

# Applications of Light-Front QCD\*

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## Abstract

Light-front Fock state wavefunctions encode the bound state properties of hadrons in terms of their quark and gluon degrees of freedom at the amplitude level. The freedom to choose the light-like quantization four-vector provides an explicitly covariant formulation of light-front quantization and can be used to determine the analytic structure of light-front wave functions. The AdS/CFT correspondence of large  $N_C$  supergravity theory in higher-dimensional anti-de Sitter space with supersymmetric QCD in 4-dimensional space-time has interesting implications for hadron phenomenology in the conformal limit, including an all-orders demonstration of counting rules for exclusive processes. String/gauge duality also predicts the QCD power-law behavior of light-front Fock-state hadronic wavefunctions with arbitrary orbital angular momentum at high momentum transfer. The form of these near-conformal wavefunctions can be used as an initial ansatz for a variational treatment of the light-front QCD Hamiltonian. I also briefly review recent work which shows that some leading-twist phenomena such as the diffractive component of deep inelastic scattering, single spin asymmetries, nuclear shadowing and antishadowing cannot be computed from the LFWFs of hadrons in isolation.

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# 1 Introduction

Light-front Fock state wavefunctions  $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$  encode the bound-state quark and gluon properties of hadrons, including their spin and flavor correlations, in the form of universal process- and frame- independent amplitudes. Because the generators of certain Lorentz boosts are kinematical, knowing the LFWFs in one frame allows one to obtain it in any other frame. LFWFs underlie virtually all areas of QCD phenomenology. The hadronic distribution amplitudes which control hard exclusive processes are computed from the valence Fock state LFWFs. Matrix elements of space-like local operators for the coupling of photons, gravitons, and the moments of deep inelastic structure functions all can be expressed as overlaps of light-front wavefunctions with the same number of Fock constituents. Similarly, the exclusive decays of heavy hadrons such as the  $B$  meson are computed from overlaps of LFWFs. The decays of heavy hadrons. The unintegrated parton distributions and generalized parton distributions measured in deeply virtual Compton scattering can be constructed from LFWFs. Hadronization phenomena such as the coalescence mechanism for leading heavy hadron production are computed from LFWF overlaps. Diffractive jet production provides another phenomenological window into the structure of LFWFs. However, some leading-twist phenomena such as the diffractive component of deep inelastic scattering, single spin asymmetries, nuclear shadowing and antishadowing cannot be computed from the LFWFs of hadrons in isolation.

Formally, the light-front expansion is constructed by quantizing QCD at fixed light-cone time [1]  $\tau = t + z/c$  and forming the invariant light-front Hamiltonian:  $H_{LF}^{QCD} = P^+ P^- - \vec{P}_{\perp}^2$  where  $P^{\pm} = P^0 \pm P^z$  [2]. The momentum generators  $P^+$  and  $\vec{P}_{\perp}$  are kinematical; *i.e.*, they are independent of the interactions. The generator  $P^- = i \frac{d}{d\tau}$  generates light-cone time translations, and the eigen-spectrum of the Lorentz scalar  $H_{LF}^{QCD}$  gives the mass spectrum of the color-singlet hadron states in QCD together with their respective light-front wavefunctions. For example, the proton state satisfies:  $H_{LF}^{QCD} |\psi_p\rangle = M_p^2 |\psi_p\rangle$ . The expansion of the proton eigensolution  $|\psi_p\rangle$  on the color-singlet  $B = 1$ ,  $Q = 1$  eigenstates  $\{|n\rangle\}$  of the free Hamiltonian  $H_{LF}^{QCD}(g = 0)$  gives the light-front Fock expansion:

$$\begin{aligned}
 |\psi_p(P^+, \vec{P}_{\perp})\rangle &= \sum_n \prod_{i=1}^n \frac{dx_i d^2 \vec{k}_{\perp i}}{\sqrt{x_i} 16\pi^3} 16\pi^3 \delta\left(1 - \sum_{i=1}^n x_i\right) \delta^{(2)}\left(\sum_{i=1}^n \vec{k}_{\perp i}\right) \\
 &\quad \times \psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i) \left|n; x_i P^+, x_i \vec{P}_{\perp} + \vec{k}_{\perp i}, \lambda_i\right\rangle.
 \end{aligned}$$

The light-cone momentum fractions  $x_i = k_i^+/P^+$  and  $\vec{k}_{\perp i}$  represent the relative momentum coordinates of the QCD constituents. The physical transverse momenta are  $\vec{p}_{\perp i} = x_i \vec{P}_{\perp} + \vec{k}_{\perp i}$ . The  $\lambda_i$  label the light-cone spin projections  $S^z$  of the quarks and gluons along the quantization direction  $z$ . The physical gluon polarization vectors  $\epsilon^{\mu}(k, \lambda = \pm 1)$  are specified in light-cone gauge by the conditions  $k \cdot \epsilon = 0$ ,  $\eta \cdot \epsilon = \epsilon^+ = 0$ .

The solutions of  $H_{LF}^{QCD} |\psi_p\rangle = M_p^2 |\psi_p\rangle$  are independent of  $P^+$  and  $\vec{P}_\perp$ ; thus given the eigensolution Fock projections  $\langle n; x_i, \vec{k}_\perp, \lambda_i | \psi_p \rangle = \psi_n(x_i, \vec{k}_\perp, \lambda_i)$ , the wavefunction of the proton is determined in any frame [3]. In contrast, in equal-time quantization, a Lorentz boost always mixes dynamically with the interactions, so that computing a wavefunction in a new frame requires solving a nonperturbative problem as complicated as the Hamiltonian eigenvalue problem itself. The LFWFs  $\psi_{n/H}(x_i, \vec{k}_\perp, \lambda_i)$  are properties of the hadron itself; they are thus universal and process independent.

One can also define the light-front Fock expansion using a covariant generalization of light-front time:  $\tau = x \cdot \omega$ . The four-vector  $\omega$ , with  $\omega^2 = 0$ , determines the orientation of the light-front plane; the freedom to choose  $\omega$  provides an explicitly covariant formulation of light-front quantization [4]: all observables such as matrix elements of local current operators, form factors, and cross sections are light-front invariants – they must be independent of  $\omega_\mu$ . In recent work, Dae Sung Hwang, John Hiller, Volodya Karmonov [5], and I have studied the analytic structure of LFWFs using the explicitly Lorentz-invariant formulation of the front form. Eigensolutions of the Bethe-Salpeter equation have specific angular momentum as specified by the Pauli-Lubanski vector. The corresponding LFWF for an  $n$ -particle Fock state evaluated at equal light-front time  $\tau = \omega \cdot x$  can be obtained by integrating the Bethe-Salpeter solutions over the corresponding relative light-front energies. The resulting LFWFs  $\psi_n^I(x_i, k_\perp)$  are functions of the light-cone momentum fractions  $x_i = k_i \cdot \omega / p \cdot \omega$  and the invariant mass squared of the constituents  $M_0^2 = (\sum_{i=1}^n k_i^\mu)^2 = \sum_{i=1}^n [\frac{k_\perp^2 + m^2}{x}]_i$  and the light-cone momentum fractions  $x_i = k \cdot \omega / p \cdot \omega$  each multiplying spin-vector and polarization tensor invariants which can involve  $\omega^\mu$ . They are eigenstates of the Karmanov–Smirnov kinematic angular momentum operator [6]. Thus LFWFs satisfy all Lorentz symmetries of the front form, including boost invariance, and they are proper eigenstates of angular momentum.

## 2 Light-Front Wavefunctions and QCD Phenomenology

Given the light-front wavefunctions, one can compute the unintegrated parton distributions in  $x$  and  $k_\perp$  which underlie generalized parton distributions for nonzero skewness. As shown by Diehl, Hwang, and myself [7], one can give a complete representation of virtual Compton scattering  $\gamma^* p \rightarrow \gamma p$  at large initial photon virtuality  $Q^2$  and small momentum transfer squared  $t$  in terms of the light-cone wavefunctions of the target proton. One can then verify the identities between the skewed parton distributions  $H(x, \zeta, t)$  and  $E(x, \zeta, t)$  which appear in deeply virtual Compton scattering and the corresponding integrands of the Dirac and Pauli form factors  $F_1(t)$  and  $F_2(t)$  and the gravitational form factors  $A_q(t)$  and  $B_q(t)$  for each quark and anti-quark constituent. We have illustrated the general formalism for the case of deeply

virtual Compton scattering on the quantum fluctuations of a fermion in quantum electrodynamics at one loop.

The integrals of the unintegrated parton distributions over transverse momentum at zero skewness provide the helicity and transversity distributions measurable in polarized deep inelastic experiments [3]. For example, the polarized quark distributions at resolution  $\Lambda$  correspond to

$$\begin{aligned}
q_{\lambda_q/\Lambda_p}(x, \Lambda) &= \sum_{n, q_a} \int \prod_{j=1}^n dx_j d^2 k_{\perp j} \sum_{\lambda_i} |\psi_{n/H}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 \\
&\times \delta\left(1 - \sum_i^n x_i\right) \delta^{(2)}\left(\sum_i^n \vec{k}_{\perp i}\right) \delta(x - x_q) \\
&\times \delta_{\lambda_a, \lambda_q} \Theta(\Lambda^2 - \mathcal{M}_n^2),
\end{aligned}$$

where the sum is over all quarks  $q_a$  which match the quantum numbers, light-cone momentum fraction  $x$ , and helicity of the struck quark. As shown by Raufeisen and myself [8], one can construct a “light-front density matrix” from the complete set of light-front wavefunctions which is a Lorentz scalar. This form can be used at finite temperature to give a boost invariant formulation of thermodynamics. At zero temperature the light-front density matrix is directly connected to the Green’s function for quark propagation in the hadron as well as deeply virtual Compton scattering. In addition, moments of transversity distributions and off-diagonal helicity convolutions are defined from the density matrix of the light-cone wavefunctions. The light-front wavefunctions also specify the multi-quark and gluon correlations of the hadron. For example, the distribution of spectator particles in the final state which could be measured in the proton fragmentation region in deep inelastic scattering at an electron-proton collider are in principle encoded in the light-front wavefunctions.

Given the  $\psi_{n/H}^{(\Lambda)}$ , one can construct any spacelike electromagnetic, electroweak, or gravitational form factor or local operator product matrix element of a composite or elementary system from the diagonal overlap of the LFWFs [9]. Exclusive semi-leptonic  $B$ -decay amplitudes involving timelike currents such as  $B \rightarrow A\ell\bar{\nu}$  can also be evaluated exactly in the light-front formalism [10]. In this case, the timelike decay matrix elements require the computation of both the diagonal matrix element  $n \rightarrow n$  where parton number is conserved and the off-diagonal  $n+1 \rightarrow n-1$  convolution such that the current operator annihilates a  $q\bar{q}'$  pair in the initial  $B$  wavefunction. This term is a consequence of the fact that the time-like decay  $q^2 = (p_\ell + p_{\bar{\nu}})^2 > 0$  requires a positive light-cone momentum fraction  $q^+ > 0$ . Conversely for space-like currents, one can choose  $q^+ = 0$ , as in the Drell-Yan-West representation of the space-like electromagnetic form factors. The light-front Fock representation thus provides an exact formulation of current matrix elements of local operators. In contrast, in equal-time Hamiltonian theory, one must evaluate connected time-ordered diagrams where the gauge particle or graviton couples to particles associated with vacuum fluctuations. Thus even if one knows the equal-time wavefunction for the initial and final hadron,

one cannot determine the current matrix elements. In the case of the covariant Bethe-Salpeter formalism, the evaluation of the matrix element of the current requires the calculation of an infinite number of irreducible diagram contributions. One can also prove that the anomalous gravitomagnetic moment  $B(0)$  vanishes for any composite system [11]. This property follows directly from the Lorentz boost properties of the light-front Fock representation and holds separately for each Fock state component.

One of the central issues in the analysis of fundamental hadron structure is the presence of non-zero orbital angular momentum in the bound-state wave functions. The evidence for a “spin crisis” in the Ellis-Jaffe sum rule signals a significant orbital contribution in the proton wave function [12, 13]. The Pauli form factor of nucleons is computed from the overlap of LFWFs differing by one unit of orbital angular momentum  $\Delta L_z = \pm 1$ . Thus the fact that the anomalous moment of the proton is non-zero requires nonzero orbital angular momentum in the proton wavefunction [9]. In the light-front method, orbital angular momentum is treated explicitly; it includes the orbital contributions induced by relativistic effects, such as the spin-orbit effects normally associated with the conventional Dirac spinors.

### 3 Complications from Final State Interactions

It is usually assumed—following the parton model—that the leading-twist structure functions measured in deep inelastic lepton-proton scattering are simply the probability distributions for finding quarks and gluons in the target nucleon. In fact, gluon exchange between the fast, outgoing quarks and the target spectators effects the leading-twist structure functions in a profound way, leading to diffractive lepton-production processes, shadowing of nuclear structure functions, and target spin asymmetries. In particular, the final-state interactions from gluon exchange lead to single-spin asymmetries in semi-inclusive deep inelastic lepton-proton scattering which are not power-law suppressed in the Bjorken limit.

A new understanding of the role of final-state interactions in deep inelastic scattering has recently emerged [14]. The final-state interactions from gluon exchange between the outgoing quark and the target spectator system lead to single-spin asymmetries in semi-inclusive deep inelastic lepton-proton scattering at leading twist in perturbative QCD; *i.e.*, the rescattering corrections of the struck quark with the target spectators are not power-law suppressed at large photon virtuality  $Q^2$  at fixed  $x_{bj}$  [15]. The final-state interaction from gluon exchange occurring immediately after the interaction of the current also produces a leading-twist diffractive component to deep inelastic scattering  $\ell p \rightarrow \ell' p' X$  corresponding to color-singlet exchange with the target system; this in turn produces shadowing and anti-shadowing of the nuclear structure functions [14, 16]. In addition, one can show that the pomeron structure function derived from diffractive DIS has the same form as the quark contribution of the gluon structure function [17]. The final-state interactions occur at a light-cone time  $\Delta\tau \simeq 1/\nu$  after the virtual photon interacts with the struck quark, producing

a nontrivial phase. Thus none of the above phenomena is contained in the target light-front wave functions computed in isolation. In particular, the shadowing of nuclear structure functions is due to destructive interference effects from leading-twist diffraction of the virtual photon, physics not included in the nuclear light-front wave functions. Thus the structure functions measured in deep inelastic lepton scattering are affected by final-state rescattering, modifying their connection to light-front probability distributions. Some of these results can be understood by augmenting the light-front wave functions with a gauge link, but with a gauge potential created by an external field created by the virtual photon  $q\bar{q}$  pair current [18]. The gauge link is also process dependent [19], so the resulting augmented LFWFs are not universal.

The shadowing and antishadowing of nuclear structure functions in the Gribov-Glauber picture is due to the destructive and constructive coherence, respectively, of amplitudes arising from the multiple-scattering of quarks in the nucleus. The effective quark-nucleon scattering amplitude includes Pomeron and Odderon contributions from multi-gluon exchange as well as Reggeon quark exchange contributions [16]. The multiscattering nuclear processes from Pomeron, Odderon and pseudoscalar Reggeon exchange leads to shadowing and antishadowing of the electromagnetic nuclear structure functions in agreement with measurements. This picture leads to substantially different nuclear effects for charged and neutral currents, particularly in anti-neutrino reactions, thus affecting the extraction of the weak-mixing angle  $\sin^2 \theta_W$  and the constant  $\rho_o$  which are determined from the ratio of charged and neutral current deep inelastic from neutrino and anti-neutrino scattering. In recent work, Schmidt, Yang, and I [20] find that a substantial part of the difference between the standard model prediction and the anomalous NuTeV result [21] for  $\sin^2 \theta_W$  could be due to the different behavior of nuclear antishadowing for charged and neutral currents. Detailed measurements of the nuclear dependence of charged, neutral and electromagnetic DIS processes are needed to establish the distinctive phenomenology of shadowing and antishadowing and to make the NuTeV results definitive.

## 4 Other Aspects of Light-Front Wavefunction Phenomenology

A number of other important phenomenological properties follow directly from the structure of light-front wavefunctions in gauge theory.

(1) Color transparency. The small transverse size fluctuations of a hadron wavefunction with a small color dipole moment will have minimal interactions in a nucleus [22, 23]. This has been verified in the case of diffractive dissociation of a high energy pion into dijets  $\pi A \rightarrow q\bar{q}A'$  in which the nucleus is left in its ground state [24]. When the hadronic jets have balancing but high transverse momentum, one studies the small size fluctuation of the incident pion. The diffractive dissociation cross section is found to be proportional to  $A^2$  in agreement with the color transparency

prediction. Color transparency has also been observed in diffractive electroproduction of  $\rho$  mesons [25] and in quasi-elastic  $pA \rightarrow ppA - 1$  scattering [26] where only the small size fluctuations of the hadron wavefunction enters the hard exclusive scattering amplitude. In the latter case an anomaly occurs at  $\sqrt{s} \simeq 5$  GeV, most likely signaling a resonance effect at the charm threshold [27].

(2) Intrinsic charm [28]. The probability for Fock states of a light hadron such as the proton to have an extra heavy quark pair decreases as  $1/m_Q^2$  in non-Abelian gauge theory [29, 30]. The relevant matrix element is the cube of the QCD field strength  $G_{\mu\nu}^3$ . This is in contrast to abelian gauge theory where the relevant operator is  $F_{\mu\nu}^4$  and the probability of intrinsic heavy leptons in QED bound state is suppressed as  $1/m_\ell^4$ . The intrinsic Fock state probability is maximized at minimal off-shellness. The maximum probability occurs at  $x_i = m_\perp^i / \sum_{j=1}^n m_\perp^j$ ; i.e., when the Constituents have equal rapidity. Thus the heaviest constituents have the highest momentum fractions and highest  $x$ . Intrinsic charm thus predicts that the charm structure function has support at large  $x_{bj}$  in excess of DGLAP extrapolations [28]; this is in agreement with the EMC measurements [31]. It predicts leading charm hadron production and fast charmonium production in agreement with measurements [32]. The production cross section for the double charmed  $\Xi_{cc}^+$  baryon [33] and the production of double  $J/\psi$ 's appears to be consistent with the dissociation and coalescence of double IC Fock states [34]. Intrinsic charm can also explain the  $J/\psi \rightarrow \rho\pi$  puzzle [35]. It also affects the extraction of suppressed CKM matrix elements in  $B$  decays [36].

## 5 Solving for Light-front Wavefunctions

In principle, one can solve for the LFWFs directly from the fundamental theory using methods such as discretized light-front quantization (DLCQ), the transverse lattice, lattice gauge theory moments, or Bethe–Salpeter techniques. DLCQ has been remarkably successful in determining the entire spectrum and corresponding LFWFs in 1+1 field theories, including supersymmetric examples. Reviews of nonperturbative light-front methods may be found in references [2, 4, 37, 38]. One can also project the known solutions of the Bethe–Salpeter equation to equal light-front time, thus producing hadronic light-front Fock wave functions. A potentially important method is to construct the  $q\bar{q}$  Green's function using light-front Hamiltonian theory, with DLCQ boundary conditions and Lippmann-Schwinger resummation. The zeros of the resulting resolvent projected on states of specific angular momentum  $J_z$  can then generate the meson spectrum and their light-front Fock wavefunctions. The DLCQ properties and boundary conditions allow a truncation of the Fock space while retaining the kinematic boost and Lorentz invariance of light-front quantization.

Even without explicit solutions, much is known about the explicit form and structure of LFWFs. They can be matched to nonrelativistic Schrodinger wavefunctions at soft scales. At high momenta, the LFWFs large  $k_\perp$  and  $x_i \rightarrow 1$  are constrained by arguments based on conformal symmetry, the operator product expansion, or pertur-

bative QCD. The pattern of higher Fock states with extra gluons is given by ladder relations. The structure of Fock states with nonzero orbital angular momentum is also constrained.

## 6 The Infrared Behavior of Effective QCD Couplings

Theoretical [39, 40, 41, 42, 43] and phenomenological [44, 45, 46] evidence is now accumulating that the QCD coupling becomes constant at small virtuality; *i.e.*,  $\alpha_s(Q^2)$  develops an infrared fixed point in contradiction to the usual assumption of singular growth in the infrared. If QCD running couplings are bounded, the integration over the running coupling is finite and renormalon resummations are not required. If the QCD coupling becomes scale-invariant in the infrared, then elements of conformal theory [47] become relevant even at relatively small momentum transfers.

Menke, Merino, and Rathsman [45] and I have presented a definition of a physical coupling for QCD which has a direct relation to high precision measurements of the hadronic decay channels of the  $\tau^- \rightarrow \nu_\tau h^-$ . Let  $R_\tau$  be the ratio of the hadronic decay rate to the leptonic one. Then  $R_\tau \equiv R_\tau^0 \left[1 + \frac{\alpha_\tau}{\pi}\right]$ , where  $R_\tau^0$  is the zeroth order QCD prediction, defines the effective charge  $\alpha_\tau$ . The data for  $\tau$  decays is well-understood channel by channel, thus allowing the calculation of the hadronic decay rate and the effective charge as a function of the  $\tau$  mass below the physical mass. The vector and axial-vector decay modes which can be studied separately. Using an analysis of the  $\tau$  data from the OPAL collaboration [48], we have found that the experimental value of the coupling  $\alpha_\tau(s) = 0.621 \pm 0.008$  at  $s = m_\tau^2$  corresponds to a value of  $\alpha_{\overline{\text{MS}}}(M_Z^2) = (0.117\text{-}0.122) \pm 0.002$ , where the range corresponds to three different perturbative methods used in analyzing the data. This result is in good agreement with the world average  $\alpha_{\overline{\text{MS}}}(M_Z^2) = 0.117 \pm 0.002$ . However, one also finds that the effective charge only reaches  $\alpha_\tau(s) \sim 0.9 \pm 0.1$  at  $s = 1 \text{ GeV}^2$ , and it even stays within the same range down to  $s \sim 0.5 \text{ GeV}^2$ . The effective coupling is close to constant at low scales, suggesting that physical QCD couplings become constant or “frozen” at low scales.

The near constancy of the effective QCD coupling at small scales helps explain the empirical success of dimensional counting rules for the power law fall-off of form factors and fixed angle scaling. As shown in the references [49, 50], one can calculate the hard scattering amplitude  $T_H$  for such processes [3] without scale ambiguity in terms of the effective charge  $\alpha_\tau$  or  $\alpha_R$  using commensurate scale relations. The effective coupling is evaluated in the regime where the coupling is approximately constant, in contrast to the rapidly varying behavior from powers of  $\alpha_s$  predicted by perturbation theory (the universal two-loop coupling). For example, the nucleon form factors are proportional at leading order to two powers of  $\alpha_s$  evaluated at low scales in addition to two powers of  $1/q^2$ ; The pion photoproduction amplitude at fixed

angles is proportional at leading order to three powers of the QCD coupling. The essential variation from leading-twist counting-rule behavior then only arises from the anomalous dimensions of the hadron distribution amplitudes.

Parisi [51] has shown that perturbative QCD becomes a conformal theory for  $\beta \rightarrow 0$  and zero quark mass. There are a number of useful phenomenological consequences of near conformal behavior: the conformal approximation with zero  $\beta$  function can be used as template for QCD analyses [52, 53] such as the form of the expansion polynomials for distribution amplitudes [47, 54]. The near-conformal behavior of QCD is also the basis for commensurate scale relations [55] which relate observables to each other without renormalization scale or scheme ambiguities [56]. An important example is the generalized Crewther relation [57]. In this method the effective charges of observables are related to each other in conformal gauge theory; the effects of the nonzero QCD  $\beta$ -function are then taken into account using the BLM method [58] to set the scales of the respective couplings. The magnitude of the corresponding effective charge [49]  $\alpha_s^{\text{exclusive}}(Q^2) = F_\pi(Q^2)/4\pi Q^2 F_{\gamma\pi^0}^2(Q^2)$  for exclusive amplitudes is connected to  $\alpha_\tau$  by a commensurate scale relation. Its magnitude:  $\alpha_s^{\text{exclusive}}(Q^2) \sim 0.8$  at small  $Q^2$ , is sufficiently large as to explain the observed magnitude of exclusive amplitudes such as the pion form factor using the asymptotic distribution amplitude.

## 7 AdS/CFT and Near-Conformal Field Theory

As shown by Maldacena [59], there is a remarkable correspondence between large  $N_C$  supergravity theory in a higher dimensional anti-de Sitter space and supersymmetric QCD in 4-dimensional space-time. String/gauge duality provides a framework for predicting QCD phenomena based on the conformal properties of the ADS/CFT correspondence. For example, Polchinski and Strassler [60] have shown that the power-law fall-off of hard exclusive hadron-hadron scattering amplitudes at large momentum transfer can be derived without the use of perturbation theory by using the scaling properties of the hadronic interpolating fields in the large- $r$  region of AdS space. Thus one can use the Maldacena correspondence to compute the leading power-law falloff of exclusive processes such as high-energy fixed-angle scattering of gluonium-gluonium scattering in supersymmetric QCD. The resulting predictions for hadron physics effectively coincide [60, 61, 62] with QCD dimensional counting rules [63, 64, 65, 66].) Polchinski and Strassler [60] have also derived counting rules for deep inelastic structure functions at  $x \rightarrow 1$  in agreement with perturbative QCD predictions [67] as well as Bloom-Gilman exclusive-inclusive duality. An interesting point is that the hard scattering amplitudes which are normally of order  $\alpha_s^p$  in PQCD appear as order  $\alpha_s^{p/2}$  in the supergravity predictions. This can be understood as an all-orders resummation of the effective potential [59, 68]. The near-conformal scaling properties of light-front wavefunctions thus lead to a number of important predictions for QCD which are normally discussed in the context of perturbation theory.

De Teramond and I [69] have shown how one can use the scaling properties of the hadronic interpolating operator in the extended AdS/CFT space-time theory to determine the scaling of light-front hadronic wavefunctions at high relative transverse momentum. The angular momentum dependence of the light-front wavefunctions also follow from the conformal properties of the AdS/CFT correspondence. The scaling and conformal properties of the correspondence leads to a hard component of the light-front Fock state wavefunctions of the form:

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i, l_{zi}) \sim \frac{(g_s N_C)^{\frac{1}{2}(n-1)}}{\sqrt{N_C}} \prod_{i=1}^{n-1} (k_{i\perp}^{\pm})^{|l_{zi}|} \left[ \frac{\Lambda_o}{M^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} + \Lambda_o^2} \right]^{n+|l_z|-1},$$

where  $g_s$  is the string scale and  $\Lambda_o$  represents the basic QCD mass scale. The scaling predictions agree with the perturbative QCD analysis given in the references [70], but the AdS/CFT analysis is performed at strong coupling without the use of perturbation theory. The form of these near-conformal wavefunctions can be used as an initial ansatz for a variational treatment of the light-front QCD Hamiltonian.

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