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Symmetry relations for heavy-to-light meson form factors at large recoil *

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The description of large-recoil heavy-to-light meson form factors is reviewed in the framework of soft-collinear effective theory. At leading power in the heavy-quark expansion, three classes of approximate symmetry relations arise. The relations are compared to experimental data for $D \to K^*$ and $D_s \to \phi$ form factors, and to light-cone QCD sum rule predictions for $B \to \pi$ and $B \to \rho$ form factors. Implications for the extraction of $|V_{ub}|$ from semileptonic $B \to \rho$ decays are discussed.

1. Introduction

Form factors describing heavy meson decays into energetic light mesons are an essential ingredient for extracting CKM parameters from experimental B decay measurements. The QCD description in this kinematic regime is complicated by the existence of multiple energy scales, and by the competition of different scattering mechanisms. The methods of effective field theory can be used to disentangle the different energy scales, and to provide a systematic heavy-quark expansion. The soft-collinear effective theory (SCET) has been developed to accomplish this task [1,2,3,4,5],[6,7].

This talk reviews the description of form factors in SCET. The following Sec. 2 outlines the representation of weak current operators in the effective theory, and the resulting form factor expressions. Sec. 3 introduces three classes of symmetry relations which emerge from the effective theory at different levels of approximation. These predictions are compared with existing experimental data on D meson decays, and with light-cone QCD sum rules (LCSRs) for B decays. The implications for extracting $|V_{ub}|$ from semileptonic $B \to \rho$ decays are briefly discussed. Sec. 4 comments on possible scaling violations of the form factors relative to the naive q^2 dependence derived from power counting in perturbation theory. A summary is presented in Sec. 5.

2. Form factors in SCET

The decay of a heavy B-meson into an energetic light meson necessarily involves the interplay of soft partons in the heavy meson, with momentum p_s of order $\Lambda_{\rm QCD}$, and "collinear" partons in the light meson, which carry a large energy E in the direction of the outgoing meson but have small virtuality, $p_c^2 \sim \Lambda_{\rm QCD}^2$. The QCD amplitudes are parameterized by matrix elements of local currents, e.g.

$$\begin{aligned}&\langle \pi(p') | \bar{q}\gamma^\mu b | \bar{B}(p) \rangle \\ &\quad = F_+(q^2)(p+p')^\mu + F_-(q^2)(p-p')^\mu \,, \end{aligned} \tag{1}$$

where $q = p - p'$. SCET describes the large-energy/heavy-quark expansion of such quantities in the limit $E \sim m_b \gg \Lambda_{\rm QCD}$.

It is convenient to work with light-cone coordinates, $(n\cdot k, \bar{n}\cdot k, k_\perp)$, where in the B rest frame (with velocity $v^\mu = (1,0,0,0)$), and with the light meson emitted in the $+z$ direction, $n^\mu = (1,0,0,1)$ and $\bar{n}^\mu = (1,0,0,-1)$. Upon integrating out hard momentum modes, $k^2 \gtrsim m_b^2$, QCD currents are matched onto two types of SCET operators relevant at leading order: [2]

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[2]The notation and conventions are as in [8,9]. See also the references before (5).

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$$\bar{q}\,\Gamma\, b = C_i^A(E)\, J_i^A + \int_0^1 du\, C_j^B(E,u)\, J_j^B(u) + \dots\,, \quad (2)$$

where

$$J_i^A = \bar{\mathfrak{X}}(0)\Gamma_i^A h(0)\,,$$
$$J_j^B(u) = \int \frac{ds}{2\pi}\, e^{-iu(2E)s}\bar{\mathfrak{X}}(s\bar{n})\,\mathcal{A}_{\perp\mu}(0)\,\Gamma_j^{B\mu}\, h(0)\,. \quad (3)$$

Here h is the heavy quark field in HQET [10]. $\mathfrak{X}$ and $\mathcal{A}$ are fermion and gluon fields in SCET containing "hard-collinear" momenta, $p_{hc}^2 \sim (p_s + p_c)^2 \sim m_b\Lambda_{\rm QCD}$. In $J^B(u)$, $\mathfrak{X}$ and $\mathcal{A}$ carry fractions u and $1-u$, respectively, of the large-component momentum $\bar{n}\cdot p = 2E$.

Upon taking matrix elements, the A-type SCET currents yield the soft-overlap form factor contributions (evaluated at renormalization scale μ),

$$\frac{\langle M(p')|\bar{\mathfrak{X}}\,\Gamma\, h|\bar{B}_v\rangle}{2E} \equiv -\zeta_M(E,\mu)\ {\rm tr}\left\{\overline{\mathcal{M}}_M(n)\Gamma\mathcal{M}(v)\right\}\,, \quad (4)$$

where $M = P$, $V_\perp$ or $V_\parallel$ denotes a light pseudoscalar or vector meson (with transverse or longitudinal polarization). $\mathcal{M}_M(n)$ and $\mathcal{M}(v)$ are spinor wavefunctions for M and B corresponding to the large-energy and heavy quark limits. The single function $\zeta_M(E)$ in (4) describes all soft-overlap contributions to form factors involving the same final-state meson [11].

The B-type SCET currents in (2) yield factorizable hard-scattering contributions to the form factors. After integrating out hard-collinear modes in a second matching step, these contributions may be expressed in terms of a perturbatively calculable hard-scattering kernel convoluted with light-cone wavefunctions, ϕ_B and ϕ_M, for the heavy and light mesons, respectively.

At leading order in the $\Lambda_{\rm QCD}/m_b$ expansion, the form factors are therefore expressed as the sum of two terms [12,13,14,15]:

$$F_i^{B\to M}(E) = C_i^A(E,\mu)\zeta_M(E,\mu) + \Delta F_i^{B\to M}(E)\,. \quad (5)$$

The hard-scattering term is given by

$$\Delta F_i^{B\to M}(E) = \frac{m_B f_B f_M(\mu)}{8EK_F(\mu)}\int_0^\infty \frac{d\omega}{\omega}\int_0^1 du \times \phi_B(\omega,\mu)\,\phi_M(u,\mu) \times \int_0^1 du' \mathcal{J}_\Gamma\left(u,u',\ln\frac{2E\omega}{\mu^2},\mu\right) C_i^B(E,u',\mu)\,. \quad (6)$$

Here $\mathcal{J}_\Gamma$ is the Wilson coefficient for the second step of matching, with $\mathcal{J}_\Gamma = \mathcal{J}_\parallel$ for $M = P, V_\parallel$ and $\mathcal{J}_\Gamma = \mathcal{J}_\perp$ for $M = V_\perp$ [8]. f_B and f_M are decay constants, and $K_F = 1 + \mathcal{O}(\alpha_s)$ relates the QCD and HQET heavy-meson decay constants [10]. The perturbative expansions of C_i^B and of $\mathcal{J}_\Gamma$ in (6) involve logarithms of the ratios μ/E and $\mu^2/2E\omega$, with $\omega \sim \Lambda_{\rm QCD}$, so that large logarithms are unavoidable in fixed-order perturbation theory. These logarithms may be resummed using renormalization-group (RG) methods [1,16,17,8].

3. Symmetry relations

Eqs. (5) and (6) can be used to relate any two $B\to M$ form factors. [3] Given the meson LCDAs, these relations are perturbatively calculable up to $\Lambda_{\rm QCD}/m_b$ corrections. However, since the B-meson LCDA is poorly constrained at present, it is useful to find relations which are independent of detailed assumptions for $\phi_B(\omega)$. Three types of relations arise, which for descriptive purposes will be called "first-", "second-" and "third-class".

Two relations describing $B\to V_\perp$ decays hold to all orders in α_s [8,12,18],

First class relations:

$$A_1(q^2) = \left(1-\hat{q}^2\right)\left(1+\hat{m}_V\right)^{-2} V(q^2)\,, \quad (7)$$
$$T_2(q^2) = \left(1-\hat{q}^2\right) T_1(q^2)\,.$$

Here $q \equiv p - p'$ for the $B(p)\to M(p')$ transition, and hatted variables are in units of m_B: $\hat{m}_V = m_V/m_B$, $\hat{q}^2 = q^2/m_B^2$, etc. Kinematic factors linear in the light mass are retained, and $q^2 = m_B^2 - 2Em_B + \mathcal{O}(m_V^2)$ is used to express the form factors as a function of q^2.

[3] Form factor conventions are as in [12,8].

At tree level the coefficients C_i^B in (6) are independent of momentum fraction u'. When radiative corrections at the hard scale are neglected, the convolutions then yield a universal function $H_M(E)$ [8],

$$\Delta F_i^{B\to M}(E) \approx -\left(\frac{m_B}{2E}\right)^2 H_M(E)\, C_i^{B(\mathrm{tree})}(E)\,. \tag{8}$$

Eq. (8) is exact to all orders in the perturbative expansion of the jet function $\mathcal{J}_\Gamma$, neglecting only hard-scale radiative corrections in C_i^B. In this approximation, the two hadronic functions, ζ_M and H_M, may be eliminated to yield relations between the three $B\to P$ and $B\to V_\parallel$ form factors,

Second class relations:

$$\begin{aligned} &F_+(q^2) = F_0(q^2) + (1+\hat{m_P})^{-1}\,\hat{q}^2 F_T(q^2)\,,\\ &(1+\hat{m}_V)^{-1}\left[V(q^2)-A_2(q^2)\right]\\ &\quad = 2\hat{m}_V A_0(q^2) + \hat{q}^2\left[T_1(q^2)-T_3(q^2)\right]\,. \end{aligned} \tag{9}$$

Finally, neglecting the hard-scattering terms ΔF_i in (5) altogether yields the "large-energy" symmetry relations obeyed by the soft-overlap terms [11],

Third class relations:

$$\begin{aligned} F_+(q^2) &= \left(1-\hat{q}^2\right)^{-1} F_0(q^2)\\ &= (1+\hat{m_P})^{-1} F_T(q^2)\,,\\ A_0(q^2) &= \left(1-\hat{q}^2\right)(2\hat{m}_V)^{-1}(1+\hat{m}_V)^{-1}\\ &\quad\times\left[V(q^2)-A_2(q^2)\right]\\ &= (2m_V)^{-1}\left(1-\hat{q}^2\right)\left[T_1(q^2)-T_3(q^2)\right]\,,\\ T_1(q^2) &= (1+\hat{m}_V)^{-1}\,V(q^2)\,. \end{aligned} \tag{10}$$

All relations in (7), (9) and (10) are expected to receive $\mathcal{O}(\Lambda_{\mathrm{QCD}}/E)$ corrections, of order $10-20\%$. Symmetry-breaking corrections in (9) and (10) due to C_i^A could be included trivially, but this effect is $\lesssim 5\%$ in all cases [12,9]. From current estimates of the hard-scattering terms, $H/\zeta \sim 0.1-0.2$, [4] radiative corrections to C_j^B also have very little effect on symmetry relations. The second-class relations (9) should then hold with similar accuracy to the first-class ones (7), whereas the third-class relations (10) ignore the hard-scattering terms entirely and so may receive larger corrections.

[4]Estimates are based on LCSR form factor predictions (cf. Sec. 3.3), or directly from sum rule analyses of the B-meson wavefunction [19,20].

Table 1
Experimental values for form factor ratios r_V, r_2 and r_3 (see text), taken from [21].

	$D\to K^*$	$D_s\to\phi$
r_V	1.62 ± 0.08	1.92 ± 0.32
r_2	0.83 ± 0.05	1.60 ± 0.24
r_3	$0.04\pm 0.33\pm 0.29$	—

3.1. D decays

D mesons, to the extent that the heavy quark expansion in $\Lambda_{\mathrm{QCD}}/m_c$ is valid, can be analyzed in precisely the same way as B mesons. Table 1 lists current experimental data for $r_V \equiv V(0)/A_1(0)$, $r_2 \equiv A_2(0)/A_1(0)$ and $r_3 \equiv \tilde{A}_3(0)/A_1(0)$. [5] For instance, r_V is determined by a first-class relation, (7), which in terms of energy becomes,

$$2\hat{E}\,V(E) = (1+\hat{m_V})^2\,A_1(E)\,. \tag{11}$$

A similarity in measured values r_V for $D\to K^*$ and $D_s\to\phi$ could suggest that this ratio is determined largely by kinematic factors, as would be the case if a relation such as (11) were valid. In contrast, the ratio r_2 appears to exhibit a large $SU(3)$ symmetry-breaking. At $q^2=0$ for $D\to K^*$ decay, (11) yields

$$r_V^{D\to K^*} \approx (m_D+m_{K^*})^2/(m_D^2+m_{K^*}^2) = 1.78\,. \tag{12}$$

The apparent agreement of (12) with the experimental value was counted as an early success of the large-energy symmetry relations [11]. However, before taking this agreement seriously, it is important to realize that quadratic meson-mass effects shift the prediction considerably. For instance, directly from the definitions of form factors V and A_1 in terms of QCD matrix elements,

[5]$\tilde{A}_3 \equiv A_2/2 + \hat{m}_V(1+\hat{m}_V)(A_0-A_3)/\hat{q}^2$, with $2\hat{m}_V A_3 = (1+\hat{m}_V)A_1 - (1-\hat{m}_V)A_2$. This form factor contributes only when lepton masses are relevant.

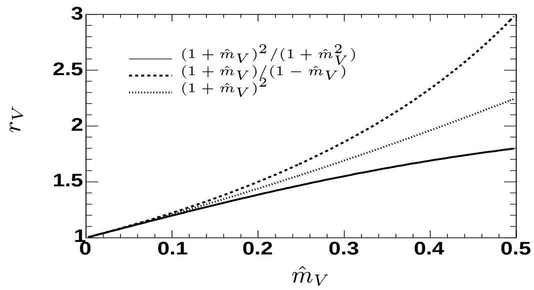

Figure 1. Predictions for r_V using the relations $(1+\hat{m}_V)^2 A_1/V = \{\, 2\hat{E}$ (solid), $2\hat{k}_V$ (dashed), $\hat{E}+\hat{k}_V$ (dotted) $\}$.

it is most natural to assume the symmetry relation,

$$2\hat{k}_V V(q^2) = (1+\hat{m}_V)^2 A_1(q^2)\,, \tag{13}$$

where $k_V \equiv \sqrt{E^2 - m_V^2} = (\bar{n}\cdot p' - n\cdot p')/2$ is the 3-momentum of the light meson. In this case,

$$r_V^{D\to K^*} \approx (m_D + m_{K^*})/(m_D - m_{K^*}) = 2.81\,. \tag{14}$$

Alternatively, it is convenient to express SCET quantities in terms of the large momentum component, $\bar{n}\cdot p' = E + k_V$. [6] The quantities E, k_V, and $(E+k_V)/2$ differ by terms of order m_V^2/E^2, and coincide in the large-energy limit. As shown in Fig. 1, the resulting difference in symmetry predictions for r_V is very small for B decays to light mesons, e.g. at $\hat{m}_V = m_\rho/m_B = 0.15$, but is large for typical D decays, e.g. at $\hat{m}_V = m_{K^*}/m_D = 0.48$. Unless such effects can be reliably accounted for, the application of large-energy symmetry relations to D decays appears problematic. This issue could be further investigated experimentally, for instance by measuring $r_V(q^2) \equiv V(q^2)/A_1(q^2)$ and comparing to the q^2 dependences predicted by (11) or (13).

SCET symmetry relations may also be used to relate B and D decay form factors involving the same final state meson. For example, in decays to pseudoscalar mesons, at the level of second-class relations,

$$\frac{F_+^{B\to P}}{F_+^{D\to P}} = \sqrt{\frac{m_B}{m_D}}\,\frac{\hat{\zeta}_P + \left(\frac{4E}{m_B}-1\right)\hat{H}_P}{\hat{\zeta}_P + \left(\frac{4E}{m_D}-1\right)\hat{H}_P}\,. \tag{15}$$

[6] In [8,9], $\bar{n}\cdot p'$ is simply denoted by $2E$.

Here $\hat{\zeta}_M$ and $\hat{H}_M$ are quantities independent of the heavy-quark mass,

$$\zeta_M \equiv \sqrt{m_B}\hat{\zeta}_M\,, \quad \left(\frac{m_B}{2E}\right)^2 H_M \equiv \sqrt{m_B}\hat{H}_M\,. \tag{16}$$

Similarly, for decays to vector mesons,

$$\begin{aligned} \frac{V^{B\to V}}{V^{D\to V}} &= \sqrt{\frac{m_D}{m_B}}\frac{m_B+m_V}{m_D+m_V}\,,\\ \frac{A_1^{B\to V}}{A_1^{D\to V}} &= \sqrt{\frac{m_B}{m_D}}\frac{m_D+m_V}{m_B+m_V}\,,\\ \frac{(V-A_2)^{B\to V}}{(V-A_2)^{D\to V}} &= \\ \sqrt{\frac{m_D}{m_B}}\frac{m_B+m_V}{m_D+m_V}&\frac{\hat{\zeta}_{V_\parallel} + \left(\frac{4E}{m_B}-1\right)\hat{H}_{V_\parallel}}{\hat{\zeta}_{V_\parallel} + \left(\frac{4E}{m_D}-1\right)\hat{H}_{V_\parallel}}\,. \end{aligned} \tag{17}$$

Relations (15) and (17) hold at the same value of the light meson energy E, or equivalently at the same value of the recoil parameter $v\cdot v' = E/m_M$, where $p' = m_M v'$. These are generalizations of corresponding relations in HQET, which counts $v\cdot v'$ as order unity; when hard-scattering corrections H_M are neglected, the leading order predictions of HQET [22] are recovered. In contrast, the SCET power counting allows $v\cdot v' = E/m_M$ to be a large parameter. This is not very relevant for D decays to vector mesons, e.g. $v\cdot v' < 1.3$ in $D\to K^*$ decays, but is important near maximum recoil in $D\to\pi$ decays, where $v\cdot v' \approx 6$.

It is possible that the heavy-quark symmetries relating B and D decays (17) might still be valid, even if the large-energy symmetry predictions applied directly to the D system, as in (11) and (13), are not useful. Using LCSR predictions for B and B_s decays [23], and neglecting hard-scattering terms, the predictions from (17) for the ratios r_V, r_2 and r_3 for $D\to K^*$ are 1.71, 0.75 and 0.38, respectively, while the predictions for the ratios r_V and r_2 for $D_s\to\phi$ are 1.74 and 0.87. These numbers are expected to receive $\sim 30\%$ corrections proportional to $1/m_c$, in addition to the $\gtrsim 15\%$ uncertainties from the B decay form factors. The agreement with experimental values in Table 1 is reasonable with the possible exception of $r_2^{D_s\to\phi}$. It would be interesting to test these relations more precisely when more data becomes available.

3.2. Semileptonic branching fractions

The differential rate for semileptonic B decay to a vector meson (e.g. $V = \rho$) is, neglecting lepton masses,

$$\frac{d\Gamma(\bar{B}^0 \to V^+ l^- \bar{\nu})}{d\hat{q}^2 d\cos\theta} = |V_{ub}|^2 \frac{G_F^2 m_B^5 \hat{k}_V \hat{q}^2}{128\pi^3} \times \left[(1-\cos\theta)^2 \frac{|H_+|^2}{2m_B^2} + (1+\cos\theta)^2 \frac{|H_-|^2}{2m_B^2} + \sin^2\theta \frac{|H_0|^2}{m_B^2} \right], \quad (18)$$

where θ is the angle between the charged lepton in the virtual W rest frame, and the direction of the W in the B rest frame. The helicity amplitudes may be expressed in terms of form factors,

$$\frac{H_\pm}{m_B} = \frac{(1+\hat{m}_V)^2 A_1 \mp 2\hat{k}_V V}{1+\hat{m}_V}, \quad \frac{H_0}{m_B} = \frac{(1+\hat{m}_V)^2 \left(\hat{E} - \hat{m}_V^2\right) A_1 - 2\hat{k}_V^2 A_2}{\hat{m}_V (1+\hat{m}_V)\sqrt{\hat{q}^2}}, \quad (19)$$

where as usual, hatted variables are in units of m_B. H_+ vanishes at leading order in $\Lambda_{\rm QCD}/E$, by the first-class symmetry relation (13). [7] In the large energy limit, the remaining helicity amplitudes are [8]

$$\frac{H_-}{m_B} = 2\left(1-\hat{q}^2\right)\zeta_{V_\perp}, \quad \frac{H_0}{m_B} = \frac{1}{\sqrt{\hat{q}^2}}\left(1-\hat{q}^2\right)\zeta_{V_\parallel}, \quad (20)$$

so that the differential rate satisfies

$$\frac{d\Gamma(B\to V l\nu)}{d\hat{q}^2} \propto (1-\hat{q}^2)^3 \int_{-1}^{1} d\cos\theta \times \left[2(1+\cos\theta)^2 \hat{q}^2 |\zeta_{V_\perp}|^2 + \sin^2\theta |\zeta_{V_\parallel}|^2 \right] = (1-\hat{q}^2)^3 \left[4\hat{q}^2 |\zeta_{V_\perp}|^2 + |\zeta_{V_\parallel}|^2 \right]. \quad (21)$$

It is apparent from (21) that without angular discrimination, the contribution from $|\zeta_{V_\perp}|^2$ is suppressed relative to that from $|\zeta_{V_\parallel}|^2$ at $\hat{q}^2 \lesssim 0.25$, or $q^2 \lesssim 7\,{\rm GeV}^2$. Unfortunately, it is $\zeta_{V_\perp}$ that can be most cleanly constrained by other measurements. For example, the value of $\zeta_{V_\perp}$ at $q^2 = 0$ can be related to the $B \to \rho\gamma$ branching fraction. [9] Reducing the uncertainty due to $\zeta_{V_\parallel}$ requires a restriction either to small θ or to larger values of q^2. The latter case, using $\hat{q}^2 \sim 0.6 - 0.7$, has been proposed to extract $|V_{ub}|$ using constraints imposed by $D \to K^*$ and $D \to \rho$ decays [24], although a good understanding of power corrections is required for this approach to yield a precision measurement.

Using the naive scaling prediction $\zeta_{V_\perp} \propto \zeta_{V_\parallel} \propto (1-\hat{q}^2)^{-2}$ in (21) gives some indication of the small q^2 behavior of $d\Gamma/d\hat{q}^2$:

$$\frac{d\Gamma}{d\hat{q}^2} \propto 1 + \left(4\frac{|\zeta_{V_\perp}|^2}{|\zeta_{V_\parallel}|^2} - 1 \right) \hat{q}^2 + \dots, \quad (22)$$

where $|\zeta_{V_\perp}|^2/|\zeta_{V_\parallel}|^2$ is evaluated at $q^2 = 0$. As discussed in the next section, LCSRs suggest that this ratio is close to unity, but also that the scaling of ζ could receive significant corrections. Determining this residual q^2 dependence from first principles is an important problem. For example, when combined with the extraction of $\zeta_{V_\perp}(q^2=0)$ from $B \to \rho\gamma$, it could provide a largely model-independent determination of $|V_{ub}/V_{td}V_{tb}|$ [25].

3.3. Light cone sum rules

LCSRs give predictions for all $B \to \pi$ and $B \to \rho$ form factors at large recoil, and it is interesting to compare these predictions with the symmetry relations (7), (9) and (10). Figs. 2, 3 and 4 show the LCSR predictions for $B \to P$, $B \to V_\parallel$ and $B \to V_\perp$ decay form factors, respectively, using the results of [23,26]. Tensor form factors are evaluated at renormalization scale $\mu = m_b = 4.8\,{\rm GeV}$. Normalizations are chosen such that at leading power, $F_i^{B\to M} = \zeta_M$ up to hard-scattering and radiative corrections. Ac-

[7] Note that (13) is the "exact" form of the symmetry relation (cf. the discussion in Sec. 3.1). In the same way, H_0 involves the "exact" version of the combination proportional to $V - A_2$.

[8] Corrections to the large-energy limit can in principle be computed, but for simplicity they are not included in the following discussion.

[9] If corrections to $SU(3)$ symmetry can be brought under control, the branching fractions for $B \to K^*\gamma$ and $B \to K^* l^+ l^-$ could also be used to obtain both $\zeta_{V_\perp}$ and $\zeta_{V_\parallel}$.

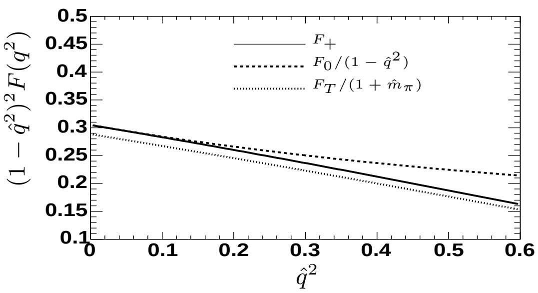

Figure 2. LCSR predictions for $B \to \pi$ form factors, taken from [26]

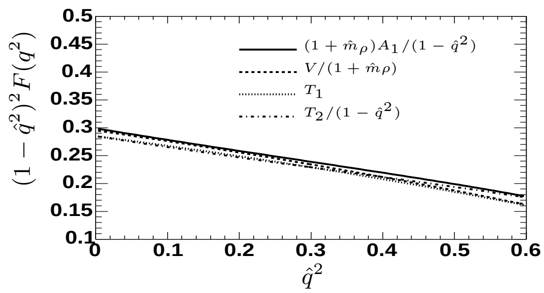

Figure 4. LCSR predictions for $B \to \rho_\perp$ form factors, taken from [23].

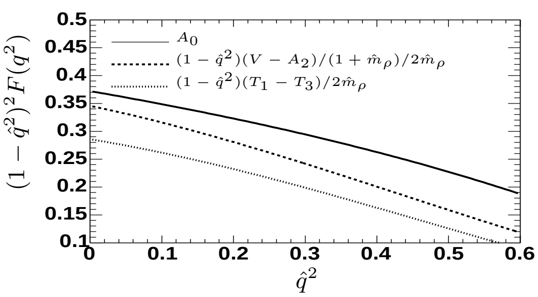

Figure 3. LCSR predictions for $B \to \rho_\parallel$ form factors, taken from [23].

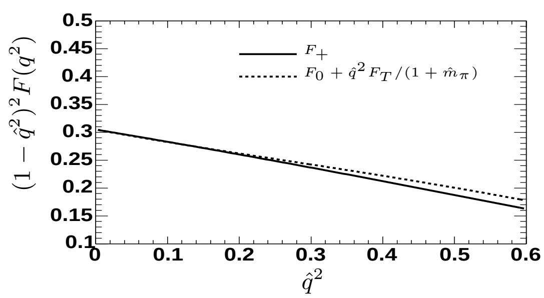

Figure 5. LCSR predictions for $B \to \pi$ form factors obeying second-class symmetry relations.

cording to the power counting obtained in perturbation theory, the soft form factors scale as $1/E^2 \propto (1-\hat{q}^2)^{-2}$, and this factor is extracted in the plots.

For the cases $B \to P$ and $B \to V_\parallel$, the form factors obey only third-class relations, so that the deviation of the curves is a measure of hard-scattering terms. In particular, since the hard-scattering corrections to A_0 and $V - A_2$ are positive, while that to $T_1 - T_3$ is negative [8], the deviations from the symmetry relations may be particularly large for these curves. For the $B \to V_\perp$ case, both A_1 and V, and T_1 and T_2, obey first-class symmetry relations, whereas A_1 and T_1 obey only third-class relations.

Figs. 5 and 6 show the form factors in (9) obeying second-class symmetry relations. Comparison of the corresponding curves in Figs. 2 and 5, and in Figs. 3 and 6, shows that in both cases the second-class relations are satisfied more accurately than the third-class relations, as expected.

4. Scaling violations

The LCSR results in Figs. 2 - 6 show a significant deviation from the $1/E^2$ dependence predicted from naive power counting. In fact, using first and second class relations to isolate the soft-overlap terms, the sum rule results give in all cases, $d(\ln\zeta)/d\hat{q}^2 \approx 1.3$ at $q^2 = 0$, to be compared with the naive scaling prediction of $d(\ln\zeta)/d\hat{q}^2 = 2$.

Perturbative contributions to scaling violations can be investigated by considering the RG evolution equation [1,16,15],

$$\frac{d\ln\zeta(E,\mu)}{d\ln\mu} = -\Gamma_{\rm cusp}(\alpha_s)\ln\frac{2E}{\mu} - \tilde{\gamma}(\alpha_s)\,, \quad (23)$$

where at one-loop order, $\Gamma_{\rm cusp}(\alpha_s) = C_F\alpha_s/\pi$ and $\tilde{\gamma}(\alpha_s) = -5C_F\alpha_s/4\pi$. The solution of (23)

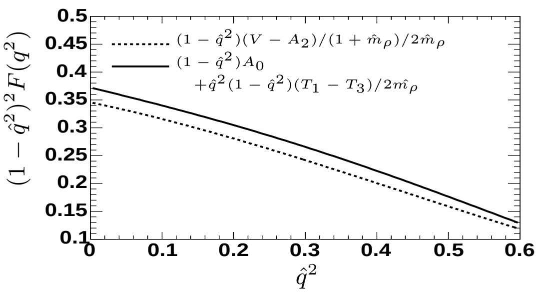

Figure 6. LCSR predictions for $B \to \rho_\parallel$ form factors obeying second-class symmetry relations.

relates $\zeta(E,\mu)$ at different scales,

$$\begin{aligned}\frac{\zeta(E,\mu)}{\zeta(E,\mu_0)} &= \left(\frac{2E}{\mu}\right)^{a(\mu,\mu_0)} \\ &\times \exp\left[S(\mu,\mu_0) - \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\tilde{\gamma}(\alpha_s(\mu'))\right] ,\end{aligned} \tag{24}$$

where

$$\begin{aligned} a(\mu,\mu_0) &= -\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\rm cusp}(\alpha_s(\mu'))\,, \\ S(\mu,\mu_0) &= -\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\rm cusp}(\alpha_s(\mu'))\ln\frac{\mu}{\mu'}\,. \end{aligned} \tag{25}$$

For instance, up to hard-scale radiative corrections, $\zeta(E,\mu=2E)$ describes the soft-overlap part of the physical form factors, [10] and may be related to $\zeta(E,\mu_0)$, for a lower, energy-independent, scale μ_0 (say $\mu_0 = 1\,\text{GeV}$). The slope at $q^2=0$ then satisfies:

$$\begin{aligned} &\frac{d}{d\hat{q}^2}\ln\zeta(E,\mu=2E)\bigg|_{q^2=0} - \frac{d}{d\hat{q}^2}\ln\zeta(E,\mu_0)\bigg|_{q^2=0} \\ &= \int_{\mu_0}^{m_b}\frac{d\mu}{\mu}\Gamma_{\rm cusp}(\alpha_s(\mu)) + \tilde{\gamma}(\alpha_s(m_b)) \\ &\approx 0.28 - 0.13\,, \end{aligned} \tag{26}$$

where the last line is evaluated at $\mu_0 = 1\,\text{GeV}$. The first term on the right-hand side of (26) is positive for all $\mu_0 < m_b$, while the second is independent of μ_0. Any large deviation of the form factor slope from the naive scaling prediction, particularly any negative correction, must therefore arise from the nonperturbative function $\zeta(E,\mu_0)$.

The situation is analogous to that for heavy-heavy meson transitions at large recoil. At leading order in the heavy-quark expansion, $B \to D$ form factors are described by the single Isgur-Wise function,

$$\frac{\langle D_{v'}|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|B_v\rangle}{\sqrt{m_B m_D}} = -\xi(v\cdot v',\mu)\,{\rm tr}\left\{\overline{\mathcal{M}}(v')\Gamma\mathcal{M}(v)\right\} , \tag{27}$$

which obeys the evolution equation,

$$\frac{d\ln\xi(v\cdot v',\mu)}{d\ln\mu} = -\Gamma_{\rm cusp}(\phi,\alpha_s)\,. \tag{28}$$

Here $\Gamma_{\rm cusp}(\phi,\alpha_s)$ is the universal cusp anomalous dimension [27], and $\phi = \text{arccosh}(v\cdot v')$ is the angle between initial and final meson velocities. At one loop order, $\Gamma_{\rm cusp}(\phi,\alpha_s) = (C_F\alpha_s/\pi)\,(\phi\coth\phi - 1)$. [11] Considering large recoil $v\cdot v' \sim m_b/\Lambda_{\rm QCD} \gg 1$, the Isgur-Wise function ξ behaves similarly to the soft SCET form factor ζ. In particular, $\xi \propto (v\cdot v')^{-2}$ up to logarithms [19]. [12] Perturbative scaling violations may again be calculated using (28), but a nonperturbative dependence on $v\cdot v'$ remains in $\xi(v\cdot v',\mu_0)$.

5. Summary

Symmetry relations in the heavy-quark/large-energy limit for heavy-to-light meson form factors provide a valuable handle on otherwise poorly understood hadronic parameters. Application of

[10] In fact, taking the hard scale $\mu = 2E$ (as opposed to $\mu = m_b$), the remaining dependence of the Wilson coefficients on energy is very mild [9], so that e.g. $\zeta_P(E,\mu=2E)$ should accurately describe the energy-dependence of the soft-overlap contribution to F_+.

[11] At large values of the cusp angle, the coefficient of $\ln v{\cdot}v'$ is defined to be the angle-independent cusp anomalous dimension [28], $\Gamma_{\rm cusp}(\alpha_s)$. In the heavy-light case, the cusp angle becomes infinite, and the resulting RG equation takes the form (23), with a non-universal, but energy-independent, anomalous dimension $\tilde{\gamma}$.

[12] Also in this case, competing "hard-scattering" terms enter at the same order. At still larger recoil $v\cdot v' \gg m_b/\Lambda_{\rm QCD}$, the hard-scattering terms dominate.

the symmetry relations to D-meson decays, especially into vector mesons, may be problematic due to large meson-mass effects; further experimental investigation could clarify this. Such effects may largely cancel in ratios relating $B \to M$ to $D \to M$ transitions at the same light-meson energy. Here SCET generalizes corresponding HQET relations to allow for large recoil energy, and would be important for relating the form factors near maximum recoil for the semileptonic $D \to \pi$ decay.

The analysis of semileptonic $B \to \rho$ decay becomes especially simple in the large-energy limit, where only two helicity amplitudes contribute. Extraction of the remaining form factors from processes such $B \to K^*\gamma$ or $B \to K^* l^+ l^-$ (with an understanding of $SU(3)$ violations) or from $B \to \rho\gamma$ (with an understanding of the q^2 dependence of form factors) have potential to provide useful CKM constraints.

A comparison of the symmetry relations to LCSR predictions shows no sign of symmetry-breaking power corrections beyond the 10% level. In addition, the LCSRs show a large correction to the perturbative scaling behavior of the soft form factors. A better understanding of such scaling violations from first principles is an important problem, as is a study of power corrections for exclusive decays in the large-energy limit. Improved measurements coming from B decays at BaBar and Belle, and D decays at CLEO-c, will provide numerous tests and applications of the SCET predictions.

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