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A model describing stable coherent synchrotron radiation in storage rings

F. Sannibale,* J. M. Byrd, Á. Loftsdóttir,† and M. Venturini
Lawrence Berkeley National Laboratory, One Cyclotron Road, Berkeley, California 94720, US

M. Abo-Bakr, J. Feikes, K. Holldack, P. Kuske, and G. Wüstefeld
BESSY mbH, Albert-Einstein-Strasse 15, 12489 Berlin, Germany

H.-W. Hübers
DLR, Rutherford Strasse 2, 12489 Berlin, Germany

R. Warnock
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
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We present a model describing high power stable broadband coherent synchrotron radiation (CSR) in the terahertz frequency region in an electron storage ring. The model includes distortion of bunch shape from the synchrotron radiation (SR), which enhances higher frequency coherent emission, and limits to stable emission due to an instability excited by the SR wakefield. It gives a quantitative explanation of several features of the recent observations of CSR at the BESSY II storage ring. We also use this model to optimize the performance of a source for stable CSR emission.

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Coherent synchrotron radiation (CSR) occurs when the electrons in a bunch emit synchrotron radiation (SR) in phase. Coherent radiation intensity is proportional to the square of the number of particles per bunch in contrast to the linear dependence of the usual incoherent radiation. Since the number of particles per bunch is typically very large ($\gtrsim 10^6$), the potential intensity gain for a CSR source is huge.

Although the possibility of CSR in circular accelerators was discussed over 50 years ago [1], it is only recently that observations have been reported. Bursts of coherent infrared radiation (IR) with a stochastic character were seen at several synchrotron light source storage rings [2–8]. Of remarkable importance is the first observation of steady state CSR when the BESSY II storage ring was tuned into a special low momentum compaction mode [9, 10]. The combination of this last experimental result and the first successful application of BESSY II as a CSR source [11], is receiving much attention among synchrotron light users and accelerator physicists because of the concrete possibility of constructing a stable broadband source with extremely high power in the terahertz domain. In fact, BESSY II now has regular scheduled shifts dedicated to CSR production, other laboratories are investigating the possibility of a CSR mode of operation [12, 13], and a design for a new ring optimized for CSR is at an advanced stage [14].

No physical explanation for the BESSY II experimental results was available. In this letter, we present a model that for the first time accounts for the BESSY II observations and provides a novel scheme for predicting and optimizing the performance of ring-based CSR sources with a broadband photon flux in the terahertz region of up to ~ 9 orders of magnitude larger than in existing 'conventional' storage rings. Stability of the photon flux is also a fundamental requirement for such a source. Our scheme incorporates a simple stability criterion that was previously used to account for IR bursts [8, 15–17]. The model is of interest not only for the design of new sources but also for the evaluation and optimization of the CSR performance in any existing electron storage ring. Another quite general result is that in the short bunch regime ($\lesssim 1$ ps), the collective effects induced by the SR become dominant.

Before describing the model we need some introduction. The SR power spectrum is given by [1, 18]:

$$\frac{dP}{d\lambda} = \frac{dp}{d\lambda}[N + N(N-1)g(\lambda)] \tag{1}$$

where λ is the wavelength of the radiation, p is the single particle radiated power, N is the number of particles per bunch and g is the CSR form factor, the absolute square of the Fourier transform of the normalized bunch distribution. Here $dp/d\lambda$ is defined to account for shielding due to the conductive vacuum chamber. The shielding effect has been studied by several authors over many years [1, 19–22]. A salient feature is that $dp/d\lambda$ drops off abruptly for λ greater than the shielding cutoff wavelength λ_0, which is estimated to be about $2h\,(h/\rho)^{1/2}$, where h is the chamber height and ρ is the dipole bending radius. The first term in Eq. (1), linear in N, is the incoherent component of the power. The second term, pro-

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*Electronic address: FSannibale@LBL.gov
†Also at Department of Physics, University of California, Davis, Davis, California 95616.

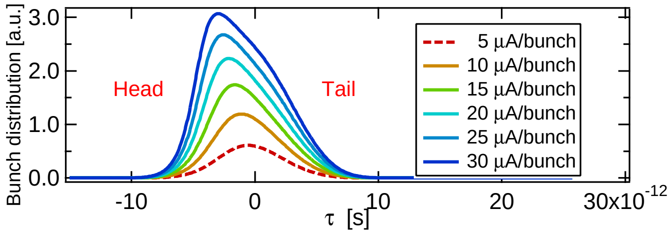

FIG. 1: Calculated equilibrium longitudinal distribution for different currents per bunch using the shielded SR wake. BESSY II case with a natural bunch length of 2.5 ps.

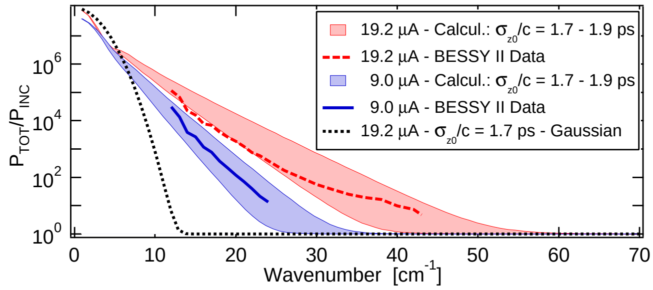

FIG. 2: CSR gain as a function of the wavenumber $1/\lambda$. The BESSY II data for two different currents per bunch are compared with the shielded SR calculation and with the curve for a Gaussian distribution of the same length.

portional to $N^2 g(\lambda)$, represents a potentially much larger coherent component. Thus, to have significant CSR at the wavelength of interest we must have $g(\lambda) > 1/N$ and also $\lambda < \lambda_0$. For the particular case of a Gaussian distribution, CSR occurs for $2\pi\sigma_z/\sqrt{\ln N} \lesssim \lambda < \lambda_0$, with σ_z the rms bunch length. In the general case, comparing at short wavelengths bunches with same rms length, sharp-edged distributions yield a g factor significantly larger than Gaussian ones. This implies that these 'distorted' bunches can have CSR emission at wavelengths remarkably shorter than in the purely Gaussian case. We have found that the BESSY II results show this pattern. In fact, streak camera measurements [23] showed, at $N \sim 2\times10^8$, asymmetric distributions with hard leading edge and with bunch length of ~ 1 mm while CSR was clearly visible down to ~ 200 μm [10].

Low current equilibrium longitudinal distributions in electron storage rings are usually Gaussian. In order to understand the BESSY II results we have systematically considered all the several possible phenomena generating bunch distortions and evaluated them. Such phenomena are classified in two main categories, nonlinear dynamics and collective effects. Radio frequency (RF) and lattice nonlinearities belong to the first group, while SR and vacuum chamber wakes fall in the second one.

For most storage rings, including BESSY II, RF nonlinearities are very small and can be neglected. Our extensive simulations showed that the distortion effects due to lattice nonlinearities are negligible as well.

In the analysis of the collective effects we start first with the wakes due to the SR in a bending magnet, and later consider the ones associated with the vacuum chamber. For the SR wake we use the analytical expressions where the vacuum chamber shielding is represented by the parallel plates model [19–22].

To find the equilibrium longitudinal bunch distribution in the presence of wakes we used the well known Haïssinski equation [24]. Figure 1 shows the numerical solutions of this equation for the case of BESSY II for different currents per bunch and using the shielded SR wake. The results show several interesting features: with growing current per bunch, the distribution leans forward with an increasingly sharp leading edge, the bunch rear becomes less steep, the bunch centroid shifts to earlier times (synchronous phase shift) and the rms length of the distribution slightly increases. The calculated asymmetric distributions with sharper leading edge are in qualitative agreement with the BESSY II streak camera measurements.

For a more quantitative comparison with the BESSY results, we can define for a given current and wavenumber, the CSR gain as the ratio between the radiation intensities when CSR is present and when the emission is completely incoherent. Figure 2 shows the CSR gain measured at BESSY II [10] and the calculated distributions as a function of the radiation wavenumber for two different currents per bunch. Also shown is the calculated CSR gain for the undistorted Gaussian distribution. The shaded areas represent the shielded SR calculations obtained by varying the natural bunch length over a 10% range. This choice can be explained as follows. The natural bunch length used as input parameter for the simulations was derived from measurements of the synchrotron frequency, of the RF voltage and of other machine quantities. The experimental error for this evaluation is consistent with a 10% uncertainty. The comparison in Fig. 2 shows the general good agreement between calculations and data and also the strong power enhancement at the higher wavenumbers that the distorted case presents with respect to the Gaussian one. In Ref. [10] a 40 μA curve is also shown, but the data are unusable for comparison because of bursting, this current being above the SR induced instability threshold [8, 15–17]. The measured data present a more irregular shape if compared with the calculations. We think that the systematic errors in the measurements are responsible for that.

Let us now introduce in our analysis the vacuum chamber wakes starting with the resistive wall (RW) one [25, 26]. Figure 3 shows, for a particular case of BESSY II, the comparison between the CSR gain curves calculated using the shielded SR wake with (dotted line) and without (dashed line) the inclusion of the RW wake. The effect of the RW wake is clearly very small and slightly decreases the CSR gain. Additional calculations using dif-

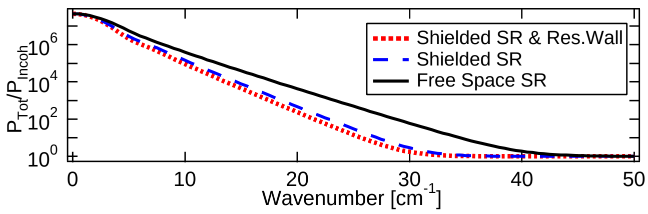

FIG. 3: CSR gain vs. radiation wavenumber calculated using the shielded SR wake with (dotted line) and without (dashed line) the resistive wall wake. The solid line shows the gain calculated with the free space SR wake. Case of BESSY II with 9.0 μA per bunch and 1.7 ps natural bunch length.

ferent models for the BESSY II vacuum chamber broadband impedance showed a totally negligible contribution from this term.

The solid line in Fig. 3 shows the CSR gain calculated using the free space (FS) SR wake [21, 22] for the interesting case where the vacuum chamber shielding is negligible. Compared to the FS case, the vacuum chamber shielding reduces the gain significantly, pointing out the important result that for maximizing the CSR gain in an optimized source the shielding effect must be kept negligible. For the case of the parallel plates model and Gaussian bunches, a simple criterion was derived [22]:

$$\Sigma = \frac{\sigma_z}{h}\left(\frac{2\rho}{h}\right)^{1/2} \lesssim 0.2 \qquad (2)$$

If $\Sigma \lesssim 0.2$, then the shielding effect is negligible and the FS SR wake can be used (note that $\Sigma \propto \sigma_z/\lambda_0$). Two examples: in the case of BESSY II with $\sigma_z \sim 1$ mm, $\rho = 4.35$ m and $h = 3.5$ cm, $\Sigma \sim 0.45$ and the shielding effect is relevant, while in a hypothetical but realistic CSR source with $\sigma_z = 300$ μm, $h = 4$ cm and $\rho = 1.33$ m, $\Sigma \sim 0.06$ and the shielding is negligible.

For a ring-based source of CSR, stability is a fundamental requirement for most applications. In particular, the mentioned SR induced instability [8, 15–17] generates IR bursts that if not controlled can jeopardize the performance of the source. In the following, we derive a novel scheme that allows to adjust the parameters of a ring such that the CSR power and bandwidth are maximized while remaining below the threshold of the SR induced instability.

Assuming that criterion (2) is fulfilled, we can use the FS SR wake and following the approach used in ref. [27] we express the bunch population N as:

$$N = A\,(B/E)^{1/3} f_{RF} V_{RF} \sigma_{z0}^{7/3}\, F(\kappa)\ , \qquad (3)$$

where $A = 6.068\times10^{-4}$ [SI units], B is the dipole magnet magnetic field, E the beam energy, f_{RF} is the storage ring RF, V_{RF} is the RF peak voltage and σ_{z0} is the natural bunch length. The numerical factor $F(\kappa) = \int y(x)dx$ is the integral of the solution of the equilibrium equation [27]:

$$y(x) = \kappa \exp\left[-x^2/2 + \mathrm{sgn}(\alpha)\int_0^\infty y(x-z) z^{-1/3} dz\right], \qquad (4)$$

which is a dimensionless form of the Haïssinski equation for the special case of the FS wake, with $x = c\tau/\sigma_{z0}$, τ the distance in time from the synchronous particle and c the speed of light. The factor $\mathrm{sgn}(\alpha)$ is the sign of the momentum compaction α. The advantage of F is that it depends only on the dimensionless normalization parameter κ. As κ increases, F, N, and the bunch distortion all increase. Using Eq. (1) for $Ng(\lambda) \gg 1$ and the expression for $dp/d\lambda$ when the wavelength is shorter than λ_0 but much larger than the SR critical wavelength (see for example [28]) we can write the power spectrum for a ring of length L and N_b bunches as:

$$\frac{dP}{d\lambda} = \frac{CN_b}{L}(f_{RF}V_{RF})^2\left(\frac{B}{E}\right)^{1/3}\left(\frac{\sigma_{z0}^2}{\lambda}\right)^{7/3} F(\kappa)^2 g(\lambda)\ , \qquad (5)$$

where $C = 2.642\times10^{-21}$ [SI units].

To optimize the intensity and spectral bandwidth given by (5), we must first be sure that the bunch population is below the threshold for the previously mentioned SR radiation induced instability, which is to say [8, 15]:

$$N \le D\,(B/E)^{1/3} f_{RF} V_{RF} \sigma_{z0}^3 \lambda^{-2/3}\ , \qquad (6)$$

where $D = 4.528\times10^{-3}$[SI units]. By combining Eq. (3) and Eq. (6) the following stability criterion is derived:

$$F \le F_{max} = G\,(\sigma_{z0}/\lambda)^{2/3}\ , \qquad (7)$$

where $G = 7.463$ is a dimensionless constant. It must be remarked that the instability theory was derived for the case of a coasting beam. Anyway, simulations and experimental results at the ALS and at BESSY II [8, 17] showed that the model works also for bunched beams and that the theory is able to predict the instability threshold when in Eq. (6) $\lambda \sim \sigma_{z0}$. By (7) the corresponding threshold for F is $F_{max} \sim G$. The value of κ for maximum F is obtained by solving (4), increasing κ to a value κ_{max} such that $F(\kappa_{max}) = G$.

Now let us put $F = G$ in Eq.(5) and examine the remaining factors. Expressing g in terms of y, we find that $g(\lambda)$ is a function only of σ_{z0}/λ and κ (the latter now set equal to κ_{max}). Thus, the spectrum extends to larger wave numbers as σ_{z0} is decreased. On the other hand, the factor $\sigma_{z0}^{14/3}$ sharply degrades the overall intensity if σ_{z0} is too small. By an appropriate compromise in the choice of σ_{z0} we get a suitable spectral bandwidth. Once that choice is made, we can still vary the other factors in (5) to maximize radiation intensity while respecting technical constraints. It must be remarked that the momentum compaction, which does not appear explicitly in Eq. (5), is used in this scheme for keeping constant σ_{z0} when the other quantities are varied.

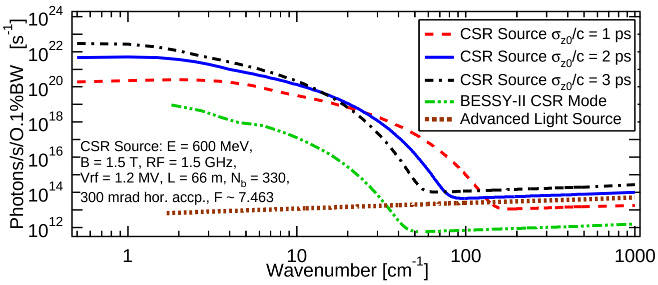

FIG. 4: Example of source optimized for the CSR production using the criteria described in this letter. The photon flux is compared with the cases of a conventional SR source (ALS) and of BESSY II CSR mode with 400 bunches, 19.2 μA/bunch, σ_{z0}/c= 1.8 ps and 60 mrad horizontal acceptance.

Figure 4 shows an example of the impressive performance that a source designed with the presented criteria can achieve. Three modes of operation, trading between power and bandwidth, are plotted and can be selected with continuity by simply tuning the lattice momentum compaction from 4.3×10^{-4} to 3.9×10^{-3}. Also shown for comparison are the curves for a 'conventional' SR source (the Advanced Light Source at Berkeley) and for BESSY II in the special CSR mode.

Criterion (7), when used in Eq.(3) with a given σ_{z0}, sets a limit on the maximum single bunch current that can be stored without experiencing the SR induced instability. The strong dependence of this threshold on σ_{z0} explains why the SR wake becomes dominant in the short bunch regime, making the longitudinal dynamics practically independent of the vacuum chamber wakes. This result has the quite general implication that very short and stable bunches can be obtained only at the cost of very small currents per bunch. For example, in our hypothetical CSR source of Fig. 4, in order to go from σ_{z0} equal to 3 ps to 1 ps, the current per bunch must drop from $\sim$1.2 mA to $\sim 90\mu$A to preserve stability.

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