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Warped Phenomenology of Higher-Derivative Gravity * †

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Abstract

We examine the phenomenological implications at colliders for the existence of higher-derivative gravity terms as extensions to the Randall-Sundrum model. Such terms are expected to arise on rather general grounds, *e.g.*, from string theory. In 5-d, if we demand that the theory be unitary and ghost free, these new contributions to the bulk action are uniquely of the Gauss-Bonnet form. We demonstrate that the usual expectations for the production cross section and detailed properties of graviton Kaluza-Klein resonances and TeV-scale black holes can be substantially altered by existence of these additional contributions. It is shown that measurements at future colliders will be highly sensitive to the presence of such terms.

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1 Introduction

The Randall-Sundrum(RS) model[1] provides a geometric solution to the hierarchy problem through an exponential warp factor whose magnitude is controlled by the separation, πr_c, of two 3-branes embedded in 5-d Anti-deSitter space, AdS_5. It has been shown that this interbrane distance can be naturally stabilized at a value necessary to produced the experimentally observed ratio of the weak and Planck scales[2]. In its simplest form, SM matter in the RS model is confined to one of the 3-branes while gravity is allowed to propagate in the bulk. A very generic signature of this kind of scenario is the existence of TeV-scale Kaluza-Klein(KK) excitations of the graviton with inverse TeV-scale couplings to the SM fields. These states will appear as a series of spin-2 resonances in a number of processes that should be observable at both hadron and e^+e^- colliders which probe the TeV-scale. The masses and couplings of these KK graviton states will be determined with reasonably high precision by future collider measurements. Another possible RS signature is the copious production of TeV scale black holes, though this is not a unique feature of the RS model[3].

One can easily imagine that this simple RS scenario is incomplete from either a top-down or bottom-up perspective. We would generally expect some 'soft' modifications to the details of the picture presented above, hopefully without disturbing the nice qualitative features of the model. One such extension of the basic RS model is the existence of higher curvature terms which might be expected on general grounds from string theory[4, 5] or other possible high-scale completions. Once we open the door to such possibilities the number of potential new terms in the action can grow rather rapidly as the number of scalars that we can form from products of the curvature and Ricci tensors as well as the Ricci scalar are enormous. A certain general class of such invariants with very interesting properties was first generally described by Lovelock[6] and, hence, are termed Lovelock invariants. (They are also sometimes referred to, apart from a factor of $\sqrt{-g}$, as Euler densities since their volume integrals are related to the Euler characteristics.) The Lovelock invariants come in fixed order, n, which we denote as $\mathcal{L}_n$, that describes the number of powers of the curvature tensor, contracted in various ways, out of which they are constructed. Given a space of dimension D the order of these invariants is constrained: For $D = 2n$, the Lovelock invariant is a topological one and leads to a total derivative in the action whereas all higher order invariants, $D \leq 2n - 1$, can be shown to vanish identically by various curvature tensor index symmetry properties. For $D \geq 2n + 1$, the $\mathcal{L}_n$ are dynamical objects that once introduced into the action for gravity can be shown to lead only to second order equations of motion as is the case for ordinary Einstein gravity, *i.e.*, no terms with derivatives higher than second will appear in the equations of motion due to their presence. Generally, arbitrary invariants formed from ever higher powers of the curvature tensor will lead to equations of motion of ever higher order, *i.e.*, ever more co-ordinate derivatives of the metric tensor and graviton field, *e.g.*, terms with quartic derivatives. Such theories will lead to very serious problems with both the presence of ghosts as well as with unitarity[4]. The Lovelock invariants are constructed in such a way as to be free of these problems making them very special and are found to be just the forms taken by the higher order curvature terms generated in perturbative string theory[4, 5]. This is just what we may have expected if string theories are to avoid these unitarity issues. Indeed, if one tracks potential ghost generating contributions through the fog of higher derivative terms in the Lovelock invariants one sees that they vanish identically.

In the case of 4-d, apart from numerical factors, $\mathcal{L}_0 = 1$ while $\mathcal{L}_1 = R$, the ordinary Ricci scalar. The invariant of the next order, $\mathcal{L}_2$, can be identified with the Gauss-Bonnet(G-B) invariant, $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$, which is a topological term as well as a total derivative and can be written *in 4-d* as $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}R_{\mu\nu}{}^{\alpha\beta}R_{\rho\sigma}{}^{\gamma\delta}$. All higher order invariants vanish. When we go to 5-d, as is the case we will discuss below, all the $\mathcal{L}_{n\geq 3}$ still vanish as in 4-d but the G-B invariant is no longer a total derivative and its presence will modify the results obtained from Einstein gravity in the RS model, altering the equations of motion. It is interesting to note that if we demand the absence of ghosts, *i.e.*, terms with no more than two derivatives of the metric, then the addition of the G-B piece to the Einstein term is the *unique* modification of the 5-d RS bulk action. What is particularly amazing is that the addition of the G-B term to the conventional RS model still allows for a solution with the same qualitative structure as is present in the traditional RS model[7]. In fact, generalizing to D dimensions, Meissner and Olechowski[8] have shown that RS-like solutions exist with the presence of $(D-2)$-branes even allowing for all of the non-zero $\mathcal{L}_n$ contributions to be present in the bulk action.

Some of the modifications of the RS model due the presence of G-B terms have been discussed by other authors (*e.g.*, Refs. [7, 8, 9, 10, 11, 12, 13]). In this paper we are interested in how the presence of the G-B term alters the detailed collider phenomenology of the RS model. These modifications may occur in many different ways and places. One can imagine, *e.g.*, that since the G-B term is of higher order, the triple graviton coupling in the RS model[14] may be sensitive to its existence. While this is certainly true, it turns out that this is not the most immediate or sensitive way to probe for the existence of G-B contributions to the action. As we will see below, the existence of G-B terms will lead to significant changes in the masses and SM matter couplings of the Kaluza-Klein gravitons themselves. This variation is described by a single new parameter that has a rather restricted range in order to avoid tachyons being introduced into the model. In addition, we will demonstrate that the anticipated production rate for black holes at colliders can be significantly increased by the existence of G-B terms in comparison to the usual RS scenario. For simplicity, we will ignore issues having to do with the radion[15] in what follows as this physics may be sensitive to the stabilization mechanism[2]. The reader must be careful, however, to insure that no tachyons arise in this sector of the theory when exploring the model parameter space. We will also restrict our analysis to the case where the SM fields are confined to the TeV-brane. Generalization to the case of bulk SM fields is straightforward[17].

The outline of the paper is as follows: In Section 2 we will present the basic Kaluza-Klein formalism and then determine the masses, couplings and wavefunctions for the graviton excitations in the RS model in the presence of G-B terms. In Section 3 we will discuss how observations of graviton resonances and measurements of their properties at colliders can be used to determine the value of the new parameter describing the G-B interactions or place stringent upper bound on its value if no deviations from the conventional RS scenario are observed. Section 4 contains a discussion of black hole production at the LHC and how it will be substantially enhanced by any appreciable G-B terms. We will also show how the modifications to the usual RS picture can lead to a significant alteration in the shape and parameter dependence of the black hole subprocess cross section. We then summarize and conclude. The Appendix outlines the changes to our basic formalism which are necessary if graviton brane kinetic terms arising from the usual Einstein action are also present.

2 Kaluza-Klein Formalism

The essential ansatz of the 5-d RS scenario is the existence of a slice of warped, Anti-deSitter space bounded by two 'branes' which we assume are fixed at the S^1/Z_2 orbifold fixed points, $y = 0, \pi r_c$, termed the Planck and TeV branes, respectively[1]. We take the 5-d metric describing this setup to be given by the conventional expression

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \,. \tag{1}$$

As usual, due to the S^1/Z_2 orbifold symmetry we require $\sigma = \sigma(|y|)$ and, in keeping with the RS solution, we expect $\sigma = k|y|$ with k a dimensionful constant of order the fundamental

Planck scale. As first shown in Ref.[7] the inclusion of G-B terms does not alter this basic setup.

The action for the model we consider takes the form

$$S = S_{bulk} + S_{branes}\,, \tag{2}$$

where

$$S_{bulk} = \int d^5x\, \sqrt{-g}\Big[\frac{M^3}{2}R - \Lambda_b + \frac{\alpha M}{2}\Big(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}\Big)\Big]\,, \tag{3}$$

describes the bulk with M being the 5-d fundamental Planck scale, Λ_b the bulk cosmological constant and α is a dimensionless constant of unknown sign which we expect to be of order unity based on naturalness arguments. Here we note that the upper case Roman indices $A, B, ...$ run over $0-4$ while Greek indices will continue to run over $0-3$. This bulk action contains not only the usual Ricci scalar, R, of Einstein gravity but also the Gauss-Bonnet quadratic curvature term. Similarly

$$S_{branes} = \sum_{i=1}^{2}\int d^4x\, \sqrt{-g_i}\Big(\mathcal{L}_i - \Lambda_i\Big)\,, \tag{4}$$

describes the two branes with g_i being the determinant of the induced metric and Λ_i the associated brane tensions; the $\mathcal{L}_i$ describe possible SM fields on the branes. In what follows we will assume as usual that the SM fields are all localized on the TeV brane at $y = \pi r_c$. For simplicity we have not considered potential contributions arising from brane kinetic terms for the graviton generated by the Ricci scalars evaluated on the two branes[16]; as we will see in the Appendix, such contributions can be included in a relatively straightforward manner. G-B like brane terms are automatically absent as the G-B invariant is at most a surface term in less than 5-d.

To go further we insert our metric ansatz into the equations of motion arising from the above action; we obtain $\Lambda_{TeV} = -\Lambda_{Planck}$ as usual together with[1]

$$
\begin{aligned}
3\sigma''\Big(1-\frac{4\alpha}{M^2}(\sigma')^2\Big) &= \frac{\Lambda_{Planck}}{M^3}\Delta(y) \\
6(\sigma')^2\Big(1-\frac{2\alpha}{M^2}(\sigma')^2\Big) &= -\frac{\Lambda_b}{M^3}\,,
\end{aligned} \tag{5}
$$

where $' = \partial_y$ and $\Delta(y)$ is the combination

$$
\Delta(y) = \delta(y) - \delta(y - \pi r_c)\,. \tag{6}
$$

As in the conventional RS model the solution indeed takes the form $\sigma = k|y|$ though greater than normal care is required to regularize the associated δ-functions. Since

$$
\sigma'' = 2k\Delta\,, \tag{7}
$$

using the set of useful relations[8]

$$
(\sigma')^n\sigma'' = \frac{2}{n+1}k^{n+1}\Delta(y)\,, \tag{8}
$$

one then finds

$$
\Lambda_{Planck} = -\Lambda_{TeV} = 6kM^3\Big(1-\frac{4\alpha k^2}{3M^2}\Big)\,. \tag{9}
$$

This differs by a factor of 1/3 in the α-dependent term from the work of Kim, Kyae and Lee[7] due to an error in their application of the proper boundary conditions. In addition, using the relation $(\sigma')^{2n} = k^{2n}$, we obtain the following explicit expression for k:

$$
k = k_\pm = \left[\frac{M^2}{2\alpha}\left(1 \pm \Big(1+\frac{4\alpha\Lambda_b}{3M^5}\Big)^{1/2}\right)\right]^{1/2}. \tag{10}
$$

Note that for $\alpha > 0$, Λ_b may in principle take either sign whereas it must be negative if $\alpha \leq 0$; the conventional RS limit can be obtained by employing the negative root in the equation above with Λ_b negative and taking the limit $\alpha \to 0$.

In order to obtain the equations of motion for the KK excitations we need to go beyond this vacuum solution; to this end we consider metric perturbations of the form

$$g_{\mu\nu} = e^{-2\sigma}\eta_{\mu\nu} + h_{\mu\nu}\,, \tag{11}$$

and employ the traceless-transverse gauge with $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$ as usual followed by the KK decomposition

$$h_{\mu\nu}(x,y) = \sum_n h^{(n)}_{\mu\nu}(x)\chi_n(y)\,. \tag{12}$$

such that the Klein-Gordon equation, $\partial^2_\lambda h^{(n)}_{\mu\nu} = -m^2_n h^{(n)}_{\mu\nu}$, yields the KK masses. The χ_n wavefunctions are then seen to satisfy[9, 11]

$$\left[\partial^2_y - 4\bar{\alpha}\sigma'^2\partial^2_y - 8\bar{\alpha}\sigma'\sigma''\partial_y - 4(\sigma')^2(1-4\bar{\alpha}\sigma'^2) + 2\sigma''(1-12\bar{\alpha}\sigma'^2)\right]\chi_n = m^2_n e^{2\sigma}(1-4\bar{\alpha}\sigma'^2+4\bar{\alpha}\sigma'')\chi_n\,, \tag{13}$$

where $\bar{\alpha} = \alpha/M^2$, as obtained by [9]. Letting $\chi_n = e^{-2\sigma}\psi_n$, the left-hand side of the expression above can be simplified using the identities in Eq.(8) to produce

$$\left(1 - 4\frac{\alpha}{M^2}\sigma'^2\right)(\psi''_n - 4\sigma'\psi'_n) - 8\frac{\alpha}{M^2}\sigma'\sigma''\psi_n\,, \tag{14}$$

which after some algebra directly leads to the G-B generalization[12] of Eq.(5) in [17] obtained long ago:

$$\partial_y\left[\left(1 - 4\frac{\alpha}{M^2}\sigma'^2\right)e^{-4\sigma}\partial_y\psi_n\right] + m^2_n e^{-2\sigma}\left(1 - 4\frac{\alpha}{M^2}\sigma'^2 + 4\frac{\alpha}{M^2}\sigma''\right)\psi_n = 0\,. \tag{15}$$

As noted by other authors[7, 9, 10, 11, 13], it is straightforward to see that this produces the same differential equation in the bulk for the KK eigenfunctions as found in the usual RS scenario[17] and thus

$$\psi_n(y) = \frac{e^{2\sigma}}{N_n}\zeta_2(m_n e^\sigma/k)\,, \tag{16}$$

with $\zeta_q = J_q + \beta_n Y_q$ being a combination of Bessel functions of order q and where N_n is a normalization factor. Following our earlier work[17] we choose to normalize the KK states over the full interval $-\pi r_c \leq y \leq \pi r_c$. For a differential equation with this truncated Sturm-Liouville form[18]

$$\mathcal{L}\psi_n = [p(y)\psi_n']' + \lambda_n w(y)\psi_n = 0\,, \tag{17}$$

the eigenfunctions are to be orthonormalized with respect to the weight function $w(y)$ which we can read off in our case from Eq.(15) above:

$$w(y) = e^{-2\sigma}\left(1 - 4\alpha\frac{k^2}{M^2} + 8\alpha\frac{k}{M^2}\Delta\right), \tag{18}$$

so that

$$\int_{-\pi r_c}^{\pi r_c} dy\; w(y)\; \psi_n(y)\psi_m(y) = \delta_{nm}\,. \tag{19}$$

Thus, making use of the orbifold symmetry, we obtain the normalization factor

$$N_n^2 = 2\int_0^{\pi r_c} dy e^{-2ky}\Big(1 - 4\alpha\frac{k^2}{M^2}\Big)\psi_n^2(y) + 8\alpha\frac{k}{M^2}\Big(\psi_n(0)^2 - \epsilon^2\psi_n(\pi r_c)^2\Big)\,. \tag{20}$$

where $\epsilon = e^{-\pi k r_c}$. Neglecting terms suppressed by powers of ϵ, for the massless zero-mode graviton this yields the explicit expression

$$N_0^2 = \frac{1}{k}\Big(1 + 4\alpha\frac{k^2}{M^2}\Big) = \frac{1}{k}\Big(1 - 4\alpha\frac{k^2}{M^2}\Big)(1 + 2\Omega)\,, \tag{21}$$

where we have defined for later purposes the combination

$$\Omega = \frac{4\alpha k^2/M^2}{1-4\alpha k^2/M^2}\,. \tag{22}$$

Knowing this normalization we can directly relate the 5-d parameters M and k to the (effective) 4-d Planck mass, $\overline{M}_{Pl}$:

$$\overline{M}_{Pl}^2 = \frac{M^3}{k}\Big(1-4\alpha\frac{k^2}{M^2}\Big)(1+2\Omega)\,, \tag{23}$$

To obtain the explicit expression for the normalization of the massive KK excitations we need to examine the two boundary conditions which are now quite different than in the usual RS model. These are obtained by the integration of Eq.(15) above around the two branes at $y=0$ and πr_c; from this procedure we arrive at

$$\psi_n(0^+)\;' + \Omega\frac{m_n^2}{k}\psi(0^+) = 0\,, \tag{24}$$

and

$$\psi_n(\pi r_c^+)\;' + \Omega\epsilon^{-2}\frac{m_n^2}{k}\psi_n(\pi r_c^+) = 0\,, \tag{25}$$

where the eigenfunctions and their derivatives are evaluated on the positive side of both branes. These boundary conditions then determine both the wavefunction coefficients β_n as well as the KK masses $m_n = x_n k\epsilon$; we obtain

$$\zeta_1(x_n) + \Omega x_n\zeta_2(x_n) = 0\,, \tag{26}$$

whose roots determine the x_n and

$$\beta_n = -\frac{J_1(x_n\epsilon)+\Omega x_n\epsilon J_2(x_n\epsilon)}{Y_1(x_n\epsilon)+\Omega x_n\epsilon Y_2(x_n\epsilon)}\,. \tag{27}$$

Note that under *most* circumstances $\beta_n \sim \epsilon^2$ and can be safely ignored as in the usual RS case; some care, however, needs to be exercised as $\Omega \rightarrow -1/2$. From these expressions we can straightforwardly obtain an explicit expression for the normalization factor for the KK excitation wavefunctions:

$$N^2_{n>0} = \frac{1}{k\epsilon^2}\zeta_2(x_n)^2\Big(1 - 4\alpha\frac{k^2}{M^2}\Big)(1 + 2\Omega + \Omega^2 x_n^2)\,, \tag{28}$$

where subleading terms in powers of ϵ have been neglected. The usual RS model is seen to correspond to the limit $\alpha, \Omega \rightarrow 0$ in the expressions above.

At this point the experienced reader may notice that the modifications to the RS expressions above due to the new G-B interactions is strikingly similar in nature to those produced by the addition of conventional R-type kinetic terms on both branes[16]. This will allow one to combine the contributions of both these new physics sources into rather simple expressions as we will see in the Appendix below.

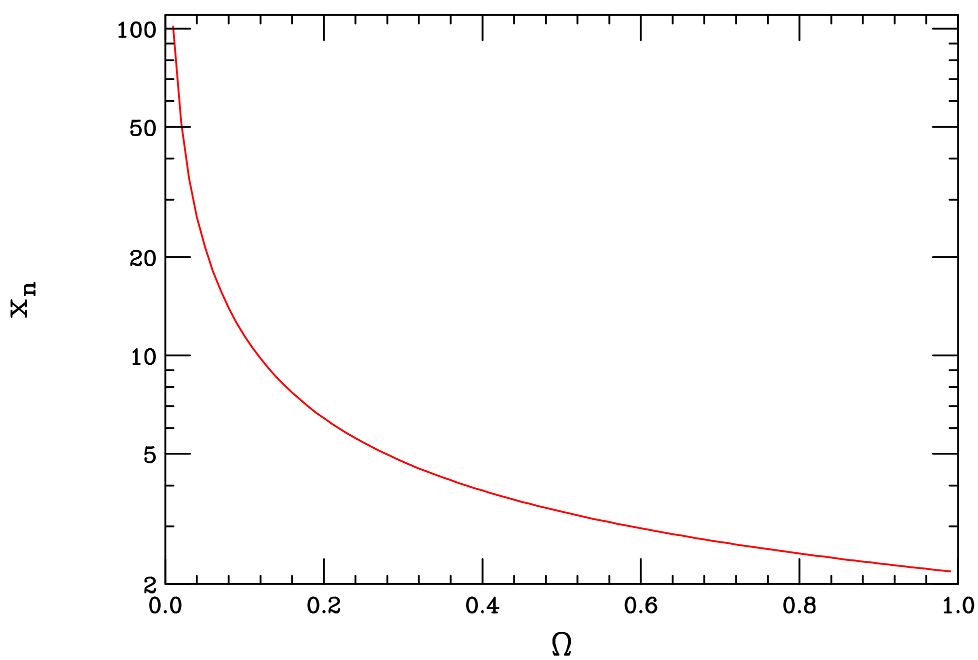

Figure 1: Imaginary root corresponding to a tachyon in Eq.(26) as a function of positive Ω.

3 Gravitons at Colliders

The discussion above tells us that the modifications of the basic RS model due to G-B terms can be described by a single parameter, Ω. To go further and see this in detail we must understand the potential range of this parameter. Since we expect α to be of order unity as well as $k \sim M$, one should expect that Ω is also of order unity. The arguments given in Ref.[12, 13] suggest that Ω cannot be positive as this leads to a tachyon in the KK spectrum. To verify this claim we have examined Eq.(26) in detail when $\Omega > 0$ for imaginary roots of the form $z_n = \pm i x_n$ with the results shown in Fig. 1. Here we see that indeed a pair of tachyonic roots do exist for $\Omega > 0$ which move off to infinity as we approach the RS limit $\Omega \to 0$. We thus agree that solutions with positive values of Ω are indeed excluded. A similar search for tachyonic roots for $\Omega < 0$ was unsuccessful so we conclude that $\Omega \leq 0$ to which we now restrict ourselves; this implies that the parameter α is also ≤ 0. Eq.(21) provides the normalization for the zero-mode graviton wavefunction which must be positive definite in order to avoid ghosts. We notice that this requires $4\alpha k^2/M^2 > -1$ which then implies, via the definition Eq.(22) and the negative Ω constraint, that Ω is confined to the range

$$-\frac{1}{2} < \Omega \leq 0\,. \tag{29}$$

A short analysis shows that the consideration of the the normalization factor for the graviton KK excitations will not improve upon this bound. Thus Ω is constrained to be in a rather narrow range which significantly increases the predictability of the model. Given the definition of Ω, Eq.(29) implies that

$$-\alpha\frac{k^2}{M^2} \leq \frac{1}{4}\,. \tag{30}$$

To begin a phenomenological analysis the first question to address is: how does the KK graviton spectrum shift as we turn on a non-zero Ω? To this end we extract the first few roots, x_n, provided by Eq.(26) and follow their Ω-dependence; this is shown in Fig. 2. Here we see that the overall Ω dependence of the roots is rather weak across the rather restricted allowed range for Ω. The values of the roots are seen to decrease slightly as Ω moves away from 0. These small variations in the root values will, however, be of interest to us below and will provide an important test of the model.

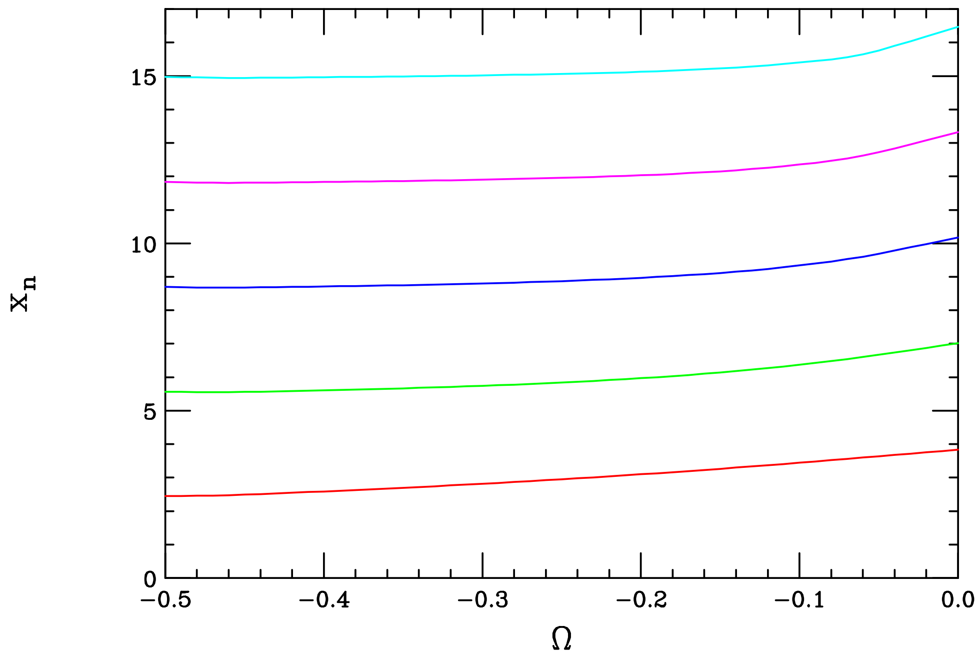

Figure 2: Behaviour of the first five roots corresponding to the graviton KK masses $x_n = m_n/k\epsilon$ as functions of the parameter Ω.

Next, we find that the interaction of the KK graviton excitations with the SM fields on the TeV brane is given by

$$\mathcal{L} = \frac{1}{\Lambda_\pi} \sum_n \left[\frac{1+2\Omega}{1+2\Omega+\Omega^2 x_n^2} \right]^{1/2} h_n^{\mu\nu} T_{\mu\nu} \,, \tag{31}$$

where as usual we define $\Lambda_\pi = \overline{M}_{Pl}\epsilon$[17]. As in the case of graviton brane kinetic terms, but *unlike* in the usual RS model, the couplings of the KK graviton tower states have become level-dependent. In particular, the KK states will always become more weakly coupled as we go up the KK tower. Furthermore, we see that the couplings of all the KK states *vanish* as $\Omega \to -0.5$. (It is interesting to note that a very similar rescaling of the conventional RS result holds for the case when SM fields are put in bulk.) Fig. 3 shows the rapid decrease in the couplings of the first five KK states as Ω decreases from 0. The couplings are observed to vanish more quickly as one goes farther up the KK tower as expected.

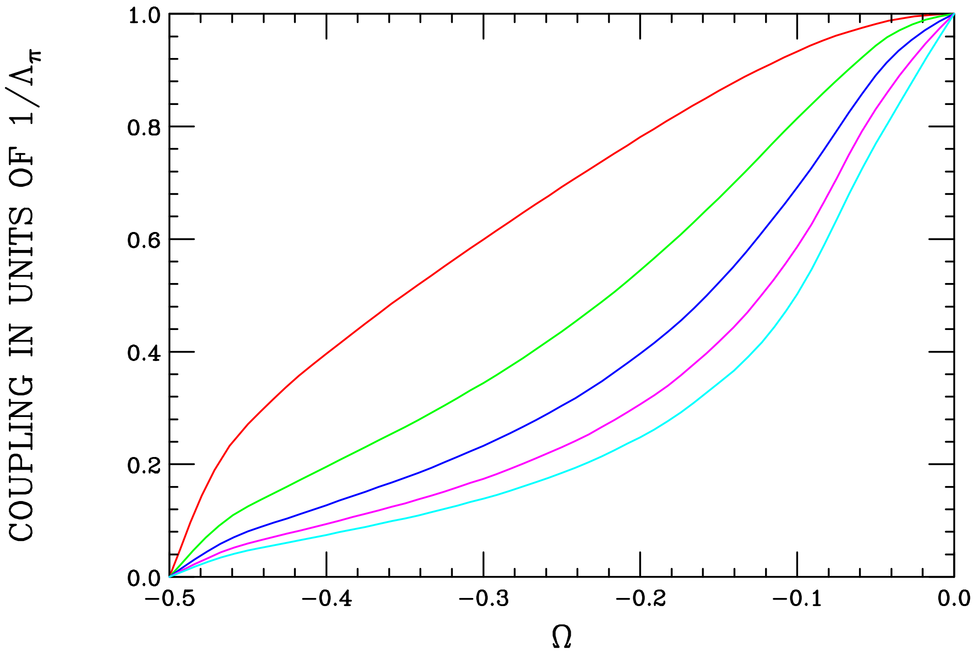

Figure 3: Coupling strengths of the first KK graviton states, from top to bottom, in units of $1/\Lambda_\pi$ as functions of the parameter Ω. Note that in the RS limit all states have the same coupling.

In addition to specifying Ω and the mass of the first KK excitation, m_1, to completely determine the phenomenology of this model we need to know the ratio $c = k/\overline{M}_{Pl}$ which enters into the expression for the KK widths. In the usual RS model this ratio is bounded by the requirement that we don't want potential quantum (*i.e.*, higher curvature) corrections to dominate over those from classical Einstein gravity. This requirement is traditionally stated as $|R| < M^2$, which implies that $k^2/M^2 \leq 1/20$ for a 5-d Anti-deSitter space; combining with Eq.(23) in the $\alpha \to 0$ limit then implies $k/\overline{M}_{Pl} \lesssim 1/10$ which is the usual result. Under the present circumstances there may be (at least) three objections to this argument. First, since we are already including G-B terms there is no longer any reason to demand small curvature and thus k^2/M^2 may be larger than 1/20. A second objection is that if we do truly want small higher-curvature terms for some reason the traditional constraint on k^2/M^2 may be too weak. To see this we note that for the RS vacuum solution we know that $R = -20k^2$ (hence, the limit above), $R_{AB}R^{AB} = 80k^4$ and $R_{ABCD}R^{ABCD} = 40k^4$ which implies that the G-B invariant has the value $120k^4$. These large numerical coefficients may imply that a stronger constraint on k^2/M^2 might be necessary. A third observation is that even if $k^2/M^2 < 1/20$ holds, for finite α from Eqs.(21) and (23) we find that

$$\frac{k^2}{\overline{M}_{Pl}^2} = \frac{k^3}{M^3}\left(1 + 4\alpha\frac{k^2}{M^2}\right)^{-1} = \frac{k^3}{M^3}\frac{(1+\Omega)}{(1+2\Omega)}\,, \tag{32}$$

so that, given the bound on negative Ω above, the ratio $k/\overline{M}_{Pl}$ can become *arbitrarily* large as Ω approaches -0.5. While we will restrict ourselves to the conventional RS range of $k/\overline{M}_{Pl}$ in what follows for purposes of comparison, we remind the reader that much larger values of this ratio may now be possible. (There are, however, other reasons to believe that this ratio may remain as constrained as in the conventional RS model as discussed in Ref.[17]). Given a values of Ω and the ratio $c = k/\overline{M}_{Pl}$, we note that the parameter α can now be uniquely determined:

$$\alpha = \frac{1}{4c^{4/3}}\frac{\Omega}{(1+2\Omega)^{2/3}(1+\Omega)^{1/3}}\,. \tag{33}$$

With these preliminaries we can now examine how these KK graviton states would appear at a collider in comparison to the expectations of the RS model. To this end we examine the process $e^+e^- \to \mu^+\mu^-$ in the energy range accessible to the International Linear Collider(ILC) and its potential upgrades. A similar analysis can, of course, be performed at

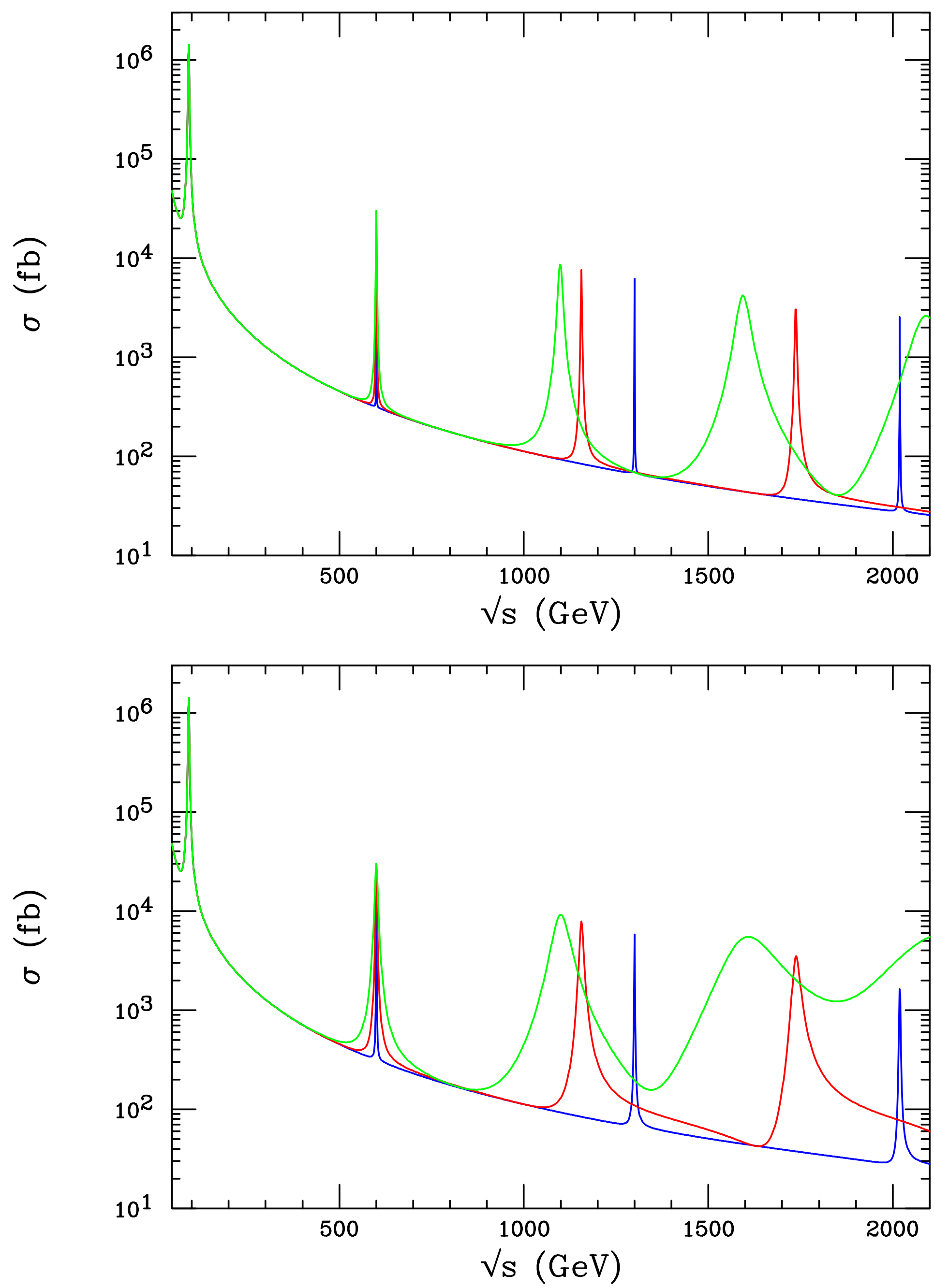

Figure 4: Cross section for $e^+e^- \to \mu^+\mu^-$ assuming $m_1 = 600$ GeV and $k/\overline{M}_{Pl} = 0.05$(top) or 0.1(bottom). The usual RS model prediction with $\Omega = 0$ is shown in green while the corresponding results in the present model with $\Omega = -0.2$(red) and -0.4(blue) are also displayed.

the LHC. To be specific for purposes of demonstration we take $m_1 = 600$ GeV and $k/\overline{M}_{Pl} = 0.05, 0.1$; the results of these calculations are displayed in Fig. 4 where we show the cases $\Omega = -0.2, -0.4$ in comparison to the standard RS model predictions with $\Omega = 0$. Several things are immediately apparent: First, as the magnitude of Ω increases, the KK states become increasingly narrow and their mass splitting is also seen to increase significantly. Second, with m_1 held fixed, the observation of a single graviton resonance and a determination of its properties would be insufficient to tell us whether or not G-B terms are present in the action. The reason for this is clear: if we just observe a single state we can attribute its width solely to the value of Λ_π. We see for example that the width of the first KK is decreased as Ω decreases from zero by the square of the factor appearing in Fig 3. The only thing we can extract from the first KK width is the value of an *effective* Λ_π which tells us nothing about the true value of Λ_π or Ω. To obtain the values of these quantities we need to also find the second KK excitation. Fig. 4 shows that the mass of the second KK increases relative to the first as the magnitude of Ω increases; it also gets much more narrow. These observations provide our best handles on Ω. If we measure the ratio of the first two KK masses, $R_1 = m_2/m_1$, this will directly tell us the value of Ω as shown in Fig. 5.

We can get a second handle on Ω as well as Λ_π (and thus $k/\overline{M}_{Pl}$ through the relation $m_n = x_n(k/\overline{M}_{Pl})\Lambda_\pi$) by comparing the decay widths of the first two KK states after removing 'phase space' factors and knowing their respective masses. Since kinematically $\Gamma_n \sim m_n^3$, the scaled ratio $R_2 = \Gamma_2 m_1^3/\Gamma_1 m_2^3$ would be unity in the usual RS model if we neglect the masses of the final state particles. This ratio depends rather strongly on the value of Ω when G-B terms are present as is shown in Fig. 5. Recall that as Ω approaches -0.5 the couplings of the KK states to SM matter on the TeV brane vanishes so that they will no longer be produced at colliders. Of course, the *ratio* of the couplings of two different KK states remains finite.

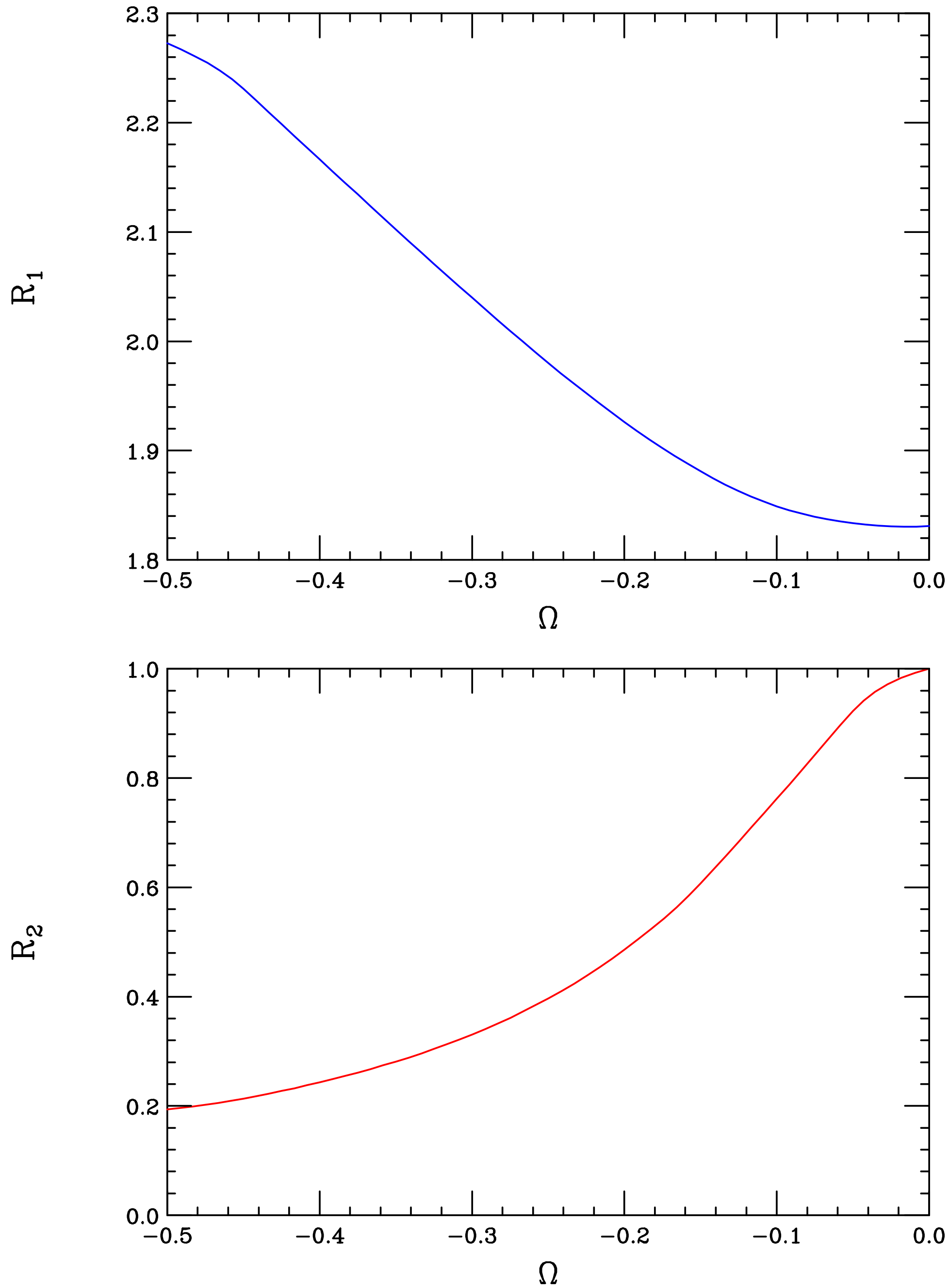

Figure 5: The ratios $R_{1,2}$ described in the text as functions of Ω.

For fixed Ω and $k/\overline{M}_{Pl}$ it is interesting to examine the widths of the graviton KK resonances as we go farther up the tower. In the usual RS scenario the widths grow quite rapidly $\Gamma_n \sim m_n^3$ and eventually the resonance structure is lost as can be seen in the lower panel of Fig. 5. Once G-B terms are present, however, the coupling decreases as m_n increases and eventually $\Gamma_n \sim m_n$. This means that for appreciable values of Ω the KK states are all becoming rather narrow as we go further and further up the tower. This can be seen more explicitly by examining the ratio Γ_n/m_n as a function of Ω for the different KK tower states as is shown in Fig. 6. Here we see that for large Ω the KK states remain narrow and have almost the same value of Γ/m independent of n.

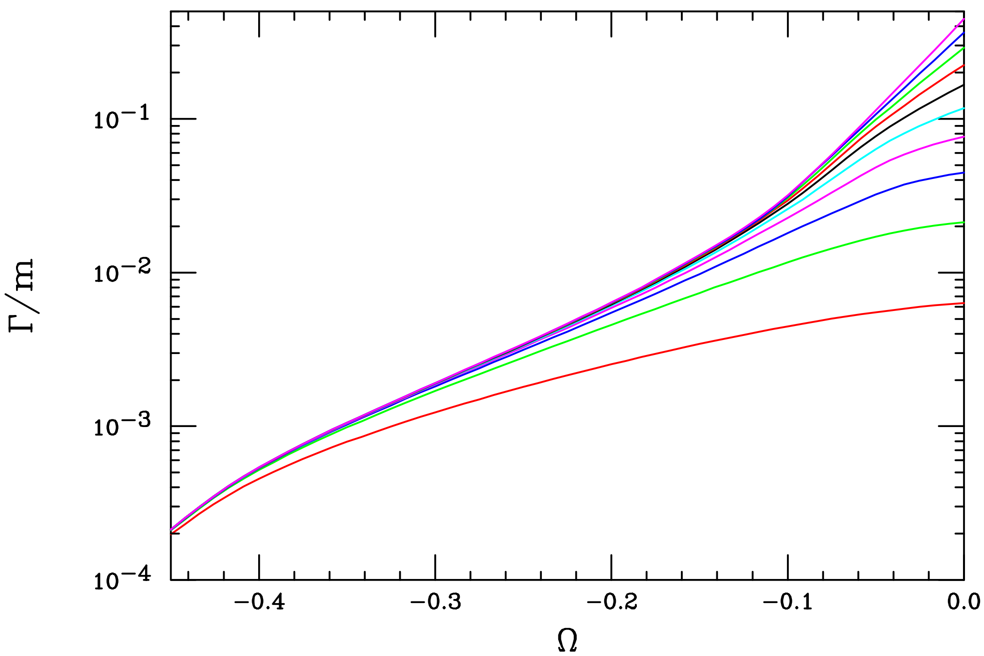

Figure 6: The ratio Γ/m as a function of Ω for the first ten KK states. The KK number goes up as we go from the bottom to the top of the figure. $k/\overline{M}_{Pl} = 0.05$ has been chosen for purposes of demonstration.

One may wonder just how small a value of Ω might be measurable. It is clear from Figs. 4 and 5 that the ILC will be highly sensitive to small values of Ω through the mass and width ratios discussed above. Note in particular that in the region near $\Omega = 0$ the Ω

dependence of R_2 is significantly greater than that for R_1. To get a feeling for the level of sensitivity we note that a determination of $R_{1,2}$ at the level of $1-2\%$, which seems rather straightforward at the ILC, will be sensitive to values of Ω of order -0.01 to -0.02 or less. Thus even small deviations from the standard RS scenario should be observable at the ILC. Of course, a detailed analysis accounting for accelerator and detector effects should be performed to confirm these conclusions.

4 Black Hole Production

The possibility that TeV scale black holes(BH) may be a copious signal for extra dimensions at future colliders has been discussed by a number of authors[19, 20] for both the flat and warped background cases. A leading approximation for the subprocess cross-section for the production of a BH of mass M_{BH} is just the geometric BH size[21, 22, 23]

$$\hat{\sigma} \simeq \pi R_s^2 \,, \tag{34}$$

where R_s is the $(4+n)$-dimensional Schwarzschild radius corresponding to the mass M_{BH}. The production of BH at the LHC has been studied in detail, including detector effects, in Refs.[24, 25] and there is reason to believe that BH arising from warped and flat backgrounds may be experimentally distinguishable[26] due to the RS S^1/Z_2 orbifold symmetry or the effects of the AdS_5 curvature. In our case of interest, we would like to examine how this cross section is influenced by including G-B terms in the RS model with $n=1$. This can be done by following, *e.g.*, the analyses presented in [27, 28, 29, 30, 31, 32]. A simple approximate expression for the cross section can be obtained through a modification of the flat space result provided that $(R_s k\epsilon)^2 << 1$ so that the bulk curvature corrections can be neglected. (Recall that in the usual flat space case $R_s << R_c$, where R_c is the compactification radius of the extra dimension.) This is just the ordinary Schwarzschild solution in an asymptotically flat 5-d background[27]. We find that this approximation works reasonably well for small values of Ω but fails when Ω nears its lower limit $\Omega = -0.5$ since the AdS curvature is larger in that case. To obtain the full expression[29] these bulk curvature terms can no longer be neglected and we must consider the BH to be embedded in AdS_5. Once we adjust for the definitions of the fundamental parameters as given in the action above, one obtains

$$\hat{\sigma} = \frac{\pi}{2\beta M_*^2}\left[-1 + \left(1 - 4\beta(2\alpha - \gamma)\right)^{1/2}\right], \tag{35}$$

where

$$\begin{aligned} \beta &= \frac{k^2}{M^2}\left(1 - 2\alpha\frac{k^2}{M^2}\right) \\ \gamma &= \frac{M_{BH}}{3\pi^2 M_*}, \end{aligned} \tag{36}$$

and where M_* is the 'warped-down' fundamental scale, $M_* = M\epsilon$. This expression is not very transparent so let us expand the square root to first order; we then find

$$\hat{\sigma} \simeq \frac{M_{BH}}{3\pi M_*^3}\left[1 - 6\pi^2\alpha\frac{M_*}{M_{BH}}\right], \tag{37}$$

which is the leading order term in the bulk curvature expansion; the usual RS result in the flat space approximation is obtained by setting $\alpha = 0$ in this expression[27].

To evaluate the cross section we note that Eq.(33) tells us that α is determined provided we know both Ω and $c = k/\overline{M}_{Pl}$. Note that for $k/\overline{M}_{Pl}$ fixed, α becomes quite large (and negative) as $\Omega \to -0.5$. Given the large prefactor of $6\pi^2$ in the approximate expression above we might expect that G-B terms can be numerically quite important even for small values of Ω. It is interesting to note that this cross section is bounded from below as can be seen from the exact expression, *i.e.*, $\hat{\sigma} \gtrsim -2\pi\alpha/M_*^2$, independent of the BH mass.

(Remember that α is negative here.) In obtaining our numerical results we will make use of the exact expression above.

Fig. 7 shows the influence of the G-B terms on the BH production cross-section at the LHC; for definiteness we have assumed $M_* = 1.5$ TeV and $k/\overline{M}_{Pl} = 0.1$ but there is nothing special about these particular values. We see, as expected, that a non-zero Ω can lead to a significant increase in the BH production cross section; for $\Omega = -0.1(-0.4)$ this increase is seen to be approximately one(two) order(s) of magnitude in comparison to the usual RS scenario.

Although we have seen a substantial increase in the BH cross section relative to the RS model due to G-B terms, the overall normalization of the cross section remains uncertain due to the complexities of BH formation by terms of order unity as has been discussed by many authors[21, 22, 23]. While overall factors may be uncertain we might expect the various scaling laws to remain at least approximately valid. In the case of flat extra dimensions, or in the RS model in the $(R_s k\epsilon)^2 << 1$ limit, the value of $n = d - 4$ uniquely determines the simple dependence of the cross section on the ratio M_{BH}/M_*: $\sigma M_*^2 \sim [M_{BH}/M_*]^{2/(n+1)}$. Here, for the case of a single warped extra dimension, the G-B terms conspire to modify the shape of the cross section as a function of M_{BH}/M_* in a rather unique manner. This can even be seen by examining the approximate expression Eq.(37): the subprocess cross section is generally no longer proportional to a single power of M_{BH}/M_*. In fact, as the G-B terms come to dominate, the subprocess cross section becomes essentially independent of M_{BH} as shown in Fig. 8 where the important feature to notice is the shape of the distribution and not the overall magnitude. Note that this effect happens quite rapidly as a non-zero value of Ω is turned on. This rather distinctive behavior may be observable in BH production at the LHC once the parton distribution luminosity is unfolded and will help to pin down the existence of G-B interactions. It should then be possible to correlate shifts in the production

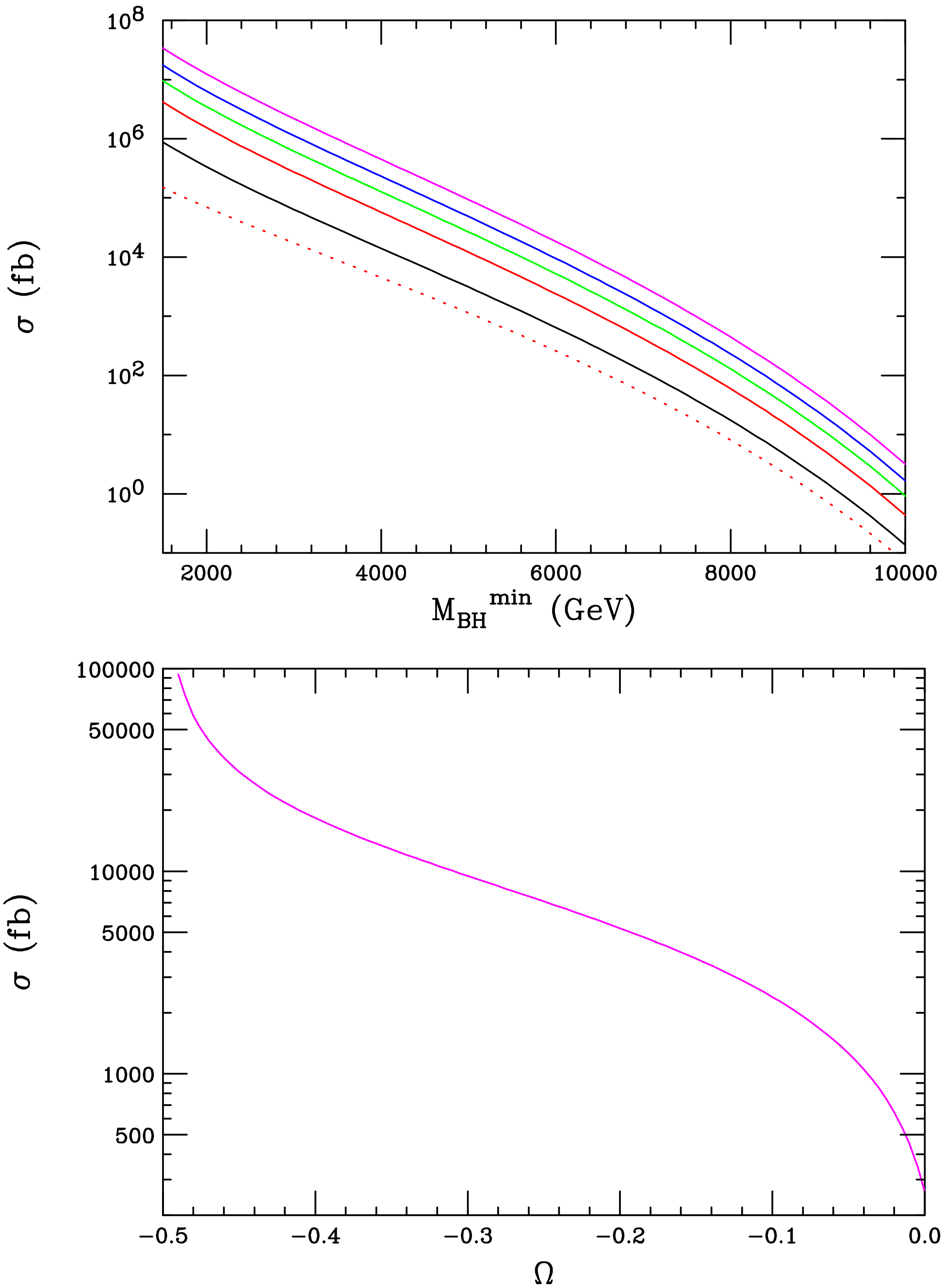

Figure 7: (Top) Black hole production cross section at the LHC assuming $M_* = 1.5$ TeV and $k/\overline{M}_{Pl} = 0.1$ as described in the text. The lowest dotted curve corresponds to the RS model prediction with $\Omega = 0$; the subsequently higher curves corresponds to $\Omega = -0.02, -0.1, -0.2, -0.3$ and -0.4, respectively in the G-B case. (Bottom) Same as in the top panel but now for a fixed minimum black hole mass of 6 TeV as a function of Ω.

properties of BH with those of graviton resonances to precisely determine the G-B parameter Ω.

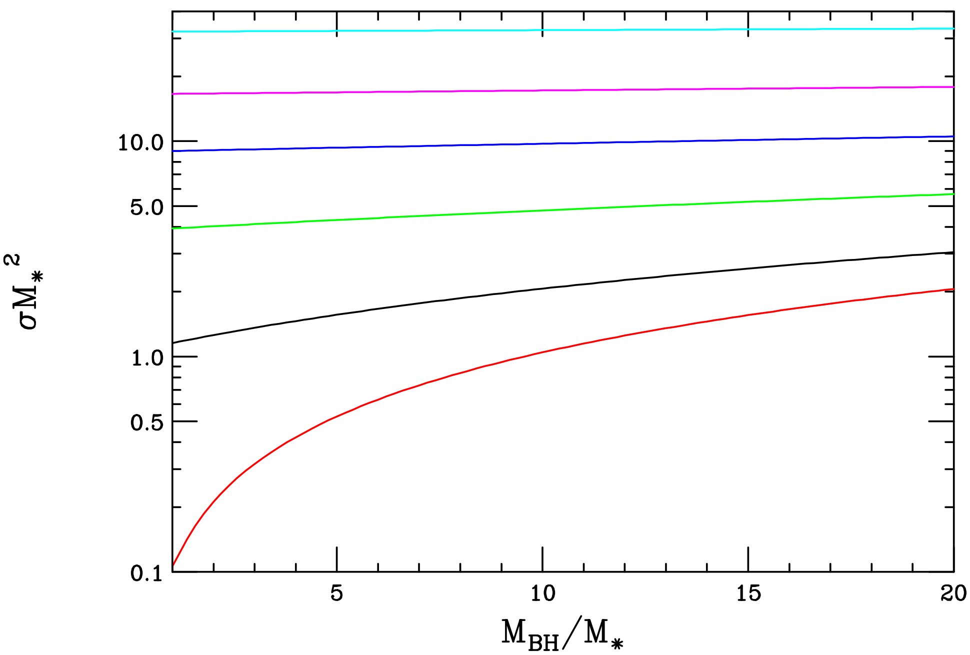

Figure 8: The BH sub-process cross section as a function of the ratio M_{BH}/M_* for different values of Ω. From bottom to top the curves correspond to $\Omega = 0, -0.03, -0.1, -0.2, -0.3$ and -0.4, respectively.

5 Conclusions

Higher-dimensional curvature corrections to Einstein gravity are to be expected on rather general grounds; the observation of any effect arising from such terms will tell us important information about the ultraviolet completion of this theory. The requirements of unitarity and the absence of ghosts in the 5-d Randall-Sundrum model uniquely determine the form such terms may take and eliminates any derivatives higher than second in the equations of motion. Consequentially, the Gauss-Bonnet invariant is singled out as the only possible new non-trivial addition to the bulk action for gravity beyond the usual Ricci scalar. The existence of G-B terms in the action has been previously shown to leave the qualitative features of the RS model invariant but the quantitative details are expected to be altered, perhaps significantly.

In this paper we have begun to examine the phenomenological implications of the existence of additional G-B terms in the RS bulk action. Two of the dominant signatures for the RS model are the production of TeV-scale graviton resonances and the production of black holes at future colliders. This paper has shown that both of these processes can be significantly altered if G-B terms are present in any appreciable amount. If no deviations from the classic RS model predictions are observed, we have shown that stringent bounds can be placed on the parameter Ω which describes the relative strength of the G-B terms.

The first step in this analysis was to determine how the graviton KK masses, couplings and wavefunctions are modified by the presence of the G-B terms. Once this was done we analyzed the production properties of the KK graviton resonances at the ILC; we showed the mass spectrum and decay widths of these resonances were substantially different than in the RS case over most of the G-B model parameter space. It was demonstrated that the KK mass and width ratio measurements at the ILC can be highly sensitive to the presence of G-B terms and can be used to precisely determine their strength or to place strong bounds on their existence if no deviations from the RS model are observed.

Next we examined the production of BH at the LHC in the RS model with G-B terms. We found that not only is the production cross section significantly enhanced in comparison to the conventional RS model but its parametric dependence on the ratio M_{BH}/M_* can be drastically altered even for small values of Ω. Any deviations observed in the production of graviton resonances at the LHC or ILC can then be directly correlated with modifications to BH production to determine the value of Ω.

Hopefully signals of warped extra dimensions will be observed at future colliders and we may be ultimately be able to probe the ultraviolet completion of quantum gravity.

Acknowledgments

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Appendix

In this Appendix we consider a modification of the action given by Eqs.(3) and (4) above to include graviton brane kinetic terms generated by the 4-d Ricci curvature scalars on the TeV and Planck branes. Such terms have been suggested in the case of G-B extensions of the RS model[13] to help assist removal of the tachyon field present[12] when $\alpha > 0$. Using the notation above as well as in Ref.[16], this amounts to adding to the action a term of the form

$$S_{BKT} = \int d^5x\sqrt{-g}\ \frac{M^3}{2}\Big[g_0 r_c \delta(y) + g_\pi r_c \delta(y - \pi r_c)\Big] R^{(4)}\,, \tag{38}$$

where $R^{(4)}$ is the induced 4-d curvature and $g_{0,\pi}$ are dimensionless constants that we expect to be of order unity. Since these are boundary terms they do not modify the bulk equation for the KK wavefunctions which remain unaltered from the traditional RS case as discussed above. Defining the combinations

$$\gamma_{0,\pi} = g_{0,\pi} \frac{kr_c}{2}\,, \tag{39}$$

as in our earlier work[16], we find that the only change induced by these new brane terms is to modify the definition of $w(y)$ given in Eq.(18) above to

$$w(y) = e^{-2\sigma}\left(1 - 4\alpha\frac{k^2}{M^2} + 8\alpha\frac{k}{M^2}\Delta + \frac{2\gamma_0}{k}\delta(y) + \frac{2\gamma_\pi}{k}\delta(y - \pi r_c)\right), \tag{40}$$

with the subsequent appropriate changes in the boundary conditions:

$$\psi_n(0^+)\,' + \Omega_0 \frac{m_n^2}{k} \psi(0^+) = 0\,, \tag{41}$$

and

$$\psi_n(\pi r_c^+)\,' + \Omega_\pi \epsilon^{-2} \frac{m_n^2}{k} \psi_n(\pi r_c^+) = 0\,, \tag{42}$$

where now we now define the combinations

$$\Omega_{0,\pi} = \frac{4\alpha k^2/M^2 \pm \gamma_{0,\pi}}{1 - 4\alpha k^2/M^2}\,. \tag{43}$$

The generalizations of the other equations in the main text can now be read off directly, *e.g.*,

$$\zeta_1(x_n) + \Omega_\pi x_n \zeta_2(x_n) = 0\,, \tag{44}$$

and

$$\beta_n = -\frac{J_1(x_n\epsilon) + \Omega_0 x_n \epsilon J_2(x_n\epsilon)}{Y_1(x_n\epsilon) + \Omega_0 x_n \epsilon Y_2(x_n\epsilon)}\,, \tag{45}$$

determine the masses and wavefunctions of the KK states. A short analysis shows that there will exist parameter ranges that are tachyon free. The collider and black hole analyses will now require a detailed exploration of this $\Omega_0 - \Omega_\pi$ parameter space which is beyond the scope of the present paper.

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