## Positron transmission and polarization in E-166 spectrometer\*

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## Abstract

The proposed experiment E-166 is designed to demonstrate the possibility of producing longitudinally polarized positrons from circularly polarized photons. It utilizes a low emittance 50 GeV electron beam passing through a helical undulator in the FFTB. Circularly polarized photons generated by the electron beam in undulator hit a target and produce electron-positron pairs. Spectrometer after positron production target includes a solenoid and bending magnets to deliver polarized positrons to a reconversion target. The results of the simulation indicate that positron transmission efficiency of 1...3 % with beam polarization of 60...80 % can be obtained in spectrometer.

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#### **1 INTRODUCTION**

Polarized positron production experiment E-166 uses strongly collimated 50 GeV electron beam to generate circularly polarized photons in a helical undulator. Photons, after interaction with target, create polarized positrons. Layout and general description of experiment are given in Ref. [1]. The purpose of a spectrometer is to select the positron beam after target from electron and photon beams and to deliver positrons to a reconversion target keeping beam polarization as high as possible.

The nondispersive spectrometer for the E-166 experiment based on two 90° bending magnets was proposed in Ref. [2]. Various modifications of the spectrometer utilizing bending magnets with smaller bending angles and additional focusing elements, were discussed in Ref [3]. An analysis of background due to the interaction of positrons, electrons, and photons with surrounding materials and optimization of bending angle were performed in Ref [4]. In Ref. [5] a detailed design of the spectrometer was accomplished. The present paper analyzes positron transmission and polarization in the proposed spectrometer.

#### 2 SPECTROMETER WITH DOUBLE 90° MAGNETS

Nondispersive spectrometers are widely used in beam optics [6]. Usually, they include two or three bending magnets with additional quadrupoles between them. In the proposed experiment E-166, the spectrometer has to shift the beam from an original accelerator axis at the distance of 45 cm to separate positron beam from a photon beam and an electron beam coming out from the target. The simple and cost-effective solution is to utilize two 90<sup>o</sup> magnets providing point-to point transformation of the beam (see Figs. 1 - 4). The parameters of the system are presented in Table 1. The spectrometer includes a focusing solenoid and two 90<sup>o</sup> bending magnets separated by a drift space of L = 40 cm. An

additional feature of the double 90° magnets design is absence of the depolarization of positrons in bending magnets, because spin rotation in the first magnet is compensated by the second magnet.

The initial distribution of positrons produced by circularly polarized photons was calculated by J.C.Sheppard using the program EGS4 modified for polarized positrons [7, 8]. Positron distribution after the target is presented in Fig. 5. The distribution is characterized by a large emittance of the positron beam and a large energy spread. The correlation between energy, polarization and transverse momentum spread is illustrated by partial distributions presented in Figs. 6 – 8. Low energy positrons are less polarized and more transversely divergent while high-energy positrons are strongly polarized and less divergent. The energy spectrum peaks near the low-energy end of the distribution.

Let us calculate the acceptance of a double 90° magnet beamline. From the first order matrix analysis, the horizontal displacement of the particle after the first bend and drift is given by:

$$x = -\frac{L}{R} x_{o} + Rx'_{o} + (R + L) \frac{p}{p_{o}} .$$
 (2.1)

The maximum deviation from the axis is equal to the radial aperture,  $a_x = 5$  cm, therefore, horizontal acceptance of the channel, <sub>x</sub>, is defined as:

$$x_{o, max} = a_x \frac{R}{L} = 3.125 \text{ cm}$$
, (2.2)

$$x'_{o, max} = \frac{a_x}{R} = 0.4$$
, (2.3)

$$x = x_{o, max} x'_{o, max} = 1.25$$
 cm rad. (2.4)

The maximum energy spread is defined from Eq. (2.1) as:

$$\left(\frac{p}{p_0}\right)_{\max} = \frac{a}{R+L} = 0.15$$
 (2.5)

In the vertical direction, particle motion is unaffected except edge defocusing at the entrance and exit of the bending magnets. The maximum vertical slope of particle trajectory,  $y'_{o, max}$ , and maximum initial vertical displacement of positrons,  $y_{o, max}$ , including edge defocusing are:

$$y'_{o, \max} = 0.027$$
, (2.6)

$$y_{0, \max} = 0.9 \text{ cm}$$
, (2.7)

therefore, vertical acceptance is

$$y = y_{0, \max} y_{0, \max} = 0.024 \text{ cm rad}$$
 (2.8)

The proposed spectrometer is characterized by large values of  $x/R \sim 0.4$  and momentum deviation  $p/p \sim 0.15$ . The linear model based on matrix multiplication is not sufficient to provide accurate estimations of positron dynamics in the spectrometer. For calculation, the code BEAMPATH was used [9]. The next section discusses the mathematical model used in the code.

## **3 NUMERICAL MODEL**

## Equation of motion

Particle trajectories are calculated in a curved system of coordinates. Single-particle Hamiltonian in curvilinear coordinates x, y, z is given by:

$$H = c_{V} \sqrt{m^{2}c^{2} + (\frac{P_{z}}{1 + \frac{X}{R(z)}} - qA_{z})^{2} + (P_{x} - qA_{x})^{2} + (P_{y} - qA_{y})^{2}} + qV, \qquad (3.1)$$

where  $P_x$ ,  $P_y$ ,  $P_z$  are components of canonical-conjugate momentum of a particle,  $A_x$ ,  $A_y$ ,  $A_z$  are components of vector-potential, V is a scalar potential of the structure and R(z) is a curvature radius of a reference trajectory. After transformation from vector potential  $\vec{A}$  and scalar potential V to electric field  $\vec{E} = -q$   $\vec{A}/t - \text{grad V}$  and magnetic field  $\vec{B} = \text{rot } \vec{A}$  and from canonical momentum  $\vec{P}$  to mechanical momentum  $\vec{p} = \vec{P} - q\vec{A}$ , the set of equations of motion is:

$$\frac{dx}{dt} = \frac{p_x}{m},$$

$$\frac{dy}{dt} = \frac{p_y}{m},$$

$$\frac{dz}{dt} = \frac{p_z}{m (1 + \frac{x}{R})}$$

$$\frac{dp_x}{dt} = \frac{p_z^2}{m (R + x)} + qE_x + \frac{q}{m} (p_y B_z - p_z B_y),$$

$$\frac{dp_y}{dt} = qE_y + \frac{q}{m} (p_z B_x - p_x B_z),$$

$$\frac{dp_z}{dt} = -\frac{p_z p_x}{m (R + x)} + qE_z + \frac{q}{m} (p_x B_y - p_y B_x).$$
(3.2)

Integration of equations (3.2) is performed with a fixed time step utilizing integrator described in Ref. [9].

## Calculation of magnetic field

The magnetic field inside a bending magnet is described by the Taylor expansion up to the terms of second order:

$$B_{x}(x,y,z) = B_{y} \left(-n \frac{y}{R} + 2 - \frac{xy}{R^{2}}\right), \qquad (3.3)$$

$$B_{y}(x,y,z) = B_{y} \left[1 - n \frac{x}{R} + \frac{n}{2} \frac{y^{2}}{R^{2}} + \frac{(x^{2} - y^{2})}{R^{2}}\right], \qquad (3.4)$$

where  $B_y$  is the vertical component of magnetic field along the reference trajectory with radius of curvature R, n is the field index and is a nonlinear coefficient in the magnetic field expansion:

$$n = -\left[\frac{R}{B_y} - \frac{B_y}{x}\right]_{x=0,y=0}, \qquad \qquad = \left[\frac{R^2}{2! B_y} - \frac{^2B_y}{x^2}\right]_{x=0,y=0}. \tag{3.5}$$

At the entrance and at the exit of the magnet, the slope of the particle trajectory is changed because of the pole angle according to the linear matrix transformation (TRANSPORT code):

$$\begin{vmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{y} \\ \mathbf{y} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ \frac{\mathbf{tg}}{\mathbf{R}} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{\mathbf{tg}(-)}{\mathbf{R}} & 1 \end{vmatrix} \begin{vmatrix} \mathbf{x}_{0} \\ \mathbf{x}_{0} \\ \mathbf{y}_{0} \\ \mathbf{y}_{0} \end{vmatrix}.$$
 (3.6)

The correction angle is given by the expression

$$= K_1 \left(\frac{g}{R}\right) \left(\frac{1 + \sin^2}{\cos}\right) \left[1 - K_1 K_2 \left(\frac{g}{R}\right) tg\right], \qquad (3.7)$$

where g is the gap of the magnet and coefficients  $K_1$ ,  $K_2$  are defined by pole geometry.

Magnetic field of solenoids and axial-symmetric permanent magnets is calculated as

$$B_{z} = B(z) - \frac{r^{2}}{4} \frac{d^{2}B}{dz^{2}}, \qquad B_{r} = -\frac{r}{2} \left( \frac{dB}{dz} - \frac{d^{3}B}{dz^{3}} \frac{r^{2}}{8} \right), \qquad (3.8)$$

where field at the axis B(z) is given at fixed points.

## Spin tracking

Particle tracking was accompanied with integration of the Thomas-BMT equation, describing the precession of the spin vector  $\vec{S}$ :

$$\frac{d\vec{S}}{dt} = \frac{e\vec{S}}{m} \times \left[ (1+G)\vec{B} + (1+G)\vec{B}_{II} + (G + \frac{1}{1+c})\vec{E}_{II} \right], \qquad (3.9)$$

where G = 0.001159652 is the anomalous magnetic moment of the positron,  $\vec{E}$  is the electrical field, and  $\vec{B}$  and  $\vec{B}_{II}$  are components of the magnetic field perpendicular and parallel to particle velocity. The spin advance at a small distance dz is described as a matrix:

$$\begin{vmatrix} S_{x} \\ S_{y} \\ S_{z} \end{vmatrix} = \begin{vmatrix} 1 - a(B^{2} + C^{2}) & ABa + Cb & ACa - Bb \\ ABa - Cb & 1 - a(A^{2} + C^{2}) & BCa + Ab \\ ACa + Bb & BCa - Ab & 1 - a(A^{2} + B^{2}) \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix},$$
(3.10)

$$A = \frac{D_x}{D_o}, \ B = \frac{D_y}{D_o}, \ C = \frac{D_z}{D_o}, \ D_o = \sqrt{D_x^2 + D_y^2 + D_z^2} ,$$
(3.11)

 $a = 1 - \cos (D_0 dz), \quad b = \sin (D_0 dz),$  (3.12)

where components  $D_x$ ,  $D_y$ ,  $D_z$  are defined by the equations:

$$D_{x} = \frac{e}{m v} \left[ (1+G)(B_{x} - x'B_{z}) + (1+G)x'B_{z} + \frac{v}{c^{2}}(\frac{1}{1+} + G)(E_{y} - y'E_{z}) \right], \quad (3.13)$$

$$D_{y} = \frac{e}{m v} \left[ (1+G)(B_{y} - y'B_{z}) + (1+G)y'B_{z} + \frac{v}{c^{2}}(\frac{1}{1+} + G)(x'E_{z} - E_{x}) \right],$$
(3.14)

$$D_{z} = \frac{e}{m v} \left[ (1+G)(-x'B_{x} - y'B_{y}) + (1+G)(x'B_{x} + B_{z} + y'B_{y}) + \frac{v}{c^{2}}(\frac{1}{1+} + G)(y'E_{x} - E_{y}x') \right] , \quad (3.15)$$

and prime means derivative over longitudinal coordinate, ' = d/dz. Matrix (3.10) describes spin precession in Cartesian coordinates. In a bending magnet with a design orbit radius R, spin is corrected according to the matrix

$$\begin{vmatrix} S_{x} \\ S_{y} \\ S_{z} \end{vmatrix} = \begin{vmatrix} \cos & 0 & -\sin \\ 0 & 1 & 0 \\ \sin & 0 & \cos \end{vmatrix} \begin{vmatrix} S_{x,0} \\ S_{y,0} \\ S_{z,0} \end{vmatrix} , \qquad (3.16)$$

which describes rotation of a system of coordinates to the angle of = - dz/R at every integration step.

Initially, the spin vector of each positron is pointed along momentum vector. During beam transport, the spin vector precesses, resulting in the depolarization of the beam. We define the longitudinal polarization as an average of the product of the longitudinal component  $S_z$  and the value of polarization, P, summed over all positrons:

$$<\mathbf{P}_{z}> = \frac{1}{N} \sum_{i=1}^{N} S_{z}^{(i)} \mathbf{P}^{(i)}$$
 (3.17)

The initial value of longitudinal polarization is  $\langle P_z \rangle = 0.41$ . After removing low-energy positrons in spectrometer, the polarization of the final beam can reach the value of 0.8.

## **4 RESULTS OF SIMULATION**

Particle trajectories in the proposed system are presented in Fig. 9. Positron transmission and polarization after transport system calculated by Eqs. (2.1) - (2.7) and by numerical simulation are illustrated by Fig. 10. Red dots in Fig. 10 represent a fraction of initial positrons successfully transported through the double 90° magnets system restricted by the following constraints: - 3.1 cm <x< 3.1 cm, - 0.9 cm < y < 0.9 cm, - 0.4 < x' < 0.4, 0.027<y'<0.027, -0.15 < p/p < 0.15. Blue dots represent the results of numerical simulations. Positron transmission has a maximum around particle energy of 6 MeV:

$$\frac{N}{N} = 10^{-2} . (4.1)$$

The appearance of the positron transmission maximum is explained by the fact that low energy positrons are strongly divergent and only a small fraction of positrons is within the transverse acceptance of the spectrometer. With increasing energy, the positron beam becomes less divergent, but the number of positrons drops.

Transmission efficiency in the spectrometer is improved by inserting a solenoid between the positron production target and the bending magnets. The longitudinal field profile in the solenoid, proposed in Ref [5], is approximated by an expression (see Fig. 12):

$$B_{z} = B_{max} \left[ 1 - \left(\frac{z - \bar{z}}{d}\right)^{2} \right]^{4} .$$
 (4.2)

The radial equation of motion of the particle in magnetic field is

$$\frac{d^2r}{dt^2} + {}^2_{L}r = 0, (4.3)$$

where L = eB/(2m) is the Larmor frequency. The focusing properties of the solenoid lens are characterized by the focal length of the lens, f, defined from Eq. (4.3) as

$$\frac{1}{f} = (\frac{e}{2p_z})^2 \int_{-d}^{d} B_z^2 dz$$
(4.4)

Fig. 13 illustrates single particle trajectories in the solenoid, depending on the value of particle momentum. For the presented field profile with the peak field of  $B_{max, o} = 1.17$  Tesla, the solenoid provides the best focusing for particles with momentum of  $p_o = 9.4$  MeV/c. In this case, the solenoid transforms the divergent beam into a parallel beam. According to Eq. (4.4), the focusing properties of the solenoid are scaled linearly with particle momentum. The optimal value of the peak field of solenoid,  $B_{max}$ , is related to particle momentum, p, and bending field, B, by the expression:

$$B_{\max} = B_{\max, o} \left(\frac{p}{p_o}\right) = B_{\max, o} \left(\frac{eBR}{p_o}\right).$$
(4.5)

Figs. 9, 11, 14 contain the results of particle tracking through a complete beamline consisting of the solenoid and two bending magnets. Parameters of the structure were selected according to Eq. (4.5). From the results of simulation, it is clear that inserting the solenoid increases the value of positron transmission through the system as a factor of 3, while positron polarization can be kept around 80%.

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# Table 1. Parameters of the structure

Bending radius, R	12.5 cm
Bending angle	90°
Pole angle,	0
Gap, g	5 cm
Fringe field coefficients: K <sub>1</sub>	0.7
$\mathbf{K}_2$	4.4
Vertical aperture, 2a <sub>y</sub>	5 cm
Horizontal aperture, 2a <sub>x</sub>	10 cm
Aperture at the exit of transport system	$5 \text{ x} 5 \text{ cm}^2$
Drift between bending magnets, L	20 cm



Fig. 1. Layout of spectrometer.







z (cm) Fig.3. Particle trajectories in bending magnets:  $p_z = 5.6 \text{ MeV/c}, \quad p/p = \pm 0.02.$ 



Fig. 4. Vertical particle trajectories in bending magnets:  $p_z = 5.6$  MeV/c, dy/dz=  $\pm 0.027$ .



Fig. 5. Initial positron distribution after target.



Fig. 6. Fraction of initial positron distribution with average energy of  $\overline{E} = 1.9$  MeV, and  $\overline{E/E} = \pm 0.15$ .



Fig. 7. Fraction of initial positron distribution with average energy of  $\overline{E} = 5.3$  MeV, and  $\overline{E/E} = \pm 0.15$ .



Fig. 8. Fraction of initial positron distribution with average energy of  $\overline{E} = 7.5$  MeV, and  $\overline{E/E} = \pm 0.15$ .



Fig. 9. Particle trajectories in the spectrometer: (left) 2 x 90  $^{\circ}$  bends, (right) solenoid + 2x90  $^{\circ}$  bends.





Fig. 10. Positron transmission and polarization after 2 x 90° bending magnets: (red) analytical, (blue) numerical.

Fig. 11. Positron transmission and polarization in beamline containing solenoid and 2x90° bending magnets.



Fig. 12. Solenoid field profile.



Fig. 13. Positron trajectories in the solenoid for different particle momentum : (1)  $p_z = 1.86 \text{ MeV/c}$ , (2)  $p_z = 3.7 \text{ MeV/c}$ , (3)  $p_z = 5.6 \text{ MeV/c}$ , (4)  $p_z = 7.5 \text{ MeV/c}$ , (5)  $p_z = 9.4 \text{ MeV/c}$ , (6)  $p_z = 11.25 \text{ MeV/c}$ .



Fig. 14. Output beam after beamline containing the solenoid with  $B_{max}=0.92$  Tesla and two 90° bends with bending field of B=0.2 Tesla.