

Partonic calculation of the two-photon exchange contribution to elastic electron-proton scattering at large momentum transfer

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We estimate the two-photon exchange contribution to elastic electron-proton scattering at large momentum transfer through the scattering off a parton in the proton. We relate the process on the nucleon to the generalized parton distributions which also enter in other wide angle scattering processes. We find that when taking the polarization transfer determinations of the form factors as input, adding in the 2 photon correction, does reproduce the Rosenbluth data.

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There are currently two experimental methods to extract the ratio of electric (G_{Ep}) to magnetic (G_{Mp}) proton form factors: unpolarized measurements employing the Rosenbluth separation technique, and experiments using a polarized electron beam measuring the ratio of the (in-plane) polarization components of the recoiling proton parallel and perpendicular to its momentum. Recent experiments at Jefferson Lab (JLab) [1, 2] confirm the earlier Rosenbluth data [3], which are at variance with the JLab measurements of G_{Ep}/G_{Mp} at larger Q^2 using the polarization transfer technique [4, 5]. This discrepancy casts doubt on electron scattering as a precision tool, and needs to be sorted out in detail.

Given that no flaws in either experimental technique have been identified to date, the most likely explanation is that 2γ exchange effects (beyond those already accounted for in the standard radiative corrections) are responsible for the discrepancy. The general structure of two- (and multi)-photon exchange contributions to the elastic electron proton scattering observables has recently been studied [6]. It was found in that work that the 2γ exchange contribution to the unpolarized cross section can be kinematically enhanced at larger Q^2 compared with the $(G_{Ep})^2$ term, while the 2γ exchange contribution to the polarization measurements need not affect the results in a significant way. A 2γ exchange amplitude at the level of a few percent may well explain the discrepancy between the two methods. To show in a quantitative way that 2γ exchange effects are indeed able to resolve this discrepancy, realistic calculations of elastic electron-nucleon scattering beyond the Born approximation are needed. One step in this direction was taken recently in [7], where the contribution to the 2γ exchange amplitude was calculated for a nucleon intermediate state. This calculation found that the 2γ exchange correction with the intermediate nucleon has the proper sign and magnitude to partially resolve the discrepancy between the two experimental techniques. In this letter, we report the first calculation of the elastic electron-nucleon

scattering at large momentum transfer through the scattering off partons in a nucleon. We relate the process on the nucleon to generalized parton distributions (GPDs), which also enter in other wide angle scattering processes.

To describe the elastic electron-nucleon scattering :

$$l(k) + N(p) \rightarrow l(k') + N(p'), \quad (1)$$

we adopt the definitions : $P = (p + p')/2$, $K = (k + k')/2$, $q = k - k' = p' - p$, and choose $Q^2 = -q^2$ and $\nu = K \cdot P$ as the independent kinematical invariants. Neglecting the electron mass, the T -matrix for elastic electron-nucleon scattering can be expressed through 3 independent Lorentz structures as [6] :

$$\begin{aligned} T_{h, \lambda'_N \lambda_N} &= \frac{e^2}{Q^2} \bar{u}(k', h) \gamma_\mu u(k, h) \\ &\times \bar{u}(p', \lambda'_N) \left(\tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2} \right) u(p, \lambda_N), \end{aligned} \quad (2)$$

where $h = \pm 1/2$ is the electron helicity and λ_N (λ'_N) are the helicities of the incoming (outgoing) nucleon. Furthermore, \tilde{G}_M , \tilde{F}_2 , \tilde{F}_3 are complex functions of ν and Q^2 , and we introduced the factor e^2/Q^2 for convenience, where $e \simeq \sqrt{4\pi/137}$ is the proton charge, and M is the nucleon mass. To separate the one- and two-photon exchange contributions, we introduce the decompositions : $\tilde{G}_M = G_M + \delta\tilde{G}_M$, and $\tilde{F}_2 = F_2 + \delta\tilde{F}_2$, where G_M (F_2) are the proton magnetic (Pauli) form factors respectively, defined from the matrix element of the electromagnetic current. The amplitudes \tilde{F}_3 , $\delta\tilde{G}_M$ and $\delta\tilde{F}_2$ originate from processes involving at least the exchange of two photons, and are of order e^2 (relative to the factor e^2 in (2)). The reduced cross section for elastic electron-nucleon scattering, including corrections up to order e^2 is given by [6]:

$$\begin{aligned} \sigma_R &= G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2 G_M \mathcal{R} \left(\delta\tilde{G}_M + \varepsilon \frac{\nu}{M^2} \tilde{F}_3 \right) \\ &+ 2 \frac{\varepsilon}{\tau} G_E \mathcal{R} \left(\delta\tilde{G}_E + \frac{\nu}{M^2} \tilde{F}_3 \right) + \mathcal{O}(e^4), \end{aligned} \quad (3)$$

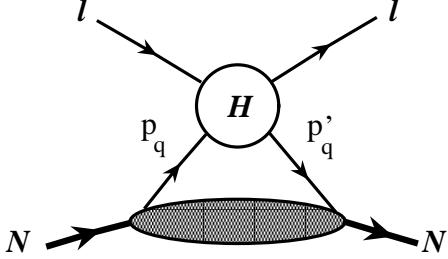


FIG. 1: Handbag diagram for the elastic lepton-nucleon scattering at large momentum transfers. In the hard scattering process H , the lepton scatters from quarks with momenta p_q and p'_q . The lower blob represents the GPDs of the nucleon.

where \mathcal{R} stands for the real part, $\tau \equiv Q^2/(4M^2)$, and $\varepsilon \equiv (\nu^2 - M^4\tau(1+\tau)) / (\nu^2 + M^4\tau(1+\tau))$. By analogy, we have defined : $\tilde{G}_E \equiv \tilde{G}_M - (1+\tau)\tilde{F}_2 = G_E + \delta\tilde{G}_E$, with G_E the proton electric form factor, and $\delta\tilde{G}_E$ the two-photon exchange correction.

An observable which is directly proportional to the 2γ exchange is given by the elastic scattering of an unpolarized electron on a proton target polarized normal to the scattering plane. The corresponding single spin asymmetry A_n is related to the absorptive part of the elastic eN scattering amplitude [8]. Since the 1γ exchange amplitude is purely real, the leading contribution to A_n is due to an interference between 1γ and 2γ exchange. It can be expressed at order $O(e^2)$ as (for $m_e = 0$) :

$$A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}} \frac{1}{\sigma_R} \left\{ -G_M \mathcal{I}\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E \mathcal{I}\left(\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right) \frac{\nu}{M^2}\tilde{F}_3\right) \right\} \quad (4)$$

where \mathcal{I} denotes the imaginary part.

To estimate the 2γ contribution to $\delta\tilde{G}_M$, $\delta\tilde{G}_E$, and \tilde{F}_3 at large Q^2 , we consider in this letter a partonic calculation as shown on Fig. 1. As a first step, we calculate elastic electron-quark scattering with massless quarks : $l(k) + q(p_q) \rightarrow l(k') + q(p'_q)$. The Mandelstam invariants are given by $\hat{s} \equiv (k + p_q)^2$, Q^2 , and $\hat{u} \equiv (k - p'_q)^2$, satisfying $\hat{s} + \hat{u} = Q^2$. The T -matrix for the 2γ part of the electron-quark hard scattering process can be written as :

$$H_{h,\lambda}^{hard} = \frac{e^2}{Q^2} \bar{u}(k', h) \gamma_\mu u(k, h) \times \bar{u}(p'_q, \lambda) \left(e_q^2 \tilde{f}_1 \gamma^\mu + e_q^2 \tilde{f}_3 \gamma \cdot K P_q^\mu \right) u(p_q, \lambda), \quad (5)$$

where $P_q \equiv (p_q + p'_q)/2$, e_q is the fractional quark charge (for a flavor q), and the quark helicity $\lambda = \pm 1/2$ is conserved in the hard scattering process. For massless quarks, there is no analog of \tilde{F}_2 in Eq. (2).

To calculate the hard amplitudes $H_{h,\lambda}^{hard}$, we consider the 2γ exchange direct and crossed box diagrams of

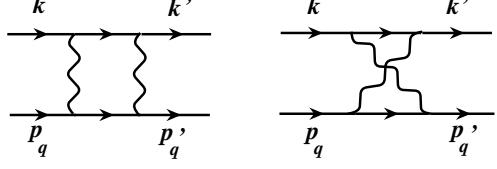


FIG. 2: Direct and crossed box diagrams to describe the two-photon exchange contribution to the lepton-quark scattering process, corresponding with the blob denoted by H in Fig. 1.

Fig. 2. The 2γ exchange contribution to the elastic electron scattering off elementary spin 1/2 particles has been calculated before. Early references include Refs. [9, 10], which we verified explicitly. For further use, we separate \tilde{f}_1 into a soft and hard part, i.e. $\tilde{f}_1 = \tilde{f}_1^{soft} + \tilde{f}_1^{hard}$, using the procedure of Ref. [11]. The soft part corresponds with the situation where one of the photons in Fig. 2 carries zero four-momentum, and is obtained by replacing the other photon's four-momentum by q in the numerator and in its propagator in the loop integral. This yields :

$$\mathcal{R}\left(\tilde{f}_1^{soft}\right) = \frac{e^2}{4\pi^2} \left\{ \ln\left(\frac{\lambda^2}{\sqrt{-\hat{s}\hat{u}}}\right) \ln\left|\frac{\hat{s}}{\hat{u}}\right| + \frac{\pi^2}{2} \right\}, \quad (6)$$

$$\begin{aligned} \mathcal{R}\left(\tilde{f}_1^{hard}\right) &= \frac{e^2}{4\pi^2} \left\{ \frac{1}{2} \ln\left|\frac{\hat{s}}{\hat{u}}\right| \right. \\ &\quad \left. + \frac{Q^2}{4} \left[\frac{1}{\hat{u}} \ln^2\left|\frac{\hat{s}}{Q^2}\right| - \frac{1}{\hat{s}} \ln^2\left|\frac{\hat{u}}{Q^2}\right| - \frac{1}{\hat{s}}\pi^2 \right] \right\}, \end{aligned} \quad (7)$$

where \tilde{f}_1^{soft} , which contains a term proportional to $\ln\lambda^2$ (λ is an infinitesimal photon mass), is IR divergent. The real part of \tilde{f}_3 from Fig. 2 is IR finite, and is given by :

$$\begin{aligned} \mathcal{R}\left(\tilde{f}_3\right) &= \frac{e^2}{4\pi^2} \frac{1}{\hat{s}\hat{u}} \left\{ \hat{s} \ln\left|\frac{\hat{s}}{Q^2}\right| + \hat{u} \ln\left|\frac{\hat{u}}{Q^2}\right| \right. \\ &\quad \left. + \frac{\hat{s} - \hat{u}}{2} \left[\frac{\hat{s}}{\hat{u}} \ln^2\left|\frac{\hat{s}}{Q^2}\right| - \frac{\hat{u}}{\hat{s}} \ln^2\left|\frac{\hat{u}}{Q^2}\right| - \frac{\hat{u}}{\hat{s}}\pi^2 \right] \right\}. \end{aligned} \quad (8)$$

The imaginary parts of \tilde{f}_1 and \tilde{f}_3 originate solely from the direct 2γ exchange box diagram of Fig. 2 and are :

$$\mathcal{I}\left(\tilde{f}_1^{soft}\right) = -\frac{e^2}{4\pi} \ln\left(\frac{\lambda^2}{\hat{s}}\right), \quad (9)$$

$$\mathcal{I}\left(\tilde{f}_1^{hard}\right) = -\frac{e^2}{4\pi} \left\{ \frac{Q^2}{2\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) + \frac{1}{2} \right\}, \quad (10)$$

$$\mathcal{I}\left(\tilde{f}_3\right) = -\frac{e^2}{4\pi} \frac{1}{\hat{u}} \left\{ \frac{\hat{s} - \hat{u}}{\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) + 1 \right\}. \quad (11)$$

Having calculated the hard subprocess, we next discuss how to embed the quarks in the nucleon. We begin by discussing the soft contributions. There are also soft contributions from processes where the photons interact with different quarks. One can show that the IR contributions from these processes, which are proportional to the products of the charges of the interacting quarks,

added to the soft contributions from the handbag diagrams give the same result as the soft contributions calculated with just a nucleon intermediate state. Thus the low energy theorem is satisfied. The hard parts when the photons couple to different quarks are subleading in Q^2 because of momentum mismatches in the wavefunctions.

For the real parts, the IR divergence arising from the direct and crossed box diagrams is cancelled when adding the bremsstrahlung interference contribution with a soft photon emitted from the electron and proton. This provides a radiative correction term proportional to the target charge Z , which may be written as :

$$\sigma_{R,soft} = \sigma_{1\gamma} \left(1 + \delta_{2\gamma}^{soft} + \delta_{brems}^{ep} \right), \quad (12)$$

where $\sigma_{1\gamma}$ is the 1γ exchange cross section. In (12), the soft-photon part of the nucleon box diagram is given by

$$\begin{aligned} \delta_{2\gamma}^{soft} = & \frac{e^2}{2\pi^2} \left\{ \ln \left(\frac{\lambda^2}{\sqrt{(s-M^2)|u-M^2|}} \right) \ln \left| \frac{s-M^2}{u-M^2} \right| \right. \\ & - L \left(\frac{s-M^2}{s} \right) - \frac{1}{2} \ln^2 \left(\frac{s-M^2}{s} \right) \\ & \left. + \mathcal{R} \left[L \left(\frac{u-M^2}{u} \right) \right] + \frac{1}{2} \ln^2 \left(\frac{u-M^2}{u} \right) + \frac{\pi^2}{2} \right\}, \end{aligned} \quad (13)$$

where L is the Spence function. The bremsstrahlung contribution δ_{brems}^{ep} we take from Ref. [12] (see their Eq. (4.14)). When comparing with elastic ep cross section data, which are usually radiatively corrected using the Mo and Tsai procedure [13], we only have to consider the difference between our above IR finite $\delta_{2\gamma}^{soft} + \delta_{brems}^{ep}$ and the expression for the Z -dependent radiative correction in [13]. This difference is predominantly given by a constant shift proportional to $\pi^2/2$ in Eq. (13).

We next discuss the hard 2γ exchange contribution. In the kinematical regime where $s = (p+k)^2$, $-u = -(p-k')^2$ and Q^2 are large compared to a hadronic scale ($s, -u, Q^2 \gg M^2$), this part of the amplitude is calculated as a convolution between an electron-quark hard scattering and a soft nucleon matrix element. It is convenient to choose a frame where $q^+ = 0$, where we introduce light-cone variables a^\pm proportional to $(a^0 \pm a^3)$ and choose $P^3 > 0$. In the frame $q^+ = 0$, the $+$ momentum fractions of electrons and partons are defined as $\eta = K^+/P^+$ and $x = P_q^+/P^+$ respectively. At large Q^2 , we can neglect the intrinsic transverse momenta of the active quarks. The Mandelstam invariants for the hard process are then given by $\hat{s} = Q^2(x+\eta)^2/(4x\eta)$ and $\hat{u} = -Q^2(x-\eta)^2/(4x\eta)$. We extend the handbag formalism [14], used in wide angle Compton scattering [15, 16], to the 2γ exchange process in elastic ep scattering, and keep the x dependence in the hard scattering amplitude. This yields the T -matrix for the process (1) as :

$$T_{h, \lambda'_N \lambda_N}^{hard} = \int_{-1}^1 \frac{dx}{x} \sum_q \frac{1}{2} \left[H_{h, +\frac{1}{2}}^{hard} + H_{h, -\frac{1}{2}}^{hard} \right]$$

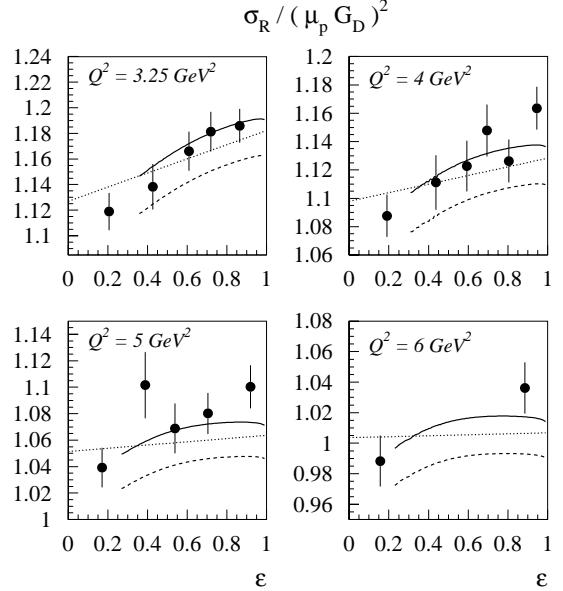


FIG. 3: Rosenbluth plots for elastic ep scattering : σ_R divided by $(\mu_p G_D)^2$, with $G_D = (1 + Q^2/0.71)^{-2}$. Dotted curves : Born approximation using G_{Ep}/G_{Mp} from polarization data [4, 5]. Dashed curves : results when adding the GPD calculation for the hard 2γ exchange correction, for the kinematical range $s, -u > M^2$. Full curves are the total results, including in addition the soft 2γ exchange correction relative to Mo and Tsai. The data are from Ref. [3].

$$\begin{aligned} & \times \left[H^q(x, 0, q^2) \bar{u}(p', \lambda'_N) \gamma \cdot n u(p, \lambda_N) \right. \\ & \left. + E^q(x, 0, q^2) \bar{u}(p', \lambda'_N) \frac{i \sigma^{\mu\nu} n_\mu q_\nu}{2M} u(p, \lambda_N) \right] \\ & + \int_{-1}^1 \frac{dx}{x} \sum_q \frac{1}{2} \left[H_{h, +\frac{1}{2}}^{hard} - H_{h, -\frac{1}{2}}^{hard} \right] \text{sgn}(x) \\ & \times \tilde{H}^q(x, 0, q^2) \bar{u}(p', \lambda'_N) \gamma \cdot n \gamma_5 u(p, \lambda_N), \end{aligned} \quad (14)$$

where H^{hard} is evaluated using \tilde{f}_1^{hard} and \tilde{f}_3 , and where $n^\mu = 2/\sqrt{M^4 - su} (-\eta P^\mu + K^\mu)$ is a Sudakov four-vector ($n^2 = 0$). Furthermore, H^q, E^q, \tilde{H}^q are the GPDs for a quark q in the nucleon.

From Eqs. (2), (5), and (14) the hard 2γ exchange contributions to $\delta\tilde{G}_M$, $\delta\tilde{G}_E$, \tilde{F}_3 are obtained as :

$$A \tilde{G}_M^{hard} = C, \quad (15)$$

$$B \tilde{G}_E^{hard} = - \left(\frac{1+\varepsilon}{2\varepsilon} \right) (A - C) + \sqrt{\frac{1+\varepsilon}{2\varepsilon}} B, \quad (16)$$

$$\tilde{F}_3 = \frac{M^2}{\nu} \left(\frac{1+\varepsilon}{2\varepsilon} \right) (A - C), \quad (17)$$

with

$$A \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u}) \tilde{f}_1^{hard} - \hat{s} \hat{u} \tilde{f}_3]}{s - u} \sum_q e_q^2 (H^q + E^q),$$

$$B \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u}) \tilde{f}_1^{hard} - \hat{s} \hat{u} \tilde{f}_3]}{(s - u)} \sum_q e_q^2 (H^q - \tau E^q),$$

$$C \equiv \int_{-1}^1 \frac{dx}{x} \tilde{f}_1^{hard} \operatorname{sgn}(x) \sum_q e_q^2 \tilde{H}^q,$$

To estimate the amplitudes of Eqs. (15)-(17), we need to specify a model for the GPDs. Following Refs. [15, 16], we use an unfactorized (valence) model for the GPDs H , E , and \tilde{H} , in terms of a forward parton distribution and a gaussian factor in x and q^2 (see Eq. (68) in second Ref. [16] with transverse size parameter $a = 0.8 \text{ GeV}^{-1}$).

In Fig. 3, we display the effect of 2γ exchange on the cross sections. For the form factors, we use the G_{Ep}/G_{Mp} ratio as extracted from the polarization transfer experiments [5]. For G_{Mp} , we adopt the parametrization of [17] (scaled by a factor 0.995, as discussed further on). Fig. 3 illustrates that the values of G_{Ep} as extracted from the polarization data are inconsistent with the slopes one extracts from a linear fit to the Rosenbluth data in the Q^2 range where data from both methods exist. By adding the 2γ exchange correction, using the GPD calculation as described above, one firstly observes that the Rosenbluth plot becomes slightly non-linear, in particular at the largest ε values. Furthermore, one sees that over most of the ε range, the slope is indeed steeper in agreement with the Rosenbluth data. Thirdly, in order to fit the data when including the 2γ exchange correction one has to slightly decrease the value of G_{Mp} of [17] (by a factor 0.995). This change in G_{Mp} is just the simplest estimate of how the additional radiative corrections would affect the extraction of G_{Mp} from the data. We see that including the 2γ exchange allows to reconcile both polarization transfer and Rosenbluth data. It is clearly worthwhile to do a global re-analysis of all large Q^2 elastic data including the 2γ correction, which is however beyond the scope of this letter. Such a re-analysis will allow one to relate the intercept and slopes of the dotted curves in Fig. 3 (obtained from the solid curves by turning off the 2γ correction) to the true values of G_{Ep} and G_{Mp} respectively.

The real part of the 2γ exchange amplitude can be accessed directly as the deviation from unity of the ratio of e^+/e^- elastic scattering. Our calculation gives an e^+/e^- ratio of about 0.98 in the range $Q^2 = 2 - 5 \text{ GeV}^2$ and at large ε values, about 1σ below the SLAC data [18]. At smaller values of ε , where no data exist at moderately large Q^2 , our prediction for the e^+/e^- ratio rises and becomes larger than 1 around $\varepsilon = 0.35$. It will be an important test of the calculation to check this trend.

A test of the imaginary part of the 2γ exchange amplitudes is obtained using A_n , shown in Fig. 4. Our partonic estimate for A_n displays a forward peaking, and reaches a significant part of 1 %. It may provide a useful cross-check of the 2γ amplitude.

In summary, we have estimated the two-photon exchange contribution to elastic ep scattering at large Q^2 in

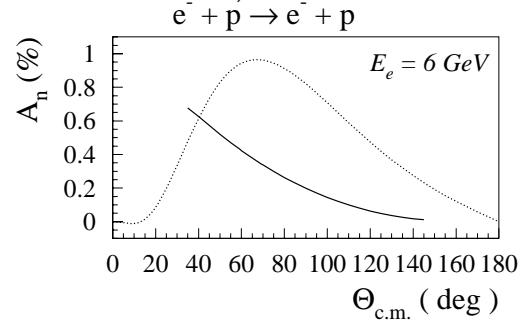


FIG. 4: Proton normal spin asymmetry for elastic ep scattering as function of the *c.m.* scattering angle. The GPD calculation is shown by the solid curve, bounded by the kinematic range where $-u, Q^2 > M^2$. For comparison, the elastic contribution (nucleon intermediate state in the two-photon exchange box diagram) is shown by the dotted curve [8].

a partonic calculation, and were able to express this amplitude in terms of the GPDs of the nucleon. We found that the 2γ exchange contribution is able to quantitatively resolve the existing discrepancy between Rosenbluth and polarization transfer experiments.

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