition ρ_{PT}, and is of the order observed for the fluctuations in the CMB (see [4] section 23.2 page 221). It is also in line with those argued by other authors[5] and papers[6]. Fluctuations in the CMB at last scattering of this order are consistent with the currently observed clustering of galaxies.

If the phase transition were to occur during the epoch of pressure-less matter domination, $z_{eq} > z_{PT} > z_{LS}$, the fluctuation amplitude will be seen to demonstrate weak dependence on the time of the phase transition. The acoustic wave has coherent phase information that is transmitted to the CMB at last scattering. There must have been a significant enough passage of time from the creation of the acoustic wave to the time of last scattering such that peaks and troughs of the various modes should be present at $\delta t > \frac{\lambda}{v_s}$, where λ is the distance scale of the longest wavelength (sound horizon), and $v_s \sim c/\sqrt{3}$ is the speed of the acoustic wave. Generally, if the phase transition occurs while the energy density is dominated by dark matter/plasma, then the amplitude satisfies

$$\begin{aligned} \sqrt{\frac{\rho_\Lambda}{\rho_{PT}}} &= \sqrt{\frac{\Omega_{\Lambda o}}{\Omega_{rad\,o}(1+z_{PT})^4+\Omega_{Mo}(1+z_{PT})^3+\Omega_{\Lambda o}}} \\ &= \sqrt{\frac{\Omega_{\Lambda o}}{(1-\Omega_{\Lambda o})(1+z_{PT})^3\left(\frac{1+z_{eq}}{2+z_{eq}}\right)\left(1+\frac{1+z_{PT}}{1+z_{eq}}\right)+\Omega_{\Lambda o}}}\,. \end{aligned} \tag{12}$$

This gives an amplitude at last scattering of the order

$$\Delta_{LS} \cong \left(\frac{1+z_{PT}}{1+z_{LS}}\right)\sqrt{\frac{\Omega_{\Lambda o}}{(1-\Omega_{\Lambda o})(1+z_{PT})^3\left(1+\frac{1+z_{PT}}{1+z_{eq}}\right)}}\,, \tag{13}$$

which varies from 2×10^{-5} if the phase transition occurs at radiation dust equality, to 4×10^{-5} if it occurs at last scattering.

As an example of a phase transition such as has been discussed, consider cold dark bosonic matter made up of particles of mass m. For non-relativistic bosonic dark matter, the relationship between number density and critical density for a free bose gas is given by

$$\frac{N}{V} = \frac{\zeta(\frac{3}{2})\Gamma(\frac{3}{2})}{(2\pi)^2\hbar^3}\left(2mk_BT_{crit}\right)^{3/2}. \tag{14}$$

Since the dynamics is assumed non-relativistic, $\rho_m \cong \frac{N}{V}mc^2$, giving the following requirement for a macroscopic quantum system made up of bose condensed cold dark matter:

$$\left(mc^2\right)^{5/2} \cong \frac{\rho_m}{\left(2k_B T_{crit}\right)^{3/2}} \frac{(2\pi)^2(\hbar c)^3}{\zeta(\frac{3}{2})\Gamma(\frac{3}{2})}. \tag{15}$$

In order for the macroscopic space-like quantum coherent state to persist, the ambient temperature must be less than the critical temperature. If the phase transition occurs while the dark matter is cold ($m > k_B T_{PT}$), its density can be assumed to depend on the redshift by $\rho_m = \rho_{mo}(1+z)^3$. The temperature of the photon gas is expected to likewise scale with the redshift when appropriate pair creation threshold affects are properly incorporated in the manner $T_\gamma(z) \approx T_{\gamma o}(1+z)(g(0)/g(z))^{1/4}$, where $g(z)$ counts number of low mass thermal degrees of freedom available at redshift z. Substitution into equation 15 gives

$$\left(mc^2\right)^{5/2} < (1+z)^{3/2} \left(\frac{g(z)}{g(0)}\right)^{3/8} \frac{\rho_{mo}}{\left(2k_B T_{\gamma o}\right)^{3/2}} \frac{(2\pi)^2\hbar^3}{\zeta(\frac{3}{2})\Gamma(\frac{3}{2})}. \tag{16}$$

Thus, the upper limit on a condensate mass roughly satisfies

$$mc^2 < (1+z)^{3/5} \left(\frac{g(z)}{g(0)}\right)^{3/20} \times (1.2 \times 10^{-11} GeV). \tag{17}$$

If the transition occurs as late as last scattering, this mass must be as low as $0.8eV$. However, during earlier epochs the mass of the condensate particles can be considerably larger.

Thus, it has been shown that if a general phase transition which freezes the cosmological effects of the dark energy (which thereafter can be represented as a cosmological constant) occurs sufficiently prior to last scattering, statistical fluctuations driven by the dark energy produce density perturbations of a magnitude that will evolve to be of the order 3×10^{-5} at last scattering. It is particularly profound that this result is independent of the particulars of the mechanism of dark energy de-coherence.

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