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Numerical Modeling and Analysis of Optical Response of Electro-optic Modulators

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Abstract **—This paper presents an analysis of a $LiNbO_3$ electro-optic modulator using the Finite Difference Time Domain (FDTD) technique, and also a new and efficient multiresolution time-domain technique for fast and accurate modeling of photonic devices. The electromagnetic fields computed by FDTD are coupled to standard electro-optic relations that characterize electo-optic interactions. This novel approach to $LiNbO_3$ electro-optic modulators using a coupled FDTD technique allows for previously unattainable investigations into device operating bandwidth and data transmission speed. On the other hand, the proposed multiresolution approach presented in this paper solves Maxwell's Equations on nonuniform self-adaptive grids, obtained by applying wavelet transforms followed by hard thresholding. The developed technique is employed to simulate a coplanar waveguide CPW, which represents an electro-optic modulator. Different numerical examples are presented showing more than 75% CPU-time reduction, while maintaining the same degree of accuracy of standard FDTD techniques.**

Index Terms— **Coplanar waveguides, electrooptic effects, electrooptic modulation, FDTD techniques, photonic devices, multiresolution analysis.**

I. Introduction

As the technological trend towards faster data transmission speeds continues, optical devices and communications systems can only grow in importance. The next generation of electro-optic devices will require powerful simulation tools that will be both accurate and capable of simulating all aspects of device operation. Without such a simulation tool, device performance will not reach maximum physical potential. Finite Difference Time Domain (FDTD) is a powerful and flexible technique that can be expected to play a central role in future developments in the simulation of electro-optic devices. However, standard FDTD techniques are not sufficient enough in terms of CPU utilization. Accordingly, there is an urgent need to present new and enhanced versions of FDTD techniques for fast and efficient modeling of microwave and photonic devices.

Published numerical work on the $LiNbO_3$ electro-optic modulator has concentrated on using static methods to optimize device geometry to meet various design constraints. Work has been done using Finite Elements Method to find optimum electrode thickness and wall angle to achieve a good traveling wave-optical velocity match [1]. Other work has included more exotic techniques like modified-step-segment method (MSSM) to analyze the optical waveguide region of this device [2]. No previous work, however, has proposed an intuitive approach to simulate complete device performance.

A radically new approach that utilizes a fully dynamic physical simulation of this device is proposed. The power of this approach is that it provides a complete simulation tool capable of being used to optimize device geometry to meet certain microwave design specifications as well as to optimize device optical performance by simulating the physical electro-optic interaction.

In section II, a brief description of the device structure is presented. Section III presents the time domain analysis of device performance and the procedure for coupling FDTD results to electro-optic effects to generate optical response. Section IV presents the new multiresolution numerical approach. Finally, results are presented and discussed in section V.

II. Electro-optic Modulator Device Structure

Briefly, the electro-optic modulator is a coplanar waveguide (CPW) structure with an anisotropic $LiNbO_3$ substrate that exhibits a Pockels electro-optic effect [3]. Electric fields applied to this substrate cause a change in its index of refraction that is proportional to the applied electric field. Inside this substrate are Ti diffused optical waveguides supporting optical signal propagation. This waveguide is split and comes in the close vicinity of a CPW structure to allow for electro-optic interaction. Electric signals traveling along the CPW structure induce electric fields in the substrate that change the phase of light traveling in the two embedded optical waveguides. After some interaction region, the optical waveguides combine allowing the optical signals to interfere. Upon interference, the induced changes in phase translate into intensity modulation. For the modulation to be optimal

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(i.e. to maximize bandwidth), the velocity of electric signals traveling along the CPW structure and the optical signals traveling inside the embedded optical waveguides must be matched. This is one of the most important considerations for designing the electrodes from a microwave point of view. Possible strategies to achieve such an electro-optic phase velocity match include the use of thick electrodes and SiO_2 buffer layers between the electrodes and the $LiNbO_3$ substrate. Furthermore, the interaction length should be properly optimized. An interaction region that is too short will result in a shallow optical response (given a fixed driving voltage), while an interaction region that is too long may suffer from phase reversal. Figure 1 provides a brief schematic of the general structure of the device.

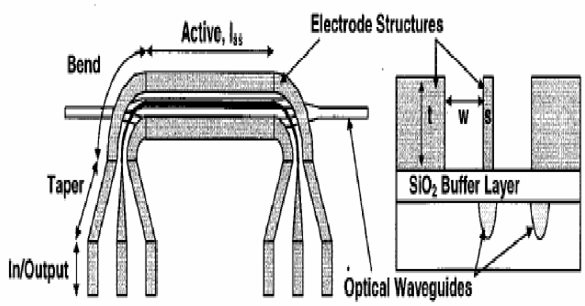

Fig. 1. Schematic showing top (left) and cross sectional (right) views of basic device structure for a z-cut electro-optic modulator. The active or interaction region is essentially a CPW structure with optical waveguides in the electro-optically active $LiNbO_3$ substrate. Electric fields propagating along CPW structure change the phase of light traveling in each optical waveguide which, upon interference, yields optical intensity modulation.

III. Time Domain Analysis of Electro-optic Modulator

The flexibility and power of the FDTD method make it ideal for the unique numerical challenges posed by this problem, which include anisotropy and non-linearity. Simulating the full optical response of the device is simply not possible using the static techniques so far applied to this problem. The power of FDTD lies in its full calculation of electric and magnetic fields. This is exploited by launching an electric Gaussian pulse along the CPW part of this device. The FDTD scheme calculates at each time step the electric field everywhere inside the device, including inside the embedded Ti diffused $LiNbO_3$ optical waveguide regions. This electric field information can then be coupled to the linear electro-optic effect using the electro-optic relations:

$$\begin{aligned}\Delta n_1 &= -\alpha n^3(r_{22}E_2 + r_{13}E_3)\\ \Delta n_2 &= -\alpha n^3(r_{22}E_2 + r_{13}E_3) \qquad (1)\\ \Delta n_3 &= -\alpha n^3 r_{33}E_3\end{aligned}$$

where α, r_{22}, r_{13} and r_{33} are constants, Δn_i is the change in index of refraction for optical fields polarized in the crystallographic i axis, and n is either the ordinary or extraordinary index of refraction [3]. If there is an electro-optically induced difference in index between the two waveguides, then light traveling in the optical waveguides will no longer be in phase and upon interference will become intensity-modulated. Because r_{33} is the largest of the above electro-optic coefficients, the design of electro-optic modulator electrodes typically try to maximize the electric field in that crystallographic direction. The design under investigation here is an x-cut design.

IV. Proposed Multiresolution Time-Domain Approach

The multiresolution numerical approach presented in this paper removes the redundancies of the FDTD original formulations. This is achieved by efficiently removing the insignificant grid points, i.e., grid points where the solution change very slowly. The basic idea is taking snapshots of the solution during the simulation and applying wavelet transform to obtain the coefficients of the details. The coefficients of the details are then normalized and threshold is applied to obtain a nonuniform multiresolution grid. The conceived nonuniform grid keeps only the important grid points, which ultimately means a faster simulator. The proposed threshold value is dependent on the variable solution at any given iteration and is given by (2).

$$T = \frac{T_0}{N}\left[\sqrt{\sum_{i=1}^{N} d_i^2}\right] \qquad (2)$$

In above equation, T_0 is the initial threshold value, d_i is the coefficient of the details, and N is the number of grid points in the *x*- or y-direction. Hence, the value of the threshold depends mainly on the variable solution at any given time rather than being fixed. The values T_0 of used in the simulation are 0.001, 0.01, 0.05, and 0.1. The grid points where the value of the normalized coefficients of the details smaller than the threshold value given by (2) are removed. Figure 2 demonstrates an example of the procedure employed to obtain the nonuniform grids for the electro-optic modulator (CPW). One observes that the proposed algorithm accurately removes grid points in the locations where variable solutions change slowly. To keep track of the wave propagation within the CPW structure. The following algorithm is developed for the grid updating of FDTD simulations:

Step 1: Construct a 3D matrix M that has only 0's and 1's, based whether or not we have a non-zero solution of the field at this location. For example, "1" is assigned if a non-zero field solution exists, and "0" elsewhere.
Step 2: Estimate the value of δ_{FDTD} (FDTD grid-updating factor) as:

$$\delta_{\mathrm{FDTD}} = \frac{\sum_{i,j,k}(M_{\mathrm{new}} \oplus M_{\mathrm{old}})_{i,j,k}}{N_{xd}N_{yd}N_{zd}} \tag{3}$$

where M_{new} and M_{old} are the matrices constructed using step one for the current and old solutions of the fields, respectively. N_{xd}, N_{yd}, and N_{zd} are the number of grid points in the x, y, and z directions, respectively.
Step 3: Check δ_{FDTD}'s value against a predefined value, for example 5%.
Step 4: If satisfied, move the grid to z=z+dz. Where dz is proportional to δ_{FDTD} .
Step 5: t=t+dt.

Three dimensional (3-D) Yee-Based FDTD code is developed for the optical modulator (CPW), with the proposed algorithm employed. In addition, a Gaussian excitation pulse is applied to evaluate the algorithm over a wide range of frequencies. Table I depicts the simulation results. One observes that as the threshold value increases, CPU-time and error introduced decreases as well. However, it was observed that when using T_0 equals to 10% introduces dispersion, which is a serious type of error. Accordingly, an initial threshold value of 5% is recommended here in terms of both CPU-time and error for efficient FDTD simulations [4].

TABLE I
COMPARISON BETWEEN STANDARD FDTD TECHNIQUE AND PROPOSED MULTIRESOLUTION TECHNIQUE FOR DIFFERENT INITIAL THRESHOLD VALUES

T_0	CPU-time	Error on Potential	
		2-norm	Infinity-norm
0% (Standard FDTD)	744.90 s		
0.1%	300.17 s	0.0873%	8.80%
1.0%	205.92 s	0.0871%	8.75%
5.0%	155.10 s	0.0778%	7.69%
10.0%	111.05 s	0.0473%	3.66%

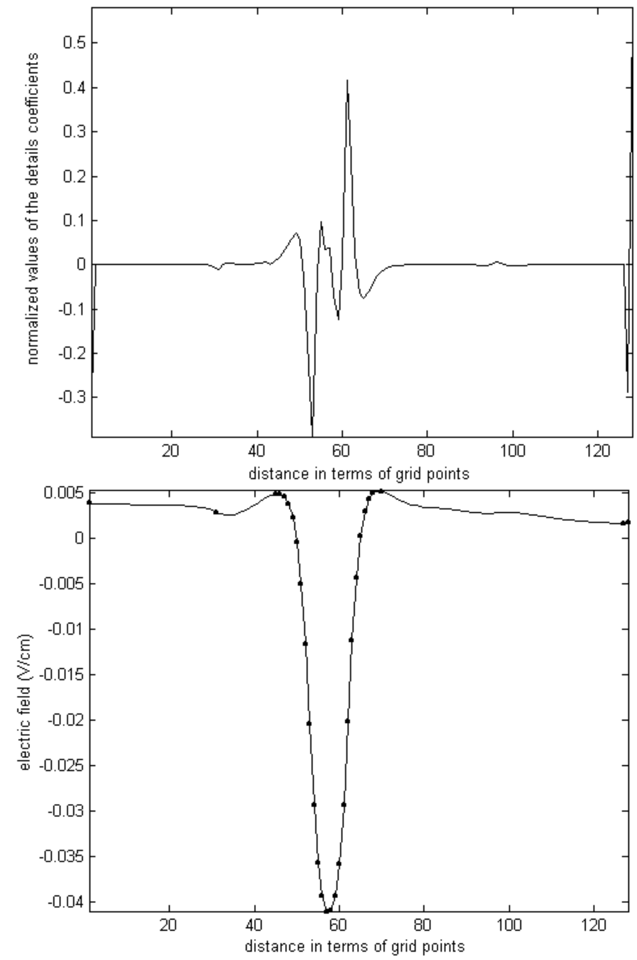

Fig. 2. (a) Normalized details coefficients for the electric field of the passive part. (b) Grid points marked on the actual curve of the electric field.

V. RESULTS AND DISCUSSIONS

To demonstrate the power of the coupling approach, two CPW structures with different phase velocities have been simulated. Figure 3 shows a direct comparison of two time domain electro-optic responses for the matched and mismatched cases. Figure 3 shows a wider unmatched optical response, which is intuitively understood since the peak electric signal applies phase shift over a larger region of the optical signal than in the matched case. Similarly, such an unmatched signal will result in a higher V_π for a given electro-optic interaction length. Figure 3 also numerically demonstrates how important electro-optic phase velocity match is to bandwidth of electro-optic modulators especially for digital applications where the wider time domain response can be clearly seen to limit the time domain proximity of two bits for data transmission.

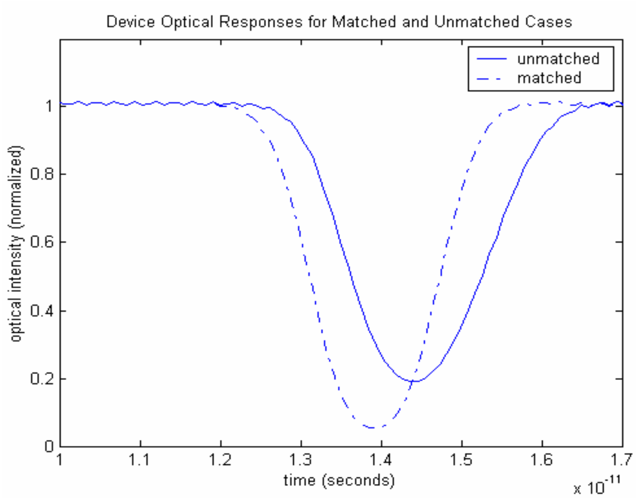

Fig. 3. Electro-optic modulator time domain optical responses to a Gaussian electric pulse for two electrode designs representing an electro-optic phase velocity match and mismatch for the same interaction length.

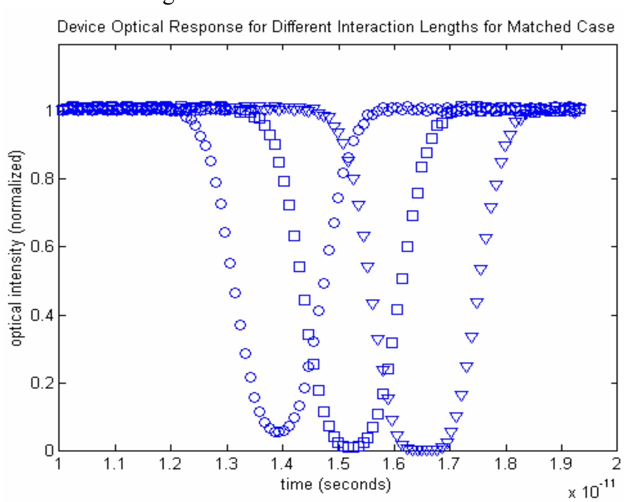

Fig. 4 Electro-optic modulator time domain optical response to a Gaussian electric pulse for an electrode design respresenting an electro-optic phase velocity match for different electro-optic interaction lengths.

Given a defined driving voltage and desired V_{π}, the interaction length of electro-optic modulators must be properly designed so as not to be too short so that the optical response is too shallow, nor too long so that the optical response will exhibit phase reversal. Figure 4 shows the optical response to an electric Gaussian pulse for different lengths of electro-optic interaction regions for the matched case. In Fig. 4, it is seen that the optical response is just beginning to exhibit phase reversal. If the interaction region is made just a bit longer, this matched electro-optic modulator will exhibit phase reversal for the given driving voltage. Figure 5 displays results for the unmatched case.

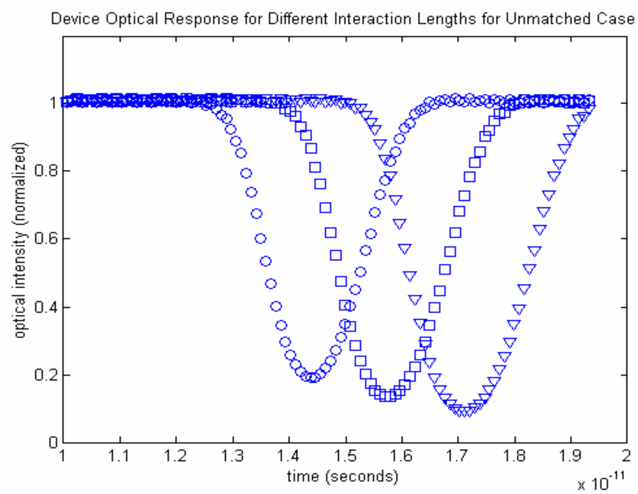

Fig. 5. Electro-optic modulator time domain optical response to a Gaussian electric pulse for an electrode design respresenting an electro-optic phase velocity mis-match for different electro-optic interaction lengths.

VI. Conclusion

FDTD provides for a fully intuitive approach to the simulation of electro-optic modulator optical response. Using FDTD solution of the E-field in time coupled to electro-optic interactions, a fully physical simulation of electro-optic modulators is possible. Also, standard FDTD techniques can be enhanced by employing multiresolution approaches. For an electro-optic modulator case, an 80% CPU-time reduction is achieved, while maintaining the same degree of accuracy of standard FDTD techniques. This precise modeling of electro-optic modulators can be expected to contribute significantly to superior device performance in the future.

Acknowledgement

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