HIP-2004-19/TH, SLAC-PUB-10416, TIFR/TH/04-10

Higher Dimensional Models of Light Majorana Neutrinos Confronted by Data

JoAnne L. Hewett*,[1,†] Probir Roy,[2,‡] and Sourov Roy[3,§]

[1] *Stanford Linear Acclerator Center, Menlo Park, CA 94025, USA*
[2] *Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai - 400 005, India*
[3] *Helsinki Institute of Physics, P.O. Box 64, FIN-00014 University of Helsinki, Finland*

(Received May 3, 2004)

We discuss experimental and observational constraints on certain models of higher Dimensional light Majorana neutrinos. Models with flavor blind brane-bulk couplings plus three or four flavor diagonal light Majorana neutrinos on the brane, with subsequent mixing induced solely by the Kaluza-Klein tower of states, are found to be excluded by data on the oscillations of solar, atmospheric and reactor neutrinos, taken together with the WMAP upper bound on the sum of neutrino masses. Extra dimensions, if relevant to neutrino mixing, need to discriminate between neutrino flavors.

PACS numbers: 11.25.Mj, 14.60.Pq, 14.60.St

Do neutrinos have anything to do with large extra dimensions [1] ? Do these extra dimensions discriminate between neutrino flavors ? These questions have engaged the attention of many authors [2] during the last few years. Let us consider the simple picture of a three-brane, containing the Standard Model (SM) fields, embedded in a higher dimensional bulk. The basic idea then is to introduce one (more than one) right chiral neutrino(s), singlet(s) with respect to the SM gauge group, in the bulk [3]. Tiny physical masses for the active $SU(2)_L$ doublet neutrinos on the brane, with mixing among themselves as well as with sterile components, then get induced. These are controlled by the size(s) of the compactified extra dimension(s) and the strength(s) of the couplings(s) between the brane and the bulk states. There is not only the zero mode, but also an accompanying tower of heavier sterile neutrinos with arithmetically progressing masses $\pm k/R$, k=1,2,... and R being the radius of the largest extra dimension. The leakage of a propagating active neutrino into these sterile states has to be kept controllably small so as not to significantly alter the conventional oscillation probability formula [5] in the standard picture of three directly mixed active neutrinos. An immediate concern, nonetheless, is whether this picture is compatible with the pattern of neutrino masses and mixing angles implied by the latest oscillation and cosmological data. Our note will address precisely this concern, focussing on the question whether the brane-bulk couplings are flavor blind or whether they are forced by all the data to have significant flavor dependence.

A relevant basic issue is whether the physical massive neutrinos are Dirac or Majorana particles. There is a relatively straightforward picture in the former case. One combination of the bulk neutrinos now becomes the right chiral partner of an active neutrino on the brane. Together they acquire a Dirac mass $m_D \sim hvM_F/M_{Pl}$, where v is the electroweak vacuum expectation value (VEV), h a Yukawa coupling strength (controlling the interaction between the brane and bulk states) which is less than of order unity (say), M_{Pl} the reduced Planck mass in (1+3) dimensions whereas M_F is the higher dimensional fundamental scale, proposedly $\sim$ 100 TeV. This scenario was given a thorough phenomenological examination in Ref. [6] with the conclusion that it is consistent with all extant experimental and observational constraints, provided $R^{-1} > 0.24$ eV. However, despite providing a natural explanation of the smallness of neutrino masses, this approach sheds no light on the observed pattern of neutrino mixing, nor on the hierarchy - if any - between the neutrino masses. These supposedly arise from the unknown structure of the Yukawa coupling matrix h in generation space. Any observed pattern of mixing and mass hierarchy among the neutrinos can be accommodated by the said unknown structure, the only requirement being the strict absence of neutrinoless double β-decay. There is a further simple generalization of this picture with heavy right chiral Majorana neutrinos in the bulk leading to light physical Majorana neutrinos via the seesaw mechanism. There also one needs to invoke (unknown) flavor dependent brane-bulk couplings.

There are higher dimensional scenarios [7–9] with a somewhat opposite approach. These assume tiny flavor diagonal Majorana masses for left chiral neutrinos on the brane with no explanation for their smallness, but invoke extra dimensions to explain their mixing pattern. Now the higher dimensional KK modes induce a seesaw leading to light physical Majorana neutrinos with flavor mixing. However, the mixing pattern of the latter gets strongly constrained. Suppose [11] that (1) no flavor dependence is assumed in couplings between the brane and bulk states and (2) no flavor mixing is allowed on the

*Work supported in part by the Department of Energy, Contract DE-AC03-76SF00515
†Electronic address: hewett@SLAC.Stanford.EDU
‡Electronic address: probir@theory.tifr.res.in
§Electronic address: roy@pcu.helsinki.fi

brane with the brane Majorana neutrinos kept flavor diagonal [7–10]. Then our observation is that the Dienes-Sarcevic model [7, 12], with three flavor diagonal neutrinos on the brane, is ruled out by the data. The allowed parameter space actually gets quite constrained [8] even when an arbitrary flavor dependence is put in the brane-bulk couplings. We also exclude that version of a model [9] with four flavor diagonal brane Majorana neutrinos, which has flavor blind brane-bulk couplings.

One attractive feature of the D–S model [7] is the attempt to provide a theory of neutrino mixing instead of just ascribing it to an unknown flavor dependence of the brane-bulk couplings. The intrinsic flavor diagonal Majorana masses on the brane for the active neutrinos ν_f (f =1,2,3) are taken to be m_f, while the brane-bulk linkage is characterized by a universal mass m appearing in all brane-bulk off-diagonal elements of the full neutrino Majorana mass matrix $\mathcal{M}$. Compactification to four dimensions leads to the zero mode as well as to a tower of sterile neutrino KK states, as mentioned earlier. After the imposition of appropriate orbifold conditions, the full neutrino Majorana mass matrix in the D-S model reads

$$\mathcal{M} = \begin{pmatrix} m_1 & 0 & 0 & m & m & m & m & m & \dots \\ 0 & m_2 & 0 & m & m & m & m & m & \dots \\ 0 & 0 & m_3 & m & m & m & m & m & \dots \\ m & m & m & 0 & 0 & 0 & 0 & 0 & \dots \\ m & m & m & 0 & 1/R & 0 & 0 & 0 & \dots \\ m & m & m & 0 & 0 & -1/R & 0 & 0 & \dots \\ m & m & m & 0 & 0 & 0 & 2/R & 0 & \dots \\ m & m & m & 0 & 0 & 0 & 0 & -2/R & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{1}$$

In consequence of (1), the active neutrinos on the brane mix both among themselves and with the sterile KK states. Thus they undergo flavor oscillations on propagation. Let U be the unitary matrix which acts on mass eigenstate neutrino fields to yield flavor eigenstate neutrino fields. Evidently, $U^T\mathcal{M}U$ is diagonal. The matrix U therefore equals the inverse of the matrix of eigenvectors of $\mathcal{M}$ and is computable as such. The time-averaged probability $\overline{P_{f\to f'}}$ that $|\nu_f\rangle$ oscillates into $|\nu_{f'}\rangle$, both f and f' being flavor indices on the brane, is given [2, 5, 7] by

$$\overline{P_{f\to f'}} = \delta_{ff'} - 2\sum_{i=2}^{N}\sum_{j=1}^{i-1}\mathrm{Re}[U_{fi}U^*_{f'i}U^*_{fj}U_{f'j}]. \tag{2}$$

In (2), i and j are neutrino mass eigenstate indices.

Let us now come to the experimental numbers for the physical neutrino mass squared differences and the average neutrino oscillation probabilities. The atmospheric neutrino data yield [13, 14]

$$1.3\times10^{-3}\ \mathrm{eV}^2 \ < \ \Delta m^2_{\mathrm{atm}} < 3.2\times10^{-3}\ \mathrm{eV}^2, \tag{3}$$
$$0.9 \ < \ \sin^2 2\theta_{\mathrm{atm}} \le 1. \tag{4}$$

Also, all combined solar neutrino observations plus the KamLAND data imply that [15–17]

$$6.1\times10^{-5}\ \mathrm{eV}^2 \ < \ \Delta m^2_{\mathrm{sol}} < 9.0\times10^{-5}\ \mathrm{eV}^2, \tag{5}$$
$$0.71 \ < \ \sin^2 2\theta_{\mathrm{sol}} < 0.91. \tag{6}$$

In addition, the data [18] from the CHOOZ reactor require that [19]

$$\sin^2 2\theta_{13} \le 0.26. \tag{7}$$

In an effectively two flavor oscillation with a mixing angle ϕ, the time-averaged oscillation probability is $\frac{1}{2}\sin^2 2\phi$. Since the oscillations can be practically regarded as decoupled, we can take

$$0.45 \ < \ \overline{P_{\mathrm{atm}}} < 0.5, \tag{8}$$
$$0.355 \ < \ \overline{P_{\mathrm{sol}}} < 0.455, \tag{9}$$
$$\overline{P_{13}} \ \le \ 0.13, \tag{10}$$

for a simple-minded analysis [20]. Finally, the recent WMAP result puts [21] an upper bound on the sum of the physical masses of stable light neutrinos that mix among one another, namely

$$\sum_i m_i < 0.71\ \mathrm{eV}. \tag{11}$$

A constraint, therefore, on any model is that there must be at least three light neutrinos with the sum of their masses not exceeding 0.71 eV.

It has been argued by Pierce and Murayama [21] that the WMAP upper bound should include any light sterile neutrino that mixes with the active ones. This mixing is, of course, restricted to be rather small by the LEP constraint on invisible Z decay. A wide window for it is nonetheless allowed by the WMAP upper bound. Suppose the active fraction of the sterile neutrino is ϵ so that $\epsilon^2 \sim \sin^2(2\theta_{mix})$ and $\epsilon \le 10^{-1}$. Such a neutrino decoupled in the early universe at a temperature T where $G_F^2 T^5 \epsilon^2 \sim T^2 M_{Pl}^{-1}$. This temperature simply has to be less than that of the QCD phase transition in order to

avoid the dilution of those neutrinos by the entropy produced at this transition. Thus the range $10^{-6} < \epsilon < 10^{-1}$ is covered. This means that in most regions of the parameter space of the models examined by us, the summation in (11) includes the contribution of light sterile neutrinos.

There are five parameters in the Dienes-Sarcevic (D-S) model : the input Majorana masses $m_{1,2,3}$ for the three neutrinos on the brane, the universal brane-bulk off-diagonal coupling m and the radius of the compact extra dimension R. For the present study we can set all mass scales in eV and R in eV^{-1}. Let us also comment on the infinite number of sterile neutrinos in the model. There are, of course, certain limiting regions of the D-S parameter space when all the sterile neutrinos are sufficiently heavy to be effectively decoupled from the phenomena leading to (3) - (11). There are other regions where one or more sterile states can become part of the light neutrino spectrum. The existence of such light sterile neutrinos is already somewhat disfavored [22] by analyses of existing neutrino data. Though the situation is far from definite, results from the ongoing mini-Boone experiment are expected to provide a clearer conclusion. At the moment, several light sterile neutrinos can still be accommodated.

We now come to a numerical investigation of the D-S model in the real five-dimensional parameter space (m_1, m_2, m_3, m, R^{-1}). An initial requirement is that there must be at least three light neutrinos with the sum of their masses obeying (11). Thus those regions of the parameter space, which correspond to less than three such neutrinos, are excluded to start with. We attribute the masses $\mu_{1,2,3}$ to the physical light neutrinos, always ordered [23] such that $\mu_3 > \mu_2 > \mu_1$. The conditions (3) and (5) then apply on the pertinent differences of squared mass eigenvalues. With those conditions, we have made a complete scan of the parameter space, by using our code in Fortran and also in Mathematica. Our quest has been to see if there is any region in which all the other constraints, i.e. (7) to (11), can be satisfied. Our answer is in the negative, i.e. there is no point in the parameter space where all the four conditions can hold simultaneously. There are parts of the parameter space where (8), (10) and (11) can be satisfied. Similarly, there are other parts where the conditions of (9), (10) and (11) can be met. But these two sets of points do not overlap. Our conclusion is that the D-S model is ruled out by the data.

In making the above scan, we have needed to take the number of contributing states N in (2) to be finite though there actually are an infinite number of states in the KK tower. We choose N = 40 but have checked that *varying this truncation does not affect our results much.* Furthermore, the choice of parameters in our numerical scanning was dictated by the eigenvalues of the mass-matrix (1). Admit, for the moment, the possibility of flavor dependent couplings for the brane neutrinos, leading to off-diagonal elements m_i' of $\mathcal{M}$. The eigenvalues of $\mathcal{M}$ are then determined by the characteristic equation

$$\frac{1}{\pi}\tan(\pi\lambda) = \sum_{i=1}^{3} \frac{m_i'^2 R^2}{\lambda - m_i R}. \tag{12}$$

Here, $\lambda = \mu R$ and μ's are the eigenvalues. Let us also define $d^2 \equiv R^2(\sum_{i=1}^{3} m_i'^2)$ so that $m_i' R = d e_i$, where $\sum_{i=1}^{3} e_i^2 = 1$. The two zeros of

$$r(\lambda) \equiv \sum_{i=1}^{3} \frac{e_i^2}{\lambda - m_i R} \tag{13}$$

are given in the limit $m_1' = m_2' = m_3' = m$ (i.e., $e_1 = e_2 = e_3 = e$) by

$$\begin{aligned} \frac{\lambda_{b,c}}{R} &= \mu_{b,c} = \frac{1}{2}(p \mp c), \\ p &= 2e^2(m_1 + m_2 + m_3), \\ c &= \left[p^2 - 4e^2(m_2 m_3 + m_3 m_1 + m_1 m_2)\right]^{1/2}. \end{aligned} \tag{14}$$

It has been shown in Ref.[8] that, as d increases, two of the eigenvalues move towards μ_b and μ_c whereas the other eigenvalues move towards half-integral values times R^{-1}. For $d \gg 1$, the two eigenvalues sit at μ_b and μ_c (independent of R), while the rest are fixed at those half-integer values times R^{-1}. We have verified this property numerically. This is why we have not included large values of m ($mR = de$) in our numerical scanning of the parameter space, since that would not allow two different squared mass differences needed to explain both the solar and the atmospheric neutrino deficits. The other significant constraint on the range of $m_{1,2,3}$ is (11). Let us now consider variations in the radius of compactification R. One should note first that, so long as the dimensionless products $m_1 R$, $m_2 R$, $m_3 R$ and mR remain the same, one would get the same time-averaged oscillation probabilities. We have further checked that, for a fixed set of (m_1, m_2, m_3 and m), our conclusions do not change with the variation in R permitted by the constraints given in (3), (5) and (11). This way R can be seen to be constrained in the range aproximately given by $10^{-6}\ \mathrm{eV}^{-1} \lesssim R \lesssim 10\ \mathrm{eV}^{-1}$. However, this is the maximal allowed range of R and will change with a different choice of other parameters.

In our convention [23], $\Delta m^2_{\mathrm{sol}} = \mu_2^2 - \mu_1^2$ and $\Delta m^2_{\mathrm{atm}} = \mu_3^2 - \mu_2^2$, for a normal ordering of neutrino masses, whereas - in the case of an inverted ordering - $\Delta m^2_{\mathrm{sol}} = \mu_3^2 - \mu_2^2$ and $\Delta m^2_{\mathrm{atm}} = \mu_2^2 - \mu_1^2$. Allowed regions, in the $m_1 - m_3$ plane for the choice of $R^{-1} = 1$ eV, are shown in Figs. 1 (a) and (b). The incompatibility of the D–S model with the presently known facts from neutrino oscillation and cosmological data is made evident by these figures. In both the plots m_1, m_2 and m_3 vary between 0.001 eV and 0.31 eV in steps of 0.002 eV, whereas m is varied between 0.001 eV and 0.075 eV in steps of 0.002 eV. Though, in the inverted case, a few points satisfying the conditions

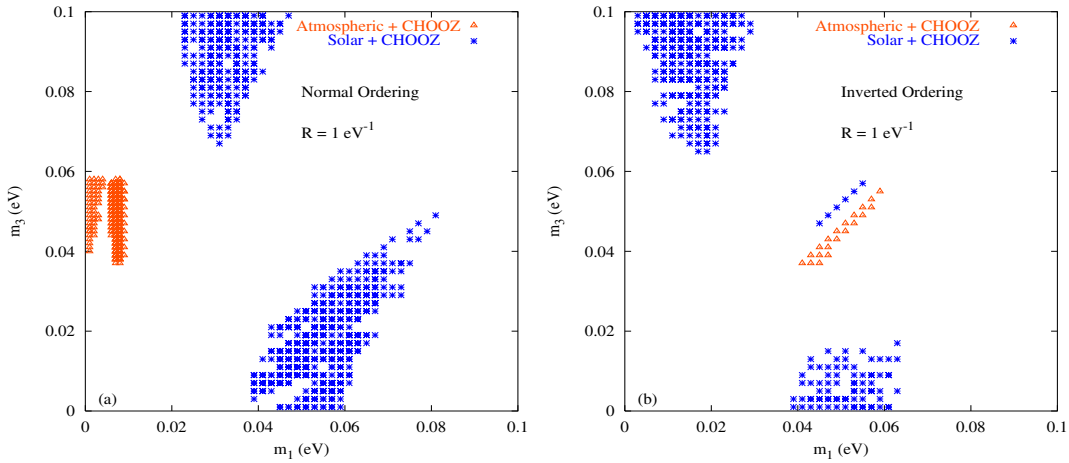

FIG. 1: Allowed regions in the m_1 and m_3 plane for (a) the normal ordering and (b) the inverted ordering of neutrino masses. Choices of parameters are described in the text.

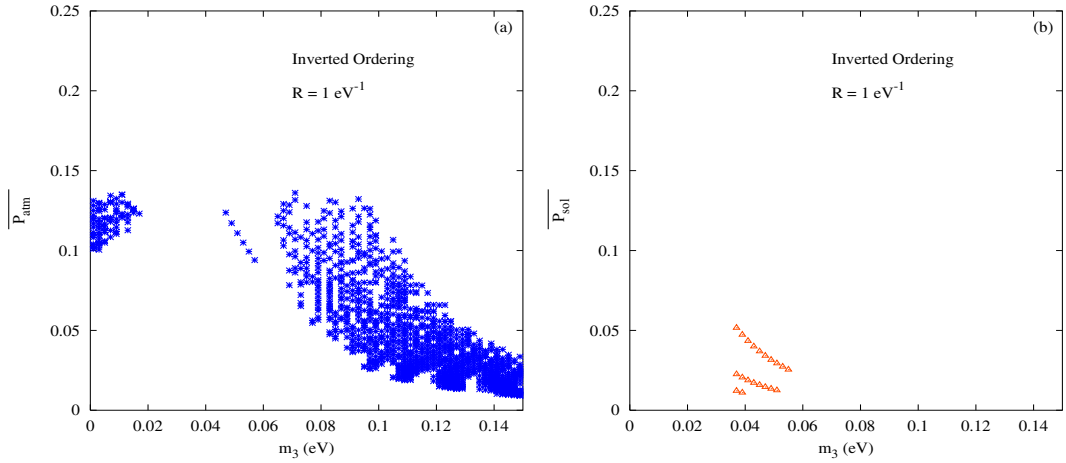

FIG. 2: Time-averaged oscillation probability (a) $\overline{P_{atm}}$ and (b) $\overline{P_{sol}}$ for the inverted ordering of neutrino masses as a function of m_3. Only those points are considered here which satisfy for (a) the Solar+CHOOZ and for (b) Atmospheric + CHOOZ constraints.

of (9), (10) do lie close to the points satisfying (8), (10), in fact there is not a single point in the five-dimensional parameter space of $(m_1, m_2, m_3, m, R^{-1})$ which is compatible with all the constraints. In order to convince the reader of this, we have plotted in Fig. 2 the time-averaged oscillation probability (a) $\overline{P_{atm}}$ and (b) $\overline{P_{sol}}$ as a function of m_3 in the inverted case for all those points satisfying the constraints in (9) and (10) for the case (a) and (8) and (10) for the case (b). Clearly, (8) is violated by the points in Fig. 2a while (9) is violated by those in Fig. 2b.

Next, we consider the model of Lam [9] which requires a sterile neutrino (with a Majorana mass m_4) on the brane in addition to three active neutrinos. In the limit of large coupling between the brane neutrinos and the bulk (i.e., $d \gg m_i R$,1) one can have three "isolated" eigenvalues which can give rise to two different Δm^2, whereas the other eigenvalues sit at half-integer values. Following Lam, we also arrange the Majorana masses of the brane neutrinos in such a way that

$$0 < m_1 < \mu_1 < m_2 < \mu_2 < m_3 < \mu_3 < m_4, \qquad (15)$$

where μ_i's are the isolated eigenvalues. Furthermore, it is assumed that the $m_i R$'s do not have integral or half-integral values. But, now we take a flavor-blind brane-bulk coupling m as in the earlier scenario. After a thourough scan in the parameter space with the above mentioned constraint in (15), we find that although in some part of the parameter space it is possible to satisfy the constraints in (3) and (5), there is no point where both the constraints in (8) and (9) can be satisfied either with a normal or with an inverted ordering of the neutrino masses.

In summary, we are able to exclude the Dienes-Sarcevic model [7] and that version of Lam's model [9] which has flavor blind off-diagonal mass terms, by using all neutrino oscillation data and the WMAP constraint on the sum of neutrino masses. Extra dimensional models of neutrino masses and mixing angles need flavor dependent brane-bulk couplings.

P.R. and S.R. acknowledge the hospitality of the Theory Group at SLAC where this work was initiated. We thank S. Choubey, S. Goswami, A.S. Joshipura and H-J. He for their helpful comments. The research of S.R. was supported by the Academy of Finland (Project number 48787). S.R. would also like to thank the Lady Davis Fellowship Trust in Technion, Israel, for financial support during the initial stages of this work.

---

[1] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys. Lett. **B429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *ibid.* **B436**, 257 (1998); L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); *ibid.* 4690 (1999).

[2] K.R. Dienes, E. Dudas and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999); G.R. Dvali and A.Y. Smirnov, Nucl. Phys. **B563**, 63 (1999); R.N. Mohapatra and A. Pérez-Lorenzana, Nucl. Phys. **B576**, 466 (2000); *ibid.* **B593**, 451 (2001); R. Barbieri, P. Creminelli and A. Strumia, Nucl. Phys. **B585**, 28 (2000); A. Lukas, P. Ramond, A. Romanino and G.G. Ross, Phys. Lett. **B498**, 230 (2001), D.O. Caldwell, R.N. Mohapatra, and S.J. Yellin, Phys. Rev. **D64**, 073001 (2001); N. Cosme, J.-M. Frère, Y. Gouverneur, F.-S. Ling, D. Monderen and V. Van Elewyck, Phys. Rev. **D63**, 113008 (2001); K. Agashe and G.H. Wu, Phys. Lett. **B498**, 230 (2001); A. Ioannisian and A. Pilaftsis, Phys. Rev. **D62**, 066001 (2000); A. Dighe and A.S. Joshipura, Phys. Rev. **D64**, 073012 (2001), N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali and J. March-Russel, Phys. Rev. **D65**, 024032 (2002); T. Appelquist, B. Dobrescu, E. Ponton and H. Yee, Phys. Rev. **D65**, 105019 (2002).

[3] There are more complicated models comprising right chiral neutrinos on the brane and/or left chiral ones in the bulk with warped geometric configurations [4], but the use of flavor dependent brane-bulk couplings to explain the observed pattern of neutrino mixing is quite generic in these.

[4] Y. Grossman and M. Neubert, Phys. Lett. **B474**, 361 (2000); G. C. McLaughlin and J.N. Ng, Phys. Lett. **B493**, 88 (2000); S.J. Huber and Q. Shafi, J. Phys. **G29**, 1807 (2003) and Phys. Lett. **B583**, 293 (2004).

[5] S.M. Bilenky and S.T. Petkov, Rev. Mod. Phys. **59**, 671 (1989).

[6] H. Davoudiasl, P. Langacker and M. Perelstein, Phys. Rev. **D65**, 105015 (2002).

[7] K.R. Dienes and I. Sarcevic, Phys. Lett. **B500**, 133 (2001).

[8] C.S. Lam and J.N. Ng, Phys. Rev. **D64**, 113006 (2001).

[9] C.S. Lam, Phys. Rev. **D65**, 053009 (2002).

[10] Q-H. Cao, S. Gopalakrishna and C.P. Yuan, hep-ph/0312339.

[11] Thus the entire flavor mixing among the light physical neutrinos is induced by the KK modes.

[12] An extension of the model of Ref. [7] with three different off-diagonal mass elements, was investigated in the limit of their very large values by Lam and Ng [8] and shown to be incompatible with the data. Our study, however, covers the entire parameter space of the Dienes-Sarcevic model [7] with a uniform off-diagonal mass.

[13] We use 90% C.L. limits throughout.

[14] M. Shiozawa, super-Kamiokande coll. in XX Intl. Conf. Neutrino Physics and Astrophysics, Munich, May 2002 (http://neutrino2002.ph.tum.de); S. Fukuda *et al.* super-Kamiokande coll. Phys. Rev. Lett. **81**, 1562 (1998); *ibid.* **85**, 3999 (2000).

[15] Q.R. Ahmad *et al.*, SNO Coll., Phys. Rev. Lett. **87**, 071301 (2001); *ibid.* **89**, 011301 (2002); S. Fukuda *et al.*, super-Kamiokande coll. Phys. Lett. **B539**, 179 (2002); B.T. Cleveland *et al.*, Astrophys J. **496**, 505 (1998); W. Hampelet *et al.* Phys. Lett. **B447**, 127 (1999); J.N. Abduradutov *et al.* JETP **95**, 181 (2002); Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).

[16] S.N. Ahmed *et al.*, nucl-ex/0309004.

[17] M.C. Gonzalez-Garcia and C. Pena-Garay, Phys. Rev. **D68**, 093003 (2003).

[18] M. Apollonio *et al.*, Phys. Lett. **B466**, 415 (1999).

[19] M. Maltoni, T. Schwetz, M.A. Tortola and J.W.F. Valle, Phys. Rev. **D68**, 113010 (2003); G.L. Fogli, E. Lisi, A. Marrone, D. Montanind, A. Palazzo and A.M. Rotunno, Phys. Rev. **D69**, 017301 (2004); A. Bandyopadhyay, S. Choubey, S. Goswami, S.T. Petkov and D.P. Roy, Phys. Lett. **B583**, 134 (2004).

[20] Our conclusions follow just from these averaged probabilities without any need to use the detailed shape of the corresponding distributions.

[21] A. Pierce and H. Murayama, Phys. Lett. **B581**, 218 (2004); G. Bhattacharyya, H. Päs, L. Song, and T. Weiler, Phys. Lett. **B564**, 175 (2003).

[22] M. Maltoni, T. Schwetz, M.A. Tortola and J.W.F. Valle, Nucl. Phys. Proc. Suppl. **114**, 203 (2003); M.C. Gonzalez-Garcia, M. Maltoni and C. Pena-Garay, Phys. Rev. **D64**, 093001 (2001).

[23] This convention of always keeping $\mu_3 > \mu_2 > \mu_1$, which is different from the more common PMNS convention of attributing $\mu_{1,2,3}$ to $|\nu_{e,\mu,\tau}\rangle$ in the 'no-mixing' limit $U \to I$, has been followed for search convenience and is inessential to the derivation of the final conclusion.