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A Simulation Study of the Sawtooth Behavior

ALEX CHAO
Stanford Linear Accelerator Center, Stanford, CA 94309, USA

The fact that bunch lengthening sometimes occurs with a sawtooth behavior has received some attention recently.[1-6] Various possible mechanisms which might explain the sawtooth behavior have been suggested. In particular, in Ref.6, Baartman and D'Yachkov proposed a mechanism that involves an interplay of synchrotron oscillation, potential well distortion (which at some moment of bunch oscillation creates a double-humped longitudinal beam distribution), quantum diffusion, and radiation damping and performed computer simulations to demonstrate this mechanism. Although this BD mechanism is not the only possible explanation of a sawtooth behavior, this note is an attempt to follow up on this trend of thought by yet another simulation study, and to draw a few tentative conclusions from this study.

The collective effect is presumably caused by some wake function $W(z)$. We assume the wake function is short-ranged and only single-turn wake needs to be considered. To enhance the BD mechanism in our simulation, a wake function model has been chosen which (a) has a range approximately equal to a few times the natural bunch length σ_z, and (b) flips sign once in this range (to make it easier to produce a second hump in potential well). In fact, we have chosen a wake function $W(z) = W_0$ when $0 < z < z_0$ and $-W_0$ when $z_0 < z < 2z_0$. Of course, $W(z) = 0$ when $z < 0$. With such a choice of wake function, a second beam distribution hump, if formed, would be approximately at a distance $\sim z_0$ behind the first hump. One expects that when $z_0 \gg \sigma_z$, the quantum diffusion (needed to transport particles from one hump to the other) is too slow to give a clear sawtooth behavior. One also expects that when $z_0 \ll \sigma_z$, the diffusion is too fast and only a chaotic behavior appears. In the simulation, parameters W_0 and z_0 are varied.

In our simulation for a damping ring, we launch a beam whose injected longitudinal emittance has rms sizes of $5\sigma_\delta$ and $5\sigma_z$. The damping time in the simulation has been chosen to be 400 turns, synchrotron oscillation tune is 0.0137. We track 1200 particles typically for 20,000 turns. We also took $\sigma_z = 5$ mm and $\sigma_\delta = 0.001$.

Figure 1 is a summary of our simulation study. The unit of W_0 is not specified. When W_0 and z_0 are varied, we found three regions in the (W_0, z_0) space. Region H is when a stable Haissinski state[7] is found. In a significant portion of the H region, the equilibrium beam distribution is double-humped (thanks to the choice of the wake function model). The S region is when there is observed a sawtooth behavior, while the C region is when the beam behaves chaotically.

Figures 2(a) to (f) show some of the simulation details for the three cases represented by the three dots in Fig.1: (a,b) $W_0 = 0.8, z_0 = 15$ mm; (c,d) $W_0 = 1, z_0 = 12$ mm; (e,f) $W_0 = 1.5, z_0 = 10$ mm. (a), (c) and (e) show the instantaneous rms bunch length as a function of the number of turns after injection. (b), (d) and (f) similarly

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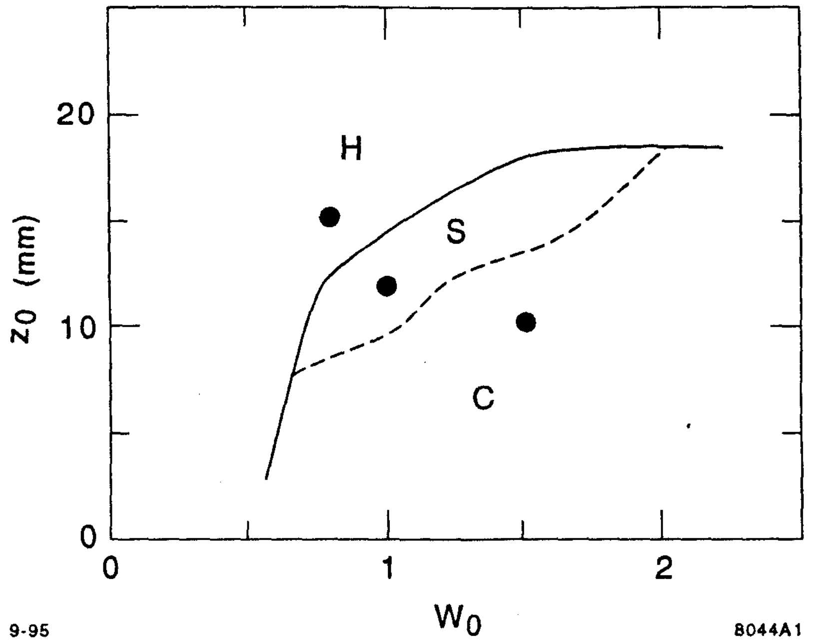

Fig. 1. Behavior of an injected beam in the (W_0, z_0) space.

show the instantaneous rms energy spread. The Haissinski steady state (a,b), the sawtooth behavior (c,d), and the chaotic behavior (e,f) are apparent. Note that in the Haissinski state, the energy spread damps down to the unperturbed value $\sigma_\delta = 0.001$, as it should, although the z-distribution of the beam has acquired distinct double humps and has an rms different from the unperturbed value.

In a separate study, a case with $W_0 = 2, z_0 = 20$ mm is first shown in Fig.3(a,b), which exhibits a chaotic behavior. (Here we used 10000 turns tracking.) When the injected beam emittance is changed from $5\sigma_z \times 5\sigma_\delta$ to $\sigma_z \times \sigma_\delta$ under otherwise the same conditions, we obtained Fig.3(c,d), which is a Haissinski state. This indicates the fact that how the beam behaves depends on the initial conditions of how the beam is injected.

Some tentative conclusions are given below:

(1) We reconfirmed the existence of the BD mechanism.

(2) However, because no double-humped distribution and no large-scale bunch shape oscillation have been observed in streak camera experiments, it seems unlikely that the BD mechanism alone is responsible for what was observed at the SLC Damping Ring.

(3) The threshold for the sawtooth behavior is not the same as the threshold when the Haissinski solution develops a second hump. It is possible to have a stable double-humped Haissinski distribution which is stable against sawtooth behavior as well as the microwave instability. This observation may be in conflict with some phenomenological models of the bunch lengthening effect.

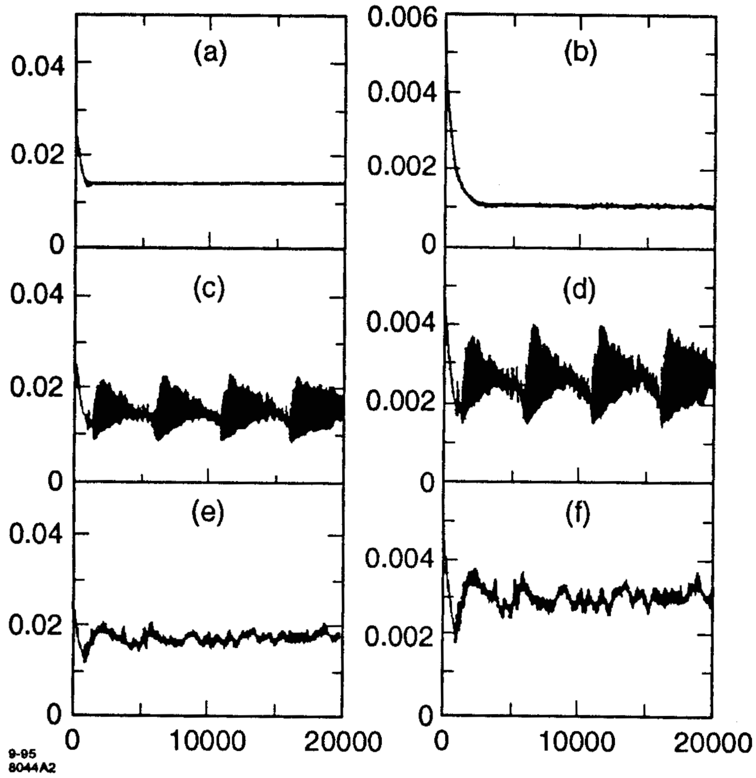

Fig. 2. Examples of beam behavior (a,b) Haissinski steady state, (c,d) sawtooth behavior, and (e,f) chaotic behavior.

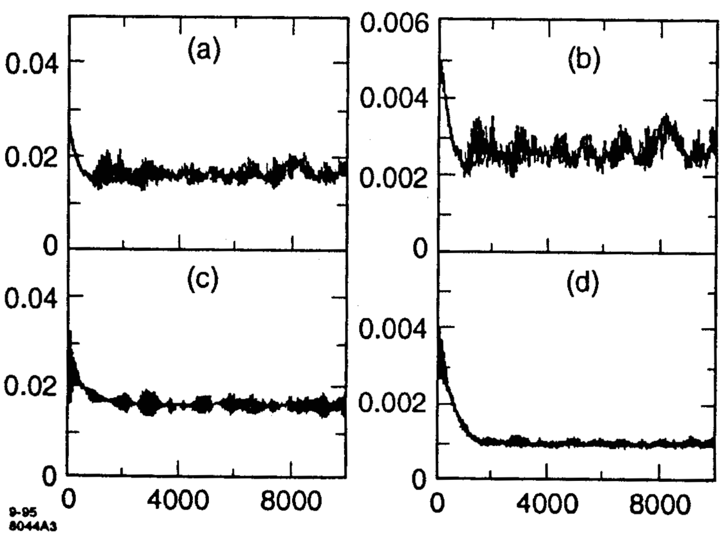

Fig. 3. Beam behavior also depends on the injected beam emittance. In case $W_0 = 2, z_0 = 20$ mm, the beam exhibits (a,b) chaotic behavior when injected with a large emittance, and (c,d) Haissinski state when injected with a small emittance.

(4) Another question arises as to whether the threshold for the sawtooth behavior is the same as the threshold of a microwave instability. More specifically, does the solid curve in Fig.1 coincide with the microwave instability threshold. This question seems to deserve a closer study and is yet to be completed.

(5) The existence and the stability of the Haissinski distribution does not guarantee a stable beam in a damping ring. The beam, injected with a much larger emittance, may choose to stay in a sawtooth state before it damps down to the Haissinski state, even when the Haissinski state is stable. This issue should receive some attention in the design of damping rings because the beam may not always reach the damped state as one might assume.

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