HOM calculations of new RF cavities for a super B-factory^{*}

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Abstract

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HOM calculations of new RF cavities for a super B-factory

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1. INTRODUCTION

In recent time, a strong interest has developed in exploring the parameter regime over which electronpositron factories may ultimately produce high 10³⁶ cm⁻² sec⁻¹ luminosity[1-2]. Electron and positron beam average current of tens of amperes are needed to achieve this luminosity. Additionally bunch length of beams must be very short, of the order of several millimeters. To compensate energy loss due to synchrotron radiation, RF power of tens megawatt is needed. The averaged power of wake fields generated in a vacuum chamber can reach substantial value, as it scales quadratically with currents and inverses with bunch length. As this power comes from the beam kinetic energy it must be compensated by additional RF power. It is of great concern for future super B-factories of very high luminosity, obtained from high beam current and short bunch length. Main part of wake fields comes from RF cavities. We can minimize these fields by choosing the right cavity shape. Here we present results of computer spectrum analyses of different kind of cavities, which are already used or can be used in the storage rings of electron-positron factories.

2. WAKE FIELDS IN RF CAVITIES

Interaction of electromagnetic field of a charged bunch with a vacuum chamber leads to excitation of additional fields. We call these fields "wake fields". There are two main sources of wake fields. They can be generated due to finite conductivity and surface roughness of the material of a vacuum chamber. In principle this part can be reduced by choosing high conductivity material, high quality surface tooling or increasing transverse dimensions of vacuum chamber. Another source is geometrical discontinuities of vacuum chamber. We can make a very smooth vacuum chamber, but we can not avoid using RF cavities which become the main non homogeneity in the smooth ring. Beams generate electromagnetic fields in a cavity. Some part of these fields stays in a cavity in the form of the main RF mode and some High Order Modes (HOM). This HOMs are usually called "HOMs bellow cut-off frequency". In a time other part of the wake field leaves a cavity and propagates in the vacuum chamber. These fields are called "HOMs above cut-off frequency". Very high frequency part of the wake field travels together with bunches for very long time. The power of the excited main mode effectively determines the power which beams take out from the cavity. The HOM power is dissipated in the walls of the cavity or vacuum chamber and absorbers as Joule losses.

Fig. 1 shows a snapshot of electric force lines of wake fields, excited in the PEP-II "empty" cavity by a single bunch. At this moment a bunch is crossing the right exit of the cavity.



Figure 1: Wake field electric force lines (green and blue lines) in the PEP-II cavity at the time when a short bunch is leaving the cavity. Bunch density is presented by a red line. Bunch length is 1.8 mm.

2.1. Green's wake function and wake potentials

The power of the wake fields is naturally the same power that beams lose. To calculate the beam power losses we need to find electromagnetic field along the trajectory of the beam. When we know electromagnetic field, then we can calculate the integral of electric longitudinal component along the bunch trajectory. This integral is called wake potential $W(\tau)$:

$$W(\tau) = \int_{-\infty}^{\infty} E_z(t,z)_{z=c(t-\tau)} dt$$

Variable τ defines time interval between particles in a beam. Sometimes we will also use distance between particles $s = c\tau$, as they travel with a speed of light c. Wake fields and wake potentials can be calculated in the time domain by integrating Maxwell equations. Wake potential of a point charge is a Green function $G(\tau)$

which can be used for calculation of the field of bunches of any charge line distribution $\rho(\tau)$

$$W(\tau) = \int_{-\infty}^{\tau} \rho(\tau') G(\tau - \tau') d\tau' = \int_{0}^{\infty} \rho(\tau - \tau') G(\tau') d\tau'$$

Because of the difficulties of Green function calculation, the wake potential of a short bunch is used as an approximation for the Green function. We use code "NOVO" [3] which gives the best results [4] for the wake fields, generated by a very short ultra relativistic bunch in an axially symmetric, perfectly conducting, vacuum chamber pipe. Wake potential of 4 mm bunch in the PEP-II cavity is shown in Fig. 2. Bunch has the Gaussian charge distribution.



Figure 2: Wake potential of 4 mm bunch in PEP-II cavity.

Beam energy loss k is calculated by

$$k = \int_{-\infty}^{\infty} W(\tau) \rho(\tau) d\tau$$

For a single bunch loss factor is usually normalized to a bunch charge and measured in V/pC.

2.2. Longitudinal impedance and frequency spectrum of wake potential

The Fourier transform of Green function gives longitudinal coupling impedance $Z(\omega)$

$$Z(\omega) = \int_{-\infty}^{\infty} G(\tau) \exp(-i\omega\tau) d\tau$$

The Fourier transform of the wake potential

$$W(\omega) = \int_{-\infty}^{\infty} W(\tau) \exp(-i\omega\tau) d\tau =$$
$$= \int_{-\infty}^{\infty} \int_{0}^{\infty} \rho(\tau - \tau') G(\tau') d\tau' \exp(-i\omega\tau) d\tau =$$
$$= \rho(\omega) \times \int_{0}^{\infty} G(\tau') \exp(i\omega\tau') d\tau' = \rho(\omega) \times Z(-\omega)$$

gives the product of impedance $Z(\omega)$ and Fourier transform of bunch charge density $\rho(\omega)$

$$\rho(\omega) = \int_{-\infty}^{\infty} \rho(\tau) \exp(-i\omega\tau) d\tau$$

For a single bunch with the Gaussian charge density and rms bunch length σ and total charge Q_b the Fourier transform is

$$\rho(\omega) = Q_b e^{-\frac{1}{2}(\omega \frac{\sigma}{c})}$$

Frequency spectrum of the wake potential of 4 mm bunch in the PEP-II cavity is shown in Fig. 3. This spectrum is calculated from the wake potential of 5000 mm length.



Figure 3: Real (blue line) and imaginary (pink line) parts of wake field potential of 4 mm bunch in the PEP-II cavity. Dotted line shows cut-off frequency.

As can be seen the spectrum has sharp peaks in low frequency range and has broad-band peak in high frequency range. This behavior reflects the fact that low frequency modes are trapped by the cavity and oscillate in the cavity. In reality the widths of these modes are determined by the loaded quality factor Q_L of a cavity due to wall conductivity and coupling with a transmission line and HOM loads. In our calculations we assume perfect conductivity of the cavity walls, so the width of these modes is determined by the time interval of the wake potential. In the high frequency range the modes after several oscillations leak out of the cavity and propagate in the beam pipe. In principle the spectrum forms a continuum in high frequency range; however trapped modes can be there too. We can assume that the frequency which separates different spectrum behavior is the cut-off frequency.

2.3. Cut-off frequency

Cut-off frequency is the maximum frequency of a captured mode in a cavity. It is determined by the size of a beam pipe. There can be several cut-off frequencies for TM and TE modes. We consider the case of longitudinal electric fields or TM modes. For a round pipe of radius a the minimum cut-off frequency is for TM01 mode

$$f_{[GHz]}^{cut-off} = \frac{c}{a} \times \frac{v_{01}}{2\pi} = \frac{0.11474}{a_{[m]}}$$

In the formula we used the first zero of the Bessel function $v_{01} = 2.4048$. Cut-off frequency is shown by a dotted line in Fig. 3.

2.4. Frequency spectrum of energy loss

The frequency spectrum of energy loss will give us information about how much energy a bunch loses for modes bellow or above cut-off frequency.

Using inverse Fourier transform we can present loss factor k_s in the following form

$$k = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_s(\omega) \rho(-\omega) d\omega =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |\rho(\omega)|^2 Z(-\omega) d\omega$$
(1)

Loss factor is a pure real function because of the properties of impedance function, whose real part is symmetric with frequency and imaginary part is antisymmetric.

Let us introduce frequency loss integral $K_s(\omega)$ as

$$K_{s}(\omega) = \operatorname{Re}\left\{\frac{1}{\pi}\int_{0}^{\omega}W_{s}(\omega)\rho(-\omega)d\omega\right\} =$$
$$=\frac{1}{\pi}\int_{0}^{\infty}\left|\rho_{s}(\omega)\right|^{2}\operatorname{Re}\left\{Z(\omega)\right\}d\omega$$

Index "s" means wake potential of a single bunch. In this definition total integration gives loss factor: $K_s(\infty) = k_s$.

Frequency loss integral for the PEP-II cavity is shown in Fig. 4 by a red line. Trapped modes make steps in loss integral. Loss factor for 4 mm bunch is 0.805 V/pC. In frequency interval up to 6 GHz energy loss achieves 75% of its value. Energy loss above cut-off frequency is 0.389 V/pC or 48%.



Figure 4: Spectrum, loss integral and intrinsic impedance for the PEP-II cavity.

Fig. 4 also shows intrinsic impedance (R/Q) of trapped modes by pink circles, which is calculated over steps ΔK of loss integral according to the formula

$$R/Q = 2\frac{\Delta k}{\omega}$$

2.5. Power loss of a train of bunches

Analytical results presented in this and following chapters are in agreement with results already achieved by other authors [5].

Because of superposition of electromagnetic fields, the total field is a linear sum of fields of each charge. If we have a train of equal bunches spaced in time by T_b , then wake potential for the N-th bunch is sum of wake potentials of previous bunches

$$W_N(\tau) = \sum_{m=0}^{N} W_s(mT_b + \tau)$$

The wake potential sum for a train with bunch spacing equal to RF wavelength is shown in Fig. 5.



Figure 5: Wake potential for a train of bunches, spaced by RF wavelength.

It can be seen that wake field is growing with number of bunches, taking the shape of the main RF cavity mode with some high frequency oscillations.

We can use equation (1) to derive formulas for the beam power loss. Energy loss of the N th bunches is

$$k_{N} = \frac{1}{\pi} \int_{0}^{\infty} \left| \rho_{N}(\omega) \right|^{2} \operatorname{Re} \{ Z(\omega) \} d\omega$$

Power loss P_N can be defined as

$$P_{N} = \frac{k_{N+1} - k_{N}}{T_{b}} =$$

$$\frac{1}{\pi} \int_{0}^{\infty} \frac{\left|\rho_{N+1}(\omega)\right|^{2} - \left|\rho_{N}(\omega)\right|^{2}}{T_{b}} \operatorname{Re}\{Z(\omega)\}d\omega$$

For equal N bunches line charge density is

$$\rho_N(t) = \sum_{m=0}^N \rho_s(t - mT_b)$$

Its Fourier transform is

$$\rho_{N}(\omega) = \rho_{s}(\omega) \times \sum_{m=0}^{N} \exp(im\omega T_{b}) =$$
$$= \rho_{s}(\omega) \times \frac{1 - \exp(iN\omega T_{b})}{1 - \exp(i\omega T_{b})}$$

Then power loss is

$$P_N = \int_0^\infty \frac{\sin \omega T_b (N + \frac{1}{2})}{\sin \frac{\omega T_b}{2}} \times \frac{\left|\rho_s(\omega)\right|^2}{\pi T_b} \operatorname{Re}\{Z(\omega)\} d\omega$$

For a long train of bunches, left function under integral goes to an infinite sum of δ -functions

$$\lim_{N \to \infty} \frac{1}{\pi} \frac{\sin \omega T_b (N + \frac{1}{2})}{\sin \frac{\omega T_b}{2}} = \frac{2}{T_b} \sum_{m=1}^{\infty} \delta(\omega - \omega_m)$$
$$\omega_m = \frac{2\pi}{T_b} m$$

and power is

$$P = \sum_{m=1}^{\infty} \frac{2}{T_b^2} \int_0^{\infty} \delta(\omega - \omega_m) \times \left| \rho_s(\omega) \right|^2 \operatorname{Re} \{ Z(\omega) \} d\omega$$

After integration we get well-known result

$$P = \sum_{m=1}^{\infty} \frac{2}{T_b^2} \left| \rho_s(\omega_m) \right|^2 \operatorname{Re} \{ Z(\omega_m) \}$$

We can analyze this formula for frequency region below cut-off frequency and above.

2.6. Power loss below cut-off frequency

For trapped modes impedance can be described a by narrow-band resonance of frequency ω_r , loaded quality

factor $Q_L^{\omega_r}$ and intrinsic impedance $Z^{\omega_r} = R/Q$

$$Z(\omega) = Z^{\omega_r} \frac{Q_L^{\omega_r}}{1 + 2iQ_L^{\omega_r} \frac{\omega - \omega_r}{\omega_r}}$$

then power loss for the beam current $I_{beam} = \frac{Q_b}{T_b}$ is

$$P = I_{beam}^2 \sum_{m=1}^{\infty} e^{-(\omega_m \frac{\sigma}{c})^2} \sum_{\omega_r} \frac{Z^{\omega_r} Q_L^{\omega_r}}{1 + (2Q_L^{\omega_r} \frac{\omega_m - \omega_r}{\omega_r})^2}$$
(2)

The way to diminish power loss is to decrease Q_L by applying HOM loads and proper detuning of trapped modes.

2.7. Power loss above cut-off frequency

The spectrum here is a more or less smooth function of frequency and we can change the series presentation back to the integral

$$P = \sum_{m=1}^{\infty} \frac{2}{T_b^2} |\rho_s(\omega_m)|^2 \operatorname{Re}\{Z(\omega_m)\} \approx$$
$$\approx \int_{\omega_{cut-off}}^{\infty} \frac{1}{\pi T_b} |\rho_s(\omega_m)|^2 \operatorname{Re}\{Z(\omega_m)\} d\omega$$

Finally, the beam power loss above cut-off frequency takes the form

$$P = I_{beam}^2 \times T_b \times (k_s - K(\omega_{cut-off})) = I_{beam}^2 \times R_{loss}$$

Let us call

$$R_{loss} = T_b \times (k_s - K(\omega_{cut-off}))$$

as loss impedance.

3. CAVIES FOR SUPER B-FACTORY

For super B-factory we need cavities with very small loss impedance. We can try to change the shape of existing cavities or the RF frequency too, in order to decrease impedance. Several shapes of a cavity are presented below.

3.1. PEP-II type cavities

The shape of the PEP-II cavity is shown in Fig.1. First modification can be larger aperture of the beam pipe for the same RF frequency. The shape of a cavity with a double size beam pipe is shown in Fig. 6 and its spectrum is shown in Fig. 7.



Figure 6: PEP cavity with double beam pipe.

There is only one HOM mode below cut-off frequency. This mode can be effectively damped by an appropriate load. The loss impedance is two times smaller than PEP-II cavity, R/Q of the main mode is also less, but only by 35%.



Figure 7: Spectrum of a cavity presented in Fig.6.

Another possibility is to increase RF frequency two times keeping the same beam pipe size. The shape of such a cavity and its spectrum is shown in Fig.8 and Fig.9.



Figure 8: 952 MHz cavity.



Figure 9: Spectrum of a cavity which is shown in Fig.8.

Loss impedance of this cavity is 3.4 times smaller than impedance of PEP-II cavity for a bunch of 1.8mm length. R/Q of the cavity is 66 Ohms. Optimization of the cavity shape (Fig. 10) improves R/Q a little. This cavity has R/Q=78 Ohm and 8% less loss impedance.



Figure 10: Optimized shape of the cavity.

3.2. Single-mode cavity

If we have only one mode (main) below cut-off and this mode is an accelerating mode with intrinsic impedance R/Q then power loss is

$$P = I_{beam}^2 \times T_b \times (k_s - \frac{\omega_0}{2} \frac{R}{Q})$$
(4)

Superconducting cavities of Cornell CESR-III and KEKB are single-mode cavities. Cavity shape and wake field of a short bunch in the CESR-III type cavity [8-9] is shown in Fig. 11.



Figure 11: Electric force lines of a short bunch in CESR-III type cavity.



Figure 12: Wake potential of 4 mm bunch in the Cornell CESR-III-type cavity.

Wake potential of a bunch of 4 mm length is shown in Fig. 12. Loss integral and intrinsic impendence for this cavity are shown in Fig. 13.



Figure 13: Spectrum, loss integral and intrinsic impedance for Cornell CESR III – type cavity. Bunch length is 4 mm.

KEKB cavity has a little bit different shape [10]. Loss integral for this cavity is shown in Fig. 14. In this example the bunch length is 1.8 mm.



Figure 14: Spectrum, loss integral and intrinsic impedance for KEKB type cavity. Bunch length is 1.8 mm.

Cavities with large beam pipe usually have tapers. Unfortunately, these tapers change wake potential dramatically. Fig. 15 shows picture of electric force lines of wake field just at the moment when bunch is in the right taper. Spectrum for this case is shown in Fig. 16



Figure 15: Electric force lines of the wake field in KEKB superconductive cavity. At this moment bunch is in the corner of the right taper.



Figure 16: Spectrum for the KEKB superconductive cavity with tapers. Bunch length is 1.8 mm.

One more example of a single-mode cavity and its spectrum are shown in Fig17 and Fig.18. This cavity has R/Q=32 Ohm and the smallest loss impedance of 220 Ohm for 1.8 mm bunch.



Figure 17: Single-mode cavity.



Figure 18: Spectrum of a single-mode cavity. Bunch length is 1.8 mm.

An additional advantage of the single mode cavity system which has an automatically low R/Q, is that it is stronger against the coupled-bunch instability [8]: it needs less frequency detuning

$$\frac{\Delta f}{f_{RF}} = \frac{I \times R/Q}{2V_c} \times \sin \phi_s$$

3.3. Short bunches

For short bunches the loss factor can be estimated by the following expression [6]:

$$k_s = \frac{Z_0 c}{2\pi^2 a} \left(\sqrt{\frac{g}{\sigma}} - 1 \right)$$

where g is the size of cavity gap, a is the radius of the beam pipe, $Z_0 = 120\pi$ is impedance of free space. Loss factor goes up with decreasing bunch length and becomes the main parameter for the power loss

$$P = I_{beam}^2 \times T_b \times k_s = I_{beam}^2 \times Z_0 \times \frac{d_b}{2\pi^2 a} \sqrt{\frac{g}{\sigma}}$$

Where we introduced he $d_b = T_b c$ as the distanced between bunches.

3.4. Minimum power loss

The minimum distance between bunches is the RF wavelength λ_{RF} . The radius of the beam pipe is limited by the main mode and can not be more than $\lambda_{RF} / 2$. The cavity gap is approximately half of the wave length, so the minimum power loss is determined by the ratio of the RF wavelength and bunch length

$$P = I_{beam}^2 \times Z_0 \times \frac{1}{\pi^2} \sqrt{\frac{\lambda_{RF}}{2\sigma}}$$

The corresponding minimum loss impedance is

$$R_{loss} = Z_0 \times \frac{1}{\pi^2} \sqrt{\frac{\lambda_{RF}}{2\sigma}}$$

This minimum impedance is shown in Fig. 19 together with calculated impedances for different cavities. It is interesting to note that single-mode cavity (PEP-II SC), presented in Fig. 17 has smaller loss impedance, but this is because of smaller gap size ($g \ll \lambda_{RF}/2$) of the cavity.

4. CONCLUSIONS

High average current, short bunch length beams needed for Super B-factories can give rise to large amounts of HOM power, as high as hundreds of kW per cavity. This power must be damped by the water cooled absorbers. Most of the power lost by the beam is in modes below 10 GHz for 2 mm bunches. Beams in a single mode cavity excite less HOM power. A single mode cavity needs to be a superconducting cavity, as it has small intrinsic impedance R/Q. Parameters of all presented cavities are compiled in the Table 1.

Future plans include more numerical studies of the problem, finding optimum shape of the cavity.



Figure 19: Minimum loss impedance (blue line) and loss impedances of different cavities.

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Table 1: Cavities parameters

HOM Power Loss in the Super B-factory Cavity								f=136kHz	h=3492	
Cavity type	Frequency [MHz]	Pipe radius [mm]	R/Q [Ohm]	Bunch length [mm]	Total Loss [V/pC]	Above cut-off [V/pC]	Beam current [A]	Bunch charge [nC]	Wake Voltage [kV]	HOM Power [kw]
PEP-II CESR-III	476 500	47.6 120	114 46.2	13 10	0.4699 0.175	0.0849 0.1014	2 2	11.14 11.14	0.95 1.13	1.89 2.26
PEP-II CESR-III	476 500	47.6 120	116 46.2	4 4	0.805 0.291	0.389 0.217	11 11	23.44 23.44	9.12 5.10	100.32 56.06
KEKB-SC with tapers	508	110 75	44.9	4	1.326	1.192	11	23.44	27.95	307.40
KEKB-SC-NT no tapers	508	110	47.7	4	0.318	0.2373	11	23.44	5.56	61.20
PEP-II-Large	476	95.25	74.9	4	0.35	0.209	11	23.44	4.90	53.90
PEP-II CESR-III KEKB-SC-NT	476 500 508	47.6 120 110	116 46.2 47.7	1.8 1.8 1.8	1.217 0.448 0.498	0.794 0.374 0.4173	15.5 15.5 15.5	33.03 33.03 33.03	26.23 12.37 13.79	406.56 191.71 213.68
PEP-II-Large PEP-II-Large	476 476	95.25 95.25	74.3 74.3	1.8 1.8	0.538 0.538	0.397 0.397	15.5 23	33.03 49.02	13.11 19.46	203.28 447.60
New PEP-II New PEP-II	952 952	47.6 47.6	66.4 66.4	1.8 1.8	0.748 0.748	0.472 0.472	15.5 23	16.52 24.51	7.80 11.57	120.84 266.08
PEP-Ellips	952	47.6	75.8	1.8	0.719	0.434	23	24.51	10.64	244.66
PEP-SC	952	77.62	31.6	1.8	0.303	0.208	23	24.51	5.10	117.25