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The Hadronic Spectrum of a Holographic Dual of QCD

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Abstract

We compute the spectrum of light hadrons in a holographic dual of QCD defined on $AdS_5 \times S^5$ which has conformal behavior at short distances and confinement at large interquark separation. Specific hadrons are identified by the correspondence of string modes with the dimension of the interpolating operator of the hadron's valence Fock state. Higher orbital excitations are matched quanta to quanta with fluctuations about the AdS background. Since only one parameter, the QCD scale Λ_{QCD}, is used, the agreement with the pattern of physical states is remarkable. In particular, the ratio of Delta to nucleon trajectories is determined by the ratio of zeroes of Bessel functions.

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The correspondence [1] between 10-dimensional string theory defined on $AdS_5 \times S^5$ and Yang-Mills theories at its conformal 3+1 space-time boundary [2] has led to important insights into the properties of QCD at strong coupling. As shown by Polchinski and Strassler [3], one can give a nonperturbative derivation of dimensional counting rules [4] for the leading power-law fall-off of hard exclusive glueball scattering in gauge theories with a mass gap dual to supergravity in warped space-times. The resulting theories have the hard behavior expected from QCD at short distances, rather than the soft behavior characteristic of string theory. Other important applications to hadron physics are the description of form factors at large transverse momentum [5] and the behavior of deep inelastic scattering structure functions at small x [6]. One can also derive the fall-off of hadronic light-front wavefunctions in QCD at large transverse momentum by matching their short-distance properties to the behavior of the string solutions in the large-r conformal region of AdS space [7]. The scale dependence of the string modes as one approaches the boundary from the interior of AdS space determines the analytic behavior of the QCD hadronic wavefunction, providing a precise counting rule for each Fock component with any number of quarks and gluons and any internal orbital angular momentum. The specific correspondence in the $k_\perp \to \infty$ and $x \to 1$ limits provides a prescription which maps string modes into boundary states with well defined number of partons [7]. The predicted orbital dependence coincides with perturbative QCD analyses [8]. The AdS/CFT derivations validate QCD perturbative results [9, 10] and also confirm the dominance of the quark interchange mechanism [11] for exclusive QCD processes at large N_C. Scaling laws and other aspects of high-energy scattering in warped backgrounds have also been addressed in [12].

The $\mathcal{N} = 4$ super Yang-Mills (SYM) theory at large N_C in four dimensions is dual to the low energy supergravity approximation to type IIB string compactified on $AdS_5 \times S^5$ [1]. However, QCD is fundamentally different from SYM theories where all of the matter fields appear in adjoint multiplets of $SU(N_C)$. The introduction of quarks in the fundamental representation is dual to the introduction of an open string sector [13], where the strings end on a brane and join together at a point inside AdS space. In the procedure introduced by Karch and Katz [14], the endpoints of open strings are supported by N_f additional D7-branes located along $1, 2, \ldots, 7$ dimensions. This system of N_C D3-branes and N_f D7-branes leads to a a calculable meson spectrum [15].

QCD is a nearly conformal theory in the ultraviolet region and a confining gauge theory in the infrared with a mass gap Λ_{QCD}, and a well-defined spectrum of color-singlet hadronic states. The isomorphism of the group SO(2,4) of conformal QCD in the limit of massless quarks and vanishing β-function [16] with the isometries of AdS space, $x^\mu \to \lambda x^\mu$, $r \to \lambda^{-1} r$, allows one to interpret the string wavefunction in the coordinate r as the extension of the corresponding hadron wavefunction into the fifth dimension. Different values of r correspond to different energy scales at which the hadron is examined: the holographic coordinate r determines the scale of the invariant separation between quarks $x_\mu x^\mu \to \lambda^2 x_\mu x^\mu$. In particular, the $r \to \infty$ boundary correspond to the $Q \to \infty$, zero separation limit. Conversely, color confinement implies that there is a maximum separation of quarks and a minimum value of r. We thus shall assume that AdS space ends at a finite value $r_o = \Lambda_{QCD} R^2$ truncating the regime where the string modes can propagate. The cutoff at r_o, breaks conformal invariance and allows the introduction of the QCD scale.

A 10-dimensional non-conformal metric dual to a confining gauge theory is written as [3]

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) + ds^2_X, \tag{1}$$

where $A(z) \to 0$ as $z = R^2/r \to 0$, and R is the AdS radius. The metric (1) behaves asymptotically as a product of AdS space and a compact manifold X. Color confinement will be described in a simplified model based on a "hard-wall" approximation where the metric factor $e^{2A(z)}$ is a step function. This provides an analog of the MIT bag model where quarks are permanently confined inside a finite region of space [17]. However, unlike bag models, the truncated boundary conditions on string modes are imposed on the holographic coordinate, not on the bag wavefunction at fixed time. The truncated AdS/CFT theory thus provides a manifestly Lorentz invariant model with confinement at large distances and conformal behavior at short distances.

The AdS/CFT correspondence can be interpreted in the present context as a classical duality between the valence state of a hadron in the asymptotic $3+1$ boundary and the lightest mass string mode in $AdS_5 \times S^5$ [7, 18]. Higher Fock components are manifestations of the quantum fluctuations of QCD; metric fluctuations of the bulk geometry about the fixed AdS background should correspond to quantum fluctuations of Fock states above the valence state. In fact, as shown by Gubser, Klebanov and Polyakov for large Lorentz spin, orbital excitations in the boundary correspond to string degrees of freedom propagating in the bulk from quantum fluctuations in the AdS sector [19]. We thus should identify the higher spin hadrons with the fluctuations around the spin 0, $\frac{1}{2}$, 1 and $\frac{3}{2}$ string solutions on $AdS_5 \times S^5$. This identification avoids the huge string dimensions associated with spin > 2, which grow as $\Delta \sim (g_s N_C)^{\frac{1}{4}}$ at large N_C. The interpolating operators $\mathcal{O}$, $\langle P|\mathcal{O}|0\rangle \neq 0$, which couple to the color-singlet hadrons at the boundary can be constructed from gauge-invariant products of local quark and gluon fields taken at the same point in four-dimensional spacetime. In contrast with the D3/D7 construction [14], we introduce quarks in the fundamental representation at the AdS boundary, and follow their wavefunctions as they propagate into the bulk. The endpoints of the open strings of the quarks of a given hadron then converge to a point in the limit $r \to \infty$.

As a first application of our procedure, consider the twist -dimension minus spin- two glueball interpolating operators $\mathcal{O}_{4+L} = F D_{\{\ell_1} \ldots D_{\ell_m\}} F$, written in terms of the symmetrized product of covariant derivatives D. The operator $\mathcal{O}_{4+L}$ has total internal spacetime orbital momentum, $L = \sum_{i=1}^{m} \ell_i$ and conformal dimension $\Delta = 4 + L$. We shall match the large r asymptotic behavior of each string mode in the bulk to the corresponding conformal dimension of the boundary operators of each hadronic state while maintaining conformal invariance [18]. In the conformal limit, an L-quantum, which is identified with a quantum fluctuation about the AdS geometry, corresponds to an effective five-dimensional mass μ in the bulk side. The allowed values of μ are uniquely determined by requiring that asymptotically the dimensions become spaced by integers, according to the spectral relation $(\mu R)^2 = \Delta(\Delta - 4)$. For large spacetime angular momentum L, we recover the string theory results for the spectrum of oscillatory exited states $\mu \simeq L/R$. The physical string modes are plane waves along the Poincaré coordinates with four-momentum P_μ and hadronic invariant mass states given by $P_\mu P^\mu = \mathcal{M}^2$. The four-dimensional mass spectrum $\mathcal{M}_L$ then follows when we impose the truncated space boundary condition $\Phi(x, z_o) = 0$ on the solutions of the AdS wave equation with effective mass μ:

$$\left[z^2\, \partial_z^2 - (d-1) z\, \partial_z + z^2\, \mathcal{M}^2 - (\mu R)^2\right] f(z) = 0, \tag{2}$$

where $\Phi(x, z) = e^{-iP\cdot x} f(z)$. The normalizable modes are

$$\Phi_{\alpha,k}(x, z) = C_{\alpha,k}\; e^{-iP\cdot x} z^2 J_\alpha\left(z \beta_{\alpha,k} \Lambda_{QCD}\right), \tag{3}$$

with $C_{\alpha,k} = \sqrt{2}\; \Lambda_{QCD}/J_{\alpha+1}(\beta_{\alpha,k}) R^{\frac{3}{2}}$, $\alpha = 2 + L$ and $\Delta = 4 + L$ for $d = 4$. For small-z, Φ scales as $z^{-\Delta}$, where the scaling dimension Δ of the string mode has the same dimension of the interpolating operator which creates a hadron. The four-dimensional mass spectrum is then determined by the zeros of Bessel functions $\beta_{\alpha,k}$:

$$\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD} \tag{4}$$

The lattice results for the lowest glueball state Θ^{++}, $\mathcal{M} \simeq 1.5$ GeV are consistent with the holographic model predictions for $\Lambda_{QCD} \simeq 0.3$ GeV.

We next consider the twist-two, dimension $3 + L$, vector-meson operators $\mathcal{O}^{\mu}_{3+L} = \overline{\psi}\gamma^{\mu} D_{\{\ell_1} \ldots D_{\ell_m\}}\psi$, dual to string modes $\Phi_\mu = e^{-iP\cdot x} f_\mu(z)$ propagating on AdS space with polarization along the Poincaré coordinates. The string wavefunctions of the vector mesons are then determined by the five-dimensional wave equation

$$\left[z^2\partial_z^2 - (d-1)z\partial_z + z^2\mathcal{M}^2 - (\mu R)^2 + d - 1\right] f_\mu(z) = 0, \tag{5}$$

in the $\Phi_z = 0$ gauge [20], with normalizable modes

$$\Phi^{\mu}_{\alpha,k}(x, z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\beta_{\alpha,k}\Lambda_{QCD}\right)\epsilon^{\mu}, \tag{6}$$

where $\alpha = 1 + L$ and $\Delta = 3 + L$. The hadronic mass spectrum follows from $\Phi_\mu(x, z_o) = 0$. Similarly, the pseudoscalar mesons are described by the operator $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \ldots D_{\ell_m\}}\psi$, dual to string modes polarized along the radial coordinate in the $\Phi_\mu = 0$ gauge.

The predicted spectrum is compared in Fig. 1 with the masses of light mesons listed by the PDG [21]. We plot the values of $\mathcal{M}^2$ as function of L for $\Lambda_{QCD} = 0.263$ GeV. The predicted masses for the lightest hadrons are too high, but otherwise the results are in good agreement with the empirical values. A string mode with a node in the coordinate r should correspond to a radial resonance with a node in the interquark separation. The first radial AdS eigenvalue has a mass 1.8 GeV which is high compared to the masses of the observed radial excited mesons, the $\pi(1300)$ or the $\rho(1450)$. These defects could possible be cured by modifying the sharp cutoff at r_o.

The study of the baryon spectrum is crucial for our understanding of bound states of strongly interacting relativistic confined particles. Different QCD-based models often disagree, even in the identification of the relevant degrees of freedom [22]. There have been recent advances with the computation of orbital excitations based on the $1/N_C$ expansion [23] and lattice gauge theory [24]. AdS/CFT provides new insights: consider the twist-three, dimension $\frac{9}{2} + L$, baryon operators $\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \ldots D_{\ell_q}\psi D_{\ell_{q+1}} \ldots D_{\ell_m\}}\psi$, dual to spin-$\frac{1}{2}$

or $\frac{3}{2}$ modes in the bulk. We need to solve the full ten-dimensional Dirac wave equation $\not{D}\hat{\Psi} = 0$, since the lowest Kaluza-Klein (KK) mode of the Dirac operator on an N-sphere is not zero. Consequently, baryons are charged under $SO(4) \sim SO(6)$ as obtained from the isometries of $X = S^5$. The field $\hat{\Psi}$ can be expanded in terms of eigenfunctions $\eta_\kappa(y)$ of the Dirac operator on the compact space X, $i\not{D}_X \eta_k(y) = \lambda_\kappa \eta_\kappa(y)$, with eigenvalues λ_κ as $\hat{\Psi}(x,z,y) = \sum_\kappa \Psi_\kappa(x,z)\eta_\kappa(y)$, where the y are coordinates of X. The AdS Dirac equation is [20]

$$\left[z^2\,\partial_z^2 - d\;z\;\partial_z + z^2\mathcal{M}^2 - (\lambda_\kappa + \mu)^2 R^2 + \frac{d}{2}\left(\frac{d}{2}+1\right) + (\lambda_\kappa + \mu) R\,\hat{\Gamma}\right] f(z) = 0 \tag{7}$$

where $\Psi(x,z) = e^{-iP\cdot x}\ f(z)$ and $\hat{\Gamma} u_\pm = \pm u_\pm$. For AdS_5, $\hat{\Gamma}$ is the four-dimensional chirality operator γ_5. The AdS mass μ is determined asymptotically by the orbital excitations in the boundary: $\mu = L/R$. The eigenvalues on S^{d+1} are $\lambda_\kappa R = \pm\left(\kappa + \frac{d}{2} + \frac{1}{2}\right)$, $\kappa = 0, 1, 2, ...$ [25]. The normalizable modes for $\kappa = 0$ are

$$\Psi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^{\frac{5}{2}} \left[J_\alpha(z\beta_{\alpha,k}\Lambda_{QCD})\ u_+(P) + J_{\alpha+1}(z\beta_{\alpha,k}\Lambda_{QCD})\ u_-(P)\right] \tag{8}$$

where $u^- = \frac{\gamma^\mu P_\mu}{P}\ u^+$, $\alpha = 2 + L$ and $\Delta = \frac{9}{2} + L$. The solution of the spin-$\frac{3}{2}$ Rarita-Schwinger equation in AdS space is more involved, but considerable simplification occurs in the $\Psi_z = 0$ gauge for polarization along Minkowski coordinates, Ψ_μ, where it becomes similar to the spin-$\frac{1}{2}$ solution [26]. The four-dimensional spectrum follows from $\Psi^\pm(x, z_o) = 0$ or $\Psi^\pm_\mu(z, z_o) = 0$

$$\mathcal{M}^+_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}, \quad \mathcal{M}^-_{\alpha,k} = \beta_{\alpha+1,k}\Lambda_{QCD}, \tag{9}$$

with a scale independent mass ratio.

It is not possible to match dimensions at the asymptotic boundary using the fully antisymmetric color-singlet representation of $SU(N_C)$ at large N_C. We use instead the 3-quark representation of color-singlet baryonic states which has two quarks in the fundamental of color $\mathbf{N_C}$ and one quark in the antisymmetric component of the tensor product $\mathbf{N_C} \otimes \mathbf{N_C}$ [27]. We then can construct the gauge invariant baryon operator $\mathcal{O}(x)_{9/2} = \psi_{\mathbf{N_C}}(x)\psi_{\mathbf{N_C}}(x)\psi_{\mathbf{N_C(N_C-1)/2}}(x)$. For $N_C = 3$ we recover the usual interpolating operator which creates a physical baryon in $QCD(3+1)$: $\mathcal{O}_{9/2} = \epsilon_{abc}\psi_a\psi_b\psi_c$.

The spin-flavor quantum numbers of baryons can be identified from the $SU(6) \supset SU(3)_{flavor} \otimes SU(2)_{spin}$ multiplet structure. The intrinsic spin S of a given hadron matches the spin of its dual string. The boundary conditions are $\Psi^+(x, z_o) = 0$ for $S = \frac{1}{2}$ nucleons

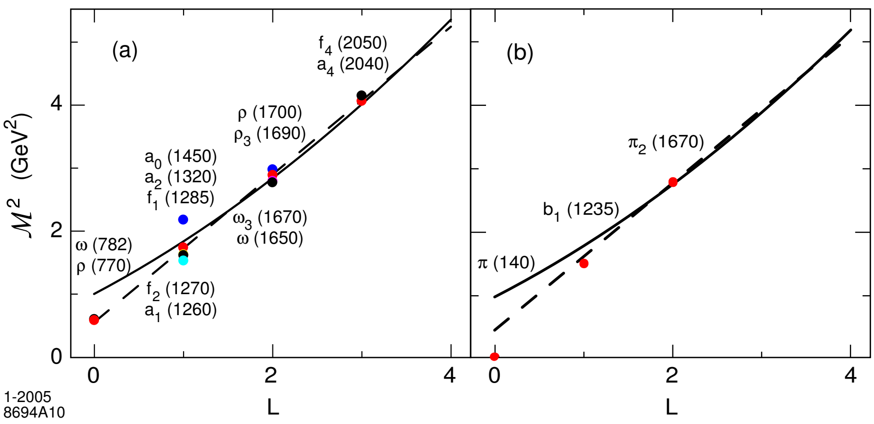

FIG. 1: Light meson orbital states for $\Lambda_{QCD} = 0.263$ GeV. Results for the vector mesons are shown in (a) and for the pseudoscalar mesons in (b). The dashed line has slope 1.16 GeV^2 and is drawn for comparison.

and $\Psi^-_\mu(x, z_o) = 0$ for $S = \frac{3}{2}$. Fig. 2 (a) shows the predicted orbital spectrum of the nucleon states and Fig. 2 (b) the Δ orbital resonances. The only parameter is the value of Λ_{QCD} which we take as 0.22 GeV. The nucleon states with intrinsic spin $S = \frac{1}{2}$ lie on a curve below the nucleons with $S = \frac{3}{2}$. In contrast to the nucleons, all the known Δ orbital states with $S = \frac{1}{2}$ and $S = \frac{3}{2}$ lie on the same trajectory. The boundary conditions in this case are imposed on Ψ^-. The predicted spectrum displays a clustering of states with the same orbital L, consistent with a strongly suppressed spin-orbit force.

Eq. (9) predicts a novel parity degeneracy between states in the parallel trajectories shown in Fig. 2 (a), as seen by displacing the upper curve by one unit of L to the right. Remarkably, the nucleon states with $S = \frac{3}{2}$ and the Δ resonances fall on the same trajectory [28]. In the quark-diquark model of Jaffe and Wilczek [29], baryon states on the lower trajectory of Fig. 2 (a), correspond to "good" diquarks, the upper to "bad" diquarks, and all the states shown in Fig. 2 (b) to "bad" diquarks. In contrast to the AdS/CFT results, quark-diquark models need to tune away the spin-orbit splittings. One difficulty for the truncated model: the first AdS radial state has a mass 1.85 GeV, so it is difficult to identify it with the Roper resonance $N\frac{1}{2}^+(1440)$.

The general agreement of the holographic model with the known light baryon spectrum is quite remarkable and nontrivial. Essential features of QCD, its near-conformal behavior at short physical distances plus color confinement at large interquark separation, are incorporated in the model. The AdS/CFT approach contains only one parameter, the QCD scale

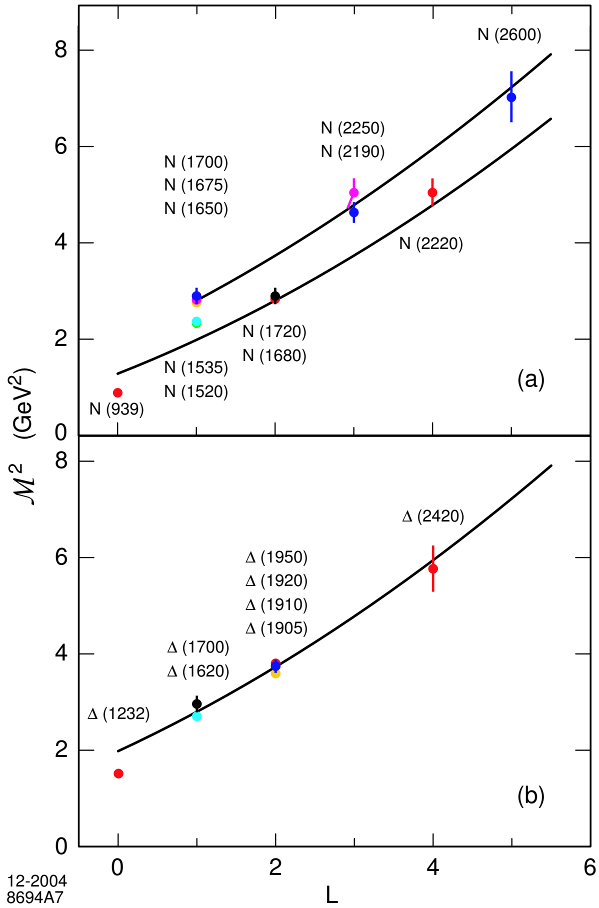

FIG. 2: Predictions for the light baryon orbital spectrum for $\Lambda_{QCD} = 0.22$ GeV. The lower curve in (a) corresponds to nucleon states dual to spin-$\frac{1}{2}$ modes and the upper to nucleon states dual to spin-$\frac{3}{2}$ modes. The Delta states dual to spin-$\frac{1}{2}$ and $\frac{3}{2}$ modes lie on the same trajectory as shown in (b).

with $\Lambda_{QCD} = 0.24 \pm 0.02$ GeV. Moreover, the ratio of the Delta to nucleon trajectories is parameter independent, depending simply on the ratios of zeroes of Bessel functions. The approach is highly successful in organizing the hadron spectrum, although in the case of mesons the holographic model underestimates the spin-orbit separations of the lowest orbital states. Our results suggest that basic features of the QCD hadron spectrum can be understood in terms of a higher dimensional dual theory.

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