SLAC-PUB-10772
astro-ph/0409238
September 2004

Casimir Effect in a Supersymmetry-Breaking Brane-World as Dark Energy

Pisin Chen[1] and Je-An Gu[2,1]

[1]*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[2]*Department of Physics, National Taiwan University, Taipei 106, Taiwan, R.O.C.*

(Dated: September 10, 2004)

A new model for the origin of dark energy is proposed based on the Casimir effect in a supersymmetry-breaking brane-world. Supersymmetry is assumed to be preserved in the bulk while broken on a 3-brane. Due to the boundary conditions imposed on the compactified extra dimensions, there is an effective Casimir energy induced on the brane. The net Casimir energy contributed from the graviton and the gravitino modes as a result of supersymmetry-breaking on the brane is identified as the observed dark energy, which in our construction is a cosmological constant. We show that the smallness of the cosmological constant, which results from the huge contrast in the extra-dimensional volumes between that associated with the 3-brane and that of the bulk, is attainable under very relaxed conditions.

PACS numbers: 11.10.Kk, 11.25.Uv, 11.30.Pb, 98.80.Es

Recent type Ia supernova (SN Ia) observations have suggested that the expansion rate of the universe is increasing [1, 2]. This mysterious accelerating expansion of the universe has stimulated various fascinating proposals over the past several years. Possible explanations of this acceleration include a form of energy which provides a significant negative pressure, such as a positive cosmological constant [3], a positive dynamical potential energy induced by a scalar field called quintessence [4, 5], a string theory induced metastable de Sitter vacuum [6], etc. There are also ideas that invoke the existence of extra spatial dimensions [7, 8] and the modification of gravity [9, 10, 11]. Dark energy, a common designation to the possible origin of this phenomenon, contributes effectively about 70% of the energy density of the present universe, as first suggested by SN Ia data and reinforced more recently by the cosmic microwave background (CMB) data from WMAP [12].

On the experimental side, the cosmological constant is perhaps the simplest to test among the dark energy candidates. It has survived through an array of updated high-precision observations and is currently in favor over other candidates. Unfortunately, on the theoretical side it suffers a serious fine-tuning problem. In particular, the idea faces a severe challenge in that the contribution of quantum fluctuations to the vacuum energy, and thus the value of the cosmological constant, is generally believed to be astronomically larger than what is suggested by observation. This is the long-standing cosmological constant problem.

The enormous vacuum energy is known to be harmless in particle physics because, instead of the "absolute energy", it is the energy differences that are relevant to observable physical effects. In particular, for a physical system with boundaries, the vacuum energy density can be set to zero when the boundary is at infinity, and only the difference between the energies associated with the finite and infinite boundaries would contribute to meaningful vacuum energy. One famous example is the Casimir effect [13] between two conducting plates in quantum electrodynamics (QED). This effect has been observed and well studied, and thereby provides a very important support of the above notion.

As for Einstein's general relativity, in which energy-momentum curves the space-time and thereby generates gravity, there is as yet no clue whether it is the absolute or the difference energy that should be responsible. If the absolute energy matters, then even the energy associated with quantum fluctuations at atomic scales would ruin structures of our universe unless it can be miraculously fine-tuned to an extremely small value. It therefore seems more reasonable to assume that in the eventual quantum theory of gravity only the energy differences would contribute to gravitational effects, much on the same footing with all other quantum theories of gauge interactions.

It is under this conviction we consider the Casimir energy as the origin for the dark energy. It is known that the Casimir energy in the ordinary $(3+1)$-dimensional space-time cannot provide negative pressure. Conversely, the Casimir energy induced from a higher-dimensional world with suitable boundary conditions in extra dimensions can in principle behave like a cosmological constant in the ordinary 3-space. Such a Casimir energy nevertheless tends to be too large for our purpose unless the size of extra dimensions can be macroscopic, as summarized by Milton [14].

We believe that additional ingredients are required for constructing a theory of dark energy following this line of thought. We note that supersymmetry (SUSY) guarantees the perfect cancellation of the vacuum energy and therefore provides a powerful tool for this purpose. Unfortunately (or fortunately, as we will see), we also know that SUSY has to be broken, at least in our 3+1 dimensional world, with the symmetry-breaking scale above TeV. Conventionally this would entail an absolute vacuum energy that is much too large for dark energy. Motivated by the brane-world scenario [15, 16] and taking advantage of the possibility of SUSY-breaking only on the brane, we find that the vacuum energy a la Casimir effect can be dramatically suppressed.

Work supported in part by the Department of Energy, contract DE-AC02-76SF00515

In this Letter we propose a new theory for dark energy as follows: We consider a (3+n+1)-dimensional space-time with n compact extra dimensions, in which the standard model fields and their superpartners are confined on a 3-brane while the gravity sector resides in the (higher-dimensional) bulk. We assume SUSY is preserved in the bulk and only broken on the 3-brane with a breaking scale M_{SUSY}. We will show that the Casimir energy so induced is wonderfully able to play the role of the dark energy in our universe.

Unbroken SUSY dictates that gravitons and gravitinos in the bulk have the same mass and the same interaction strength, while its breakage on and near the 3-brane, in our assumption, induces a mass-square difference μ^2 between them. The 3-brane in general has a nonzero thickness δ that, motivated by string theory, is characterized by the string length l_s. The thickness δ also corresponds to the effective range of SUSY-breaking. Consequently, the vacuum energy as well as the Casimir energy vanish in the bulk but is nontrivial in the extra-dimensional volume that encompasses the brane with thickness δ.

As a demonstration of how the mass shift on the brane modifies the Casimir energy, we first consider the case of a scalar field and its superpartner, "*calar*", in a Minkowskian ordinary (3+1)-dimensional space-time ($\mathcal{M}^4$) and a n-torus extra space ($\mathcal{T}^n$) with a size a. Later we will generalize this derivation to the true graviton-gravitino case and to the situation where there are more SUSY fields in the bulk.

Let us employ the form

$$\Delta m^2(y) = m^2 e^{-2|y|/\delta} \tag{1}$$

to characterize the mass-square shift, where y is the extra-dimension coordinate and we have set the location of the 3-brane at $y = 0$. With the scalar-field part of the action

$$S = \int d^4x d^ny \sqrt{|g|} \left[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\left(m_0^2 + \Delta m^2\right)\phi^2 \right], \tag{2}$$

we treat the mass-square-shift term $\Delta m^2 \phi^2$ as a perturbation and calculate the Casimir energy shift to the first order.

In $\mathcal{M}^4 \times \mathcal{T}^n$ the renormalized Casimir energy density $\rho_{\text{v}}^{(\text{ren})}(m^2, a)$ is the difference of the vacuum energy densities at a and infinity:

$$\rho_{\text{v}}^{(\text{ren})}(m^2, a) = \rho_{\text{v}}(m^2, a) - \rho_{\text{v}}(m^2, a \to \infty)\,. \tag{3}$$

Up to $\mathcal{O}\left(\Delta m^2 \phi^2\right)$, the shift of the Casimir energy density due to SUSY-breaking is

$$\begin{aligned} \delta\rho_{\text{v}}(\Delta m^2, a) &\equiv \rho_{\text{v}}^{(\text{ren})}(m^2, a) - \rho_{\text{v}}^{(\text{ren})}(m^2 = 0, a) && (4) \\ &\cong C_n \cdot \frac{a^2}{a^{4+n}} \cdot \Delta m^2(y)\,. && (5) \end{aligned}$$

As a demonstration of how C_n varies under different boundary conditions, geometries and dimensionalities, selected values of C_n for a real scalar field (with small m_0, i.e. $m_0 a \ll 1$) are listed in Table I, where the cases for the periodic (PBC) and the anti-periodic (APBC) boundary conditions for $\mathcal{M}^4 \times \mathcal{T}^n$ and that of a conformally coupled scalar field in $\mathcal{M}^4 \times \mathcal{S}^n$(n-sphere) are investigated.

TABLE I: Selected values of C_n for a real scalar field

n	$\mathcal{T}^n_{\text{PBC}}$	$\mathcal{T}^n_{\text{APBC}}$	n	$\mathcal{T}^n_{\text{PBC}}$	$\mathcal{T}^n_{\text{APBC}}$	n	$\mathcal{S}^n$(conformal)
1	0.015	−0.011	12	0.27	−0.21	3	-6.4×10^{-5}
2	0.024	−0.012	13	0.41	−0.34	5	1.4×10^{-5}
3	0.031	−0.013	14	0.65	−0.57	7	-3.0×10^{-6}
4	0.038	−0.015	15	1.1	−0.96	9	6.4×10^{-7}
5	0.045	−0.018	16	1.8	−1.7	11	-1.4×10^{-7}
6	0.054	−0.023	17	3.2	−3.0	13	3.0×10^{-8}
7	0.065	−0.031	18	5.7	−5.5	15	-6.6×10^{-9}
8	0.080	−0.043	19	10	−10	17	1.5×10^{-9}
9	0.10	−0.061	20	20	−19	19	-3.2×10^{-10}
10	0.14	−0.089	21	38	−38	21	7.3×10^{-11}
11	0.19	−0.14	22	76	−75		

When the extra-dimensional space is integrated over, the Casimir energy density in $\mathcal{M}^4$ is reduced to

$$\begin{aligned} \delta\rho_{\text{v}}^{(4)} &\cong C_n \cdot \frac{1}{a^4} \cdot m^2 a^2 \cdot \frac{\pi^{n/2}\Gamma(n)}{2^{n-1}\Gamma(n/2)} \left(\frac{\delta}{a}\right)^n && (6) \\ &= C_n \cdot \frac{1}{a^4} \cdot m^2 a^2 \cdot n! \left(\frac{V_\delta}{V_a}\right), && (7) \end{aligned}$$

where V_δ is the volume of the extra-dimensional space inside which SUSY is broken and V_a is the total volume of the extra space. We see that if $V_\delta \ll V_a$, the ratio of these two volumes would provide a powerful suppression to the Casimir energy.

Dictated by the nature of SUSY, the Casimir energy density (shift) of a superpartner has the same functional form (and therefore the same constant C_n), but with an opposite sign. So the above calculations can be directly applied to the calar field. That is,

$$\delta\rho_{\text{calar}}(\Delta\widetilde{m}^2, a) = -\delta\rho_{\text{scalar}}(\Delta\widetilde{m}^2, a)\,. \tag{8}$$

Consequently the net Casimir energy density contributed from the scalar-calar system induced by SUSY-breaking is

$$\begin{aligned} \delta\rho_{\not{S}} &= \delta\rho_{\text{scalar}}(\Delta m^2, a) + \delta\rho_{\text{calar}}(\Delta\widetilde{m}^2, a) && (9) \\ &= \delta\rho_{\text{scalar}}(\Delta m^2, a) - \delta\rho_{\text{scalar}}(\Delta\widetilde{m}^2, a), && (10) \end{aligned}$$

which, up to the first order, is equal to $\delta\rho_{\text{scalar}}(\mu^2, a)$, where $\mu^2 \equiv \Delta m^2 - \Delta\widetilde{m}^2$. We note that among possible geometries and boundary conditions there exist ample choices where $C_n\mu^2 > 0$, such that the positivity of the resultant Casimir energy is ensured.

We are actually more interested in the Casimir energy density induced by the gravity sector under SUSY-breaking, where the gravitino is assumed to have acquired a mass shift on the brane. To apply the above results to the graviton-gravitino system in the bulk, which possesses more degrees of freedom than that of the scalar-calar system, we introduce a numerical factor N to account for their extra contributions. So for the graviton-gravitino system we have

$$\delta\rho^{(4)}_{\not{s}} \cong NC_n \cdot \frac{1}{a^4} \cdot \mu^2\, a^2 \cdot n! \left(\frac{V_\delta}{V_a}\right). \qquad (11)$$

Generally there maybe more than one pair of SUSY fields in the bulk. In that case the total Caimir energy density becomes

$$\delta\rho^{(4)}_{\not{s},\mathrm{total}} \;\cong\; \left(\sum_i N_i C_n \mu_i^2\right) \cdot \frac{1}{a^2} \cdot n! \left(\frac{V_\delta}{V_a}\right) \qquad (12)$$

$$\;=\; \alpha_n \cdot \mu^2_{\mathrm{MAX}} \delta^n a^{-(n+2)}\,, \qquad (13)$$

$$\alpha_n \;\equiv\; \left(\sum_i N_i C_n \frac{\mu_i^2}{\mu^2_{\mathrm{MAX}}}\right) \cdot \frac{\pi^{n/2}\Gamma(n)}{2^{n-1}\Gamma(n/2)}\,, \qquad (14)$$

where the subscript 'i' denotes the i-th SUSY field and μ^2_{MAX} denotes the largest mass-square shift among those SUSY fields in the bulk. As a result, the SUSY fields which possess overwhelmingly large mass-square shifts dominate the Casimir energy.

By insisting $\rho^{(4)}_{\not{s},\mathrm{total}}$ as dark energy, we are in effect imposing a constraint on several relevant fundamental physical quantities. Using the relation between the Planck scale M_{pl}, the fundamental gravity scale M_{G}, and the extra-dimension size a, $M^2_{\mathrm{pl}} = M_{\mathrm{G}}^{n+2} a^n$, assuming $\delta \sim l_{\mathrm{S}} = M_{\mathrm{S}}^{-1}$, and introducing the ratio η of the (dominant) mass shift μ_{MAX} to the SUSY-breaking scale M_{SUSY}, we rewrite Eq. (13) as

$$\delta\rho^{(4)}_{\not{s},\mathrm{total}} \sim \alpha_n \eta^2 M^2_{\mathrm{SUSY}} M_{\mathrm{S}}^{-n} M_{\mathrm{pl}}^{-2(n+2)/n} M_{\mathrm{G}}^{(n+2)^2/n}. \qquad (15)$$

We further identify the Casimir energy as the dark energy with the density $\sim 3 \times 10^{-11}\,\mathrm{eV}^4$. We then arrive at the following constraint among the several energy scales:

$$\left(\frac{M_{\mathrm{S}}}{M_{\mathrm{pl}}}\right)^{-n} \left(\frac{M_{\mathrm{G}}}{M_{\mathrm{pl}}}\right)^{(n+2)^2/n} \left(\frac{M_{\mathrm{SUSY}}}{M_{\mathrm{pl}}}\right)^2 \sim 10^{123} \cdot \alpha_n^{-1} \eta^{-2}. \qquad (16)$$

This constraint is quite loose, i.e., it can be satisfied by a wide range of M_{S}, M_{G} and M_{SUSY}. Its looseness indicates that the smallness of the dark energy can be easily achieved in our model. In the following we will see that this constraint remains flexible even after additional conditions are imposed.

In our model, three undetermined energy scales have been invoked: The string scale M_{S}, the fundamental gravity scale M_{G}, and the SUSY-breaking scale M_{SUSY}. Although there is no a priori reason why these scales should be related, in the spirit of unified field theories one naturally expects additional connections to reduce their arbitrariness. In particular, it is highly desirable to reduce the large hierarchy among various energy scales. With this in mind, we impose further conditions to illustrate the constraint derived in Eq. (16): (a) $M_{\mathrm{SUSY}} \sim M_{\mathrm{G}}$, i.e. bridging the hierarchy between the SUSY-breaking scale and the fundamental gravity scale; (b) $M_{\mathrm{S}} \sim M_{\mathrm{G}} \sim M_{\mathrm{SUSY}}$, i.e. insisting that there is only one mass scale in our physics.

First we focus on the scenario where the mass shift is dominated by that of the gravitino, which is suppressed by the Planck scale: $\mu \sim M^2_{\mathrm{SUSY}}/M_{\mathrm{pl}}$ (i.e. $\eta \sim M_{\mathrm{SUSY}}/M_{\mathrm{pl}}$). Let us further assume that the values of α_n do not vary too drastically. Then in case (a) our general constraint, Eq. (16), is reduced to a more specific constraint on M_{S} and M_{G}, under different choices of the extra-dimensionality, as represented by solid curves in Fig. 1. If one further insists on condition (b), then the solutions further reduce to the intersects between the line for $M_{\mathrm{S}} = M_{\mathrm{G}}$ and the solid curves. Concentrating on the

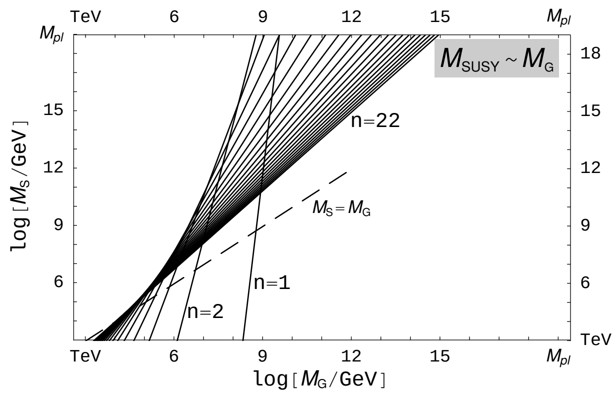

FIG. 1: Constraint on M_{S} and M_{G} under the assumption of gravitino dominance: $\mu_{\mathrm{MAX}} \sim M^2_{\mathrm{SUSY}}/M_{\mathrm{pl}}$ (i.e. $\eta \sim M_{\mathrm{SUSY}}/M_{\mathrm{pl}}$). The solid curves correspond to solutions under the further assumption of $M_{\mathrm{SUSY}} = M_{\mathrm{G}}$ and the dashed line indicates the condition $M_{\mathrm{S}} = M_{\mathrm{G}}$

energies between TeV and the Planck scale, we find that in case (a) M_{G} cannot exceed $10^{15}\,\mathrm{GeV}$ while the string scale M_{S} is barely restricted. In case (b), the specified value of these quantities is restricted in the range between TeV and $10^9\,\mathrm{GeV}$, and is getting close to TeV, a soon-to-be testable scale, when the number of extra dimensions n becomes larger.

It is also possible that the dominant mass shift μ_{MAX} is roughly of the same order of the SUSY-breaking scale M_{SUSY} (i.e. $\eta \sim 1$). So we repeat the same exercise but replace the condition $\eta \sim M_{\mathrm{SUSY}}/M_{\mathrm{pl}}$ by $\eta \sim 1$. The results are shown in Fig. 2. Case (a) possesses similar qualitative features as the previous case regarding the gravitino mass shift. Roughly speaking, M_{G} is quite restricted (especially for small n) while M_{S} is barely restricted. In case (b), the constraint is so severe that only the case of $n = 2$ survives. In this case (n=2) all fundamental scales are merely of the order of a TeV and therefore can be tested in the near future. We note that in the case of one extra dimension, in which M_{G} is required to be around $10^9\,\mathrm{GeV}$ (corresponding to the extra-dimension size around 100 meters), the solution is already ruled out.

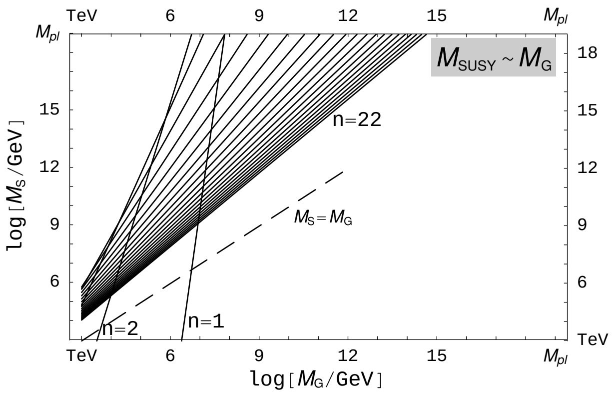

FIG. 2: Constraint on $M_{\rm S}$ and $M_{\rm G}$ under the assumption $\mu_{\rm MAX} \sim M_{\rm SUSY}$ (i.e. $\eta \sim 1$). The solid curves correspond to solutions under the further assumption of $M_{\rm SUSY} = M_{\rm G}$ and the dashed line indicates the condition $M_{\rm S} = M_{\rm G}$

We have shown that the Casimir energy in a SUSY-breaking brane-world can be small enough to play the role of the dark energy with reasonable values of $M_{\rm G}$, $M_{\rm S}$, and $M_{\rm SUSY}$. We suggest that it is the difference, instead of the absolute value, of the vacuum energy, which is relevant to gravity. Motivated by this we invoke Casimir effect and SUSY as the two basic ingredients of our model. The Casimir energy induced by extra dimensions behaves like a cosmological constant and is therefore qualitatively a reasonable candidate for dark energy. To further substantiate this notion so that our model can explain dark energy even quantitatively, we further invoke supersymmetry. SUSY, which in our case is preserved in the bulk and only broken on the brane, helps to dramatically suppress the largeness of the Casimir energy. We stress, however, that our scenario is general and does not rely on specific models or details of SUSY-breaking.

There exist various hierarchy problems in physics, such as the weakness of gravity and the cosmological constant problem. To reveal the fundamental laws of nature, it is often desirable to relate the origins of the hierarchies to more profound physics, such as invoking the largeness of extra dimension size (for diluting gravity) in the ADD model [15] or the warpage of geometry in the RS model [16] to explain the weakness of gravity.

In this regard our model follows the same spirit as that of the ADD and the RS models. The sharp contrast between the volume of the SUSY-broken brane and that of the SUSY-preserved bulk, in our model, is invoked as the origin of the smallness of the cosmological constant. Note that by further invoking $M_{\rm SUSY} \sim M_{\rm G}$, our model manages to solve two hierarchy problems at once. To turn the issue around, according to our model the smallness of the cosmological constant may actually be a manifestation of the huge difference between these two volumes in the extra space. It is interesting to note that all three models mentioned above (ADD, RS, and ours) involve extra dimensions. In particular our model suggests that the observed dark energy may be another evidence of the existence of the extra dimensions and the SUSY-breaking brane world.

We thank Keshav Dasgupta for useful discussion and comments. This work is supported by the US Department of Energy under Contract No. DE-AC03-76SF00515, and in part by the Ministry of Education (MoE 89-N-FA01-1-4) and the National Science Council (NSC 93-2811-M-002-033; NSC 93-2112-M-002-047) of Taiwan, R.O.C. JAG wishes to thank SLAC for its support and hospitality for his visit, during which this research was carried out.

[1] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999).

[2] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998).

[3] L. M. Krauss and M. S. Turner, Gen. Rel. Grav. **27**, 1137 (1995); J. P. Ostriker and P. J. Steinhardt, Nature **377**, 600 (1995); A. R. Liddle, D. H. Lyth, P. T. Viana and M. White, Mon. Not. Roy. Astron. Soc. **282**, 281 (1996).

[4] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).

[5] Je-An Gu and W-Y. P. Hwang, Phys. Lett. B **517**, 1 (2001); L. A. Boyle, R. R. Caldwell and M. Kamionkowski, Phys. Lett. B **545**, 17 (2002).

[6] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).

[7] C. Deffayet, G. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002).

[8] Je-An Gu and W-Y. P. Hwang, Phys. Rev. D **66**, 024003 (2002); Je-An Gu, in *Proceedings of CosPA 2002 Symposium*, Taiwan, 2002 (World Scientific, 2003); Je-An Gu, W-Y. P. Hwang and Jr-Wei Tsai, to appear in Nucl. Phys. B [arXiv:astro-ph/0403641].

[9] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).

[10] A. Lue, R. Scoccimarro and G. Starkman, Phys. Rev. D **69**, 044005 (2004).

[11] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, JHEP **0405**, 074 (2004).

[12] C. L. Bennett *et al.*, Astrophys. J. Suppl. **148**, 1 (2003); D. N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).

[13] H. B. G. Casimir, Kon. Ned. Akad. Wetensch. Proc. **51**, 793 (1948).

[14] K. A. Milton, Grav. Cosmol. **9**, 66 (2003).

[15] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998); I. Antoniadis, Phys. Lett. B **246**, 377 (1990).

[16] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).