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# LOW EMITTANCE E-/E+ STORAGE RING DESIGN USING BENDING MAGNETS WITH LONGITUDINAL GRADIENT

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*Abstract*

Low emittance is desired in most synchrotron light source and damping ring designs. The work presented in this paper demonstrates that by introducing dipoles with a longitudinal gradient, the minimum emittance can be lowered without increasing the number of dipoles. The minimum I5/I2 is limited by the ratio of maximum and average field in the dipole; by setting that ratio to 3, a factor of 4-5 in I5/I2 reduction can be achieved in TME-type lattice. In practical lattice designs, the reduction factor still can be 2-4 depending on configurations and constraints.

## 1 INTRODUCTION

Low emittance $e^-/e^+$ storage rings are necessary in several applications such as synchrotron radiation light sources, which deliver high spectral brightness photon beam, and damping rings for linear colliders, which provide high luminosity.

The design strategy in synchrotron light sources and damping rings is similar when choosing and optimising the storage ring lattice for low emittance. A major difference between the two kinds of designs is that light sources need many dispersion free (or near free) straight sections for insertion devices while damping rings need fewer straight sections (usually only 2). So for light sources, a multi-bend achromat (MBA) lattice is usually used, and the theoretical minimum emittance (TME) lattice is a good choice for damping rings. The damping rings also require short damping times while the light sources prefer slower damping to reduce the RF load.

The natural emittance of an $e^-/e^+$ storage ring is determined by the balance between radiation damping and quantum excitation. It can be expressed as: [1]

$$\varepsilon_x = C_q \gamma^2 \frac{I_5}{I_2 - I_4} \qquad (1)$$

$$C_q = \frac{55\hbar}{32\sqrt{3}mc} = 3.84\times10^{-13}\,(\text{m-rad})$$

where $I_2$, $I_4$ and $I_5$ are synchrotron integrals:

$$I_2 = \oint \frac{1}{\rho^2} ds\,,\ I_4 = \oint \frac{(1-2n)\eta}{\rho^3} ds$$

$$I_5 = \oint \frac{H}{\rho^3} ds$$

$$H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$$

In conventional storage ring designs, uniform dipoles are used. By optimizing the optics parameters at the entrance and/or center of dipoles, the minimum emittance that can be achieved in different types of lattices can be expressed as:

$$\varepsilon_x = F(\nu_x, lattice)\frac{E^2(GeV)}{J_x N_d^3}\,(\text{m-rad}) \quad (2)$$

where F is a factor determined by different boundary conditions and the phase advance of the cell, and $N_d$ is the number of dipoles in the storage ring. A realistic lattice usually has a slightly higher emittance than this minimum.

To further reduce the emittance, one can vary the bending field along the dipole. By setting the bending field higher where H(s) is small and vice versa, the emittance can be reduced. The solution requires minimizing Eq. (1) by varying ρ(s) and the initial condition of optics. We solved this problem numerically using a modified version of the MAD program, which allows fitting of the synchrotron integrals as well as the optics. As discussed below we found significant improvement in both TME and DBA lattices.

## 2 MINIMUM EMITTANCE OF ISOLATED DIPOLE

### 2.1 Uniform Dipole

For a TME lattice using uniform field dipoles, there is no constraint on the optics parameters at both ends of the dipoles. The minimum emittance is $\varepsilon = C_q\gamma^2\theta_b^3/12\sqrt{15}J_x$, which is achieved when $\alpha_x=\eta'_x=0$, $\beta_x=L_b/\sqrt{60}$, and $\eta_x=L_b\theta_b/24$ at center of dipole [2].

For a DBA lattice using uniform field dipoles, zero $\eta_x$ and $\eta'_x$ is required at the entrance of dipole. In this case, the minimum emittance is $\varepsilon=C_q\gamma^2\theta_b^3/4\sqrt{15}J_x$, which is achieved when $\beta_x=6L_b/\sqrt{15}$, and $\alpha_x=\sqrt{15}$ at the entrance of dipole [2].

### 2.2 Variational Dipole

Our results show that by optimising the dipole field distribution along the beam orbit without constraint on the damping rate or the maximum dipole field, the minimum emittance can be reduced by a factor of 7 in the TME lattice and a factor of 6 in the DBA; results for the TME lattice are shown in Table 1. The numerical calculations are performed by splitting the dipole into 32 equal length slices. In this case, the optimised TME emittance can be expressed as $\varepsilon=2.98\times10^{-3}C_q\gamma^2\theta_b^3/J_x$ with $\beta_x=0.015L_b$ and $\eta_x=1.38\times10^{-3}L_b\theta_b$ at the center of dipole. Similarly, the optimised DBA emittance is $\varepsilon=1.11\times10^{-2}C_q\gamma^2\theta_b^3/J_x$.

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Splitting the dipole into more slices yields slightly lower emittance.

Figure 1 shows the field distribution of a variational TME bend which has the approximate form $1/\rho=b/(a+s)$, where $a$ and $b$ are constants and $s$ is the longitudinal distance. Variation of the bend length and angle do not change the form of the optimised field when normalized to the average bend field. The field of a variational bend in a DBA lattice has a similar form as in the TME case.

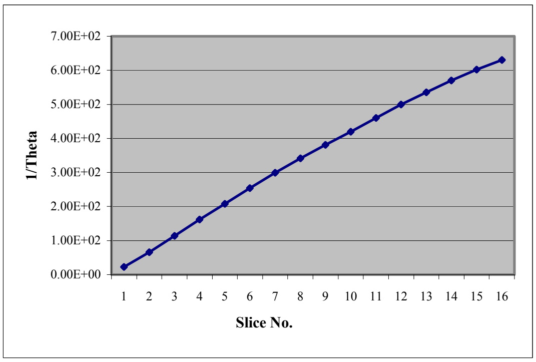


Fig. 1 Field distribution of variational TME bend

We also made an empirical analysis with the assumption that the optimised dipole field scales as $b/(1+as)^l$. The result shows good agreement with numerical simulation with $l\sim1$. Theoretically, zero $I5/I2/\theta^3$ can be achieved for certain values of $l$ if no limits on the maximum field or bend length are applied.

## 3 APPLICATION IN NLC DAMPING RING AND COMPARISON WITH CONVENTIONAL DESIGNS

We have applied the variational dipole to the NLC damping ring lattice. A practical storage ring lattice has constraints such as the bend field, cell tunes, damping time and the length of a cell. These constraints cause the minimum I5/I2 to be larger than in an isolated bend.

To compare with the conventional 1.98GeV 36-cell combined-function TME damping ring design [3], the variational bend TME lattice has a similar cell length and tune, and the same number of cells. The maximum bend field is set at 2 Tesla and the bend length is limited to 1.5 meters. With these constraints, the minimum I5/I2 value is still a factor of 4 smaller than in the conventional case.

The variational TME dipole requires much smaller $\beta_{x0}$ and $\eta_{x0}$ than in a conventional bend lattice, which results in higher tunes and larger beta functions at the ends of bend. In a real lattice, without limiting the tunes or the length of the cell, the value of I5/I2 can approach that of the isolated variational bend, although the chromaticities become high. Constraining the parameters (include cell length, tune and chromaticity, etc) of the variational bend lattice to similar values as in the conventional lattice, the variational bend TME lattice has an emittance that is roughly 50% that of the conventional case. The longitudinal profile of one quarter of a dipole magnet that has the desired field profile is shown in Fig. 2.

Table 1. Minimum I5/I2 for isolated bend

| | Uniform Bend | Variational Bend | |
|---|---|---|---|
| Max B (T) | 1.2 | 2.0 | 5.10 |
| Length (m) | 0.96 | 1.5 | 1.5 |
| Bend angle | $2\pi/36$ | $2\pi/36$ | $2\pi/36$ |
| I2 | 3.173e-2 | 3.139e-2 | 6.887e-2 |
| I5/I2 | 1.142e-4 | 2.945e-5 | 1.5793e-5 |
| $\beta_{x0}$ (m) | 0.124 | 0.0803 | 0.0222 |
| $\eta_{x0}$ (m) | 6.98e-3 | 1.40e-3 | 3.62e-4 |

Table 2. Comparison between conventional lattice and variational bend lattice with separated function bend

| | Uniform Bend | Variational Bend |
|---|---|---|
| Cell length | 5.12m | 5.12m |
| Bend length | 0.96m | 1.5m |
| $B_{max}$ (T) | 1.2 | 2.0 |
| I2 | 3.17e-2 | 3.17e-2 |
| I5/I2 | 1.58e-4 | 0.84e-4 |
| $\mu_x/\mu_y$ | 0.63/0.24 | 0.63/0.24 |
| $\xi_x/\xi_y$ | -0.949/-1.664 | -0.773/-1.537 |
| $\beta_{max,\,x/y}$ (m) | 4.99/27.59 | 4.31/25.40 |
| Sext coeff | -18.9/26.9 | -19.3/26.1 |

Table 3. Comparison between conventional lattice and variational bend lattice with combined function bend

| | Uniform Bend | Variational Bend |
|---|---|---|
| $K_b$ | -1.0 | -0.64(avg) |
| Cell length | 5.12m | 5.12m |
| Bend length | 0.96 | 1.5m |
| $B_{max}$ (T) | 1.2 | 2.0 |
| I2 | 3.17e-2 | 3.17e-02 |
| I5/(I2-I4) | 1.61e-4 | 7.72e-5 |
| $\mu_x/\mu_y$ | 0.625/0.268 | 0.63/0.24 |
| $\xi_x/\xi_y$ | -0.868/-0.629 | -0.831/-0.586 |
| $\beta_{max,\,x/y}$ (m) | 4.35/ 10.33 | 4.35/9.83 |
| Sext coeff | -17.7/23.8 | -23.8/28.6 |

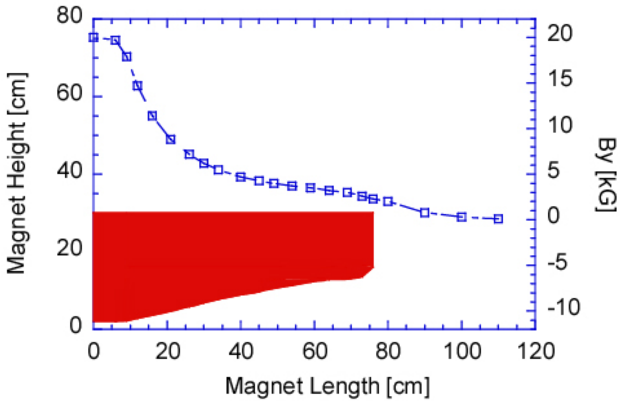


Fig. 2 Profile and field distribution of variational bend in Table 1 with 2.0 Tesla max B, without transverse gradient

A vertical focusing gradient in the dipoles is helpful in reducing the vertical chromaticity and increasing the dynamic aperture. The present NLC damping ring design is based upon combined-function dipoles. Clearly a combined-function magnet will be more difficult in a magnet that has a longitudinal gradient. To study this, we assume that the local transverse focusing gradient along the length of the magnet is simply proportional to the local bending field. The optics of the resulting cell are plotted in Fig. 3 while Table 3 lists the parameters of the comparison – again a 50% decrease in the emittance was found.

Finally, one might assume that the conventional and variational bend lattices might have comparable dynamic apertures based on the similar values of the chromaticity in the two lattices. Figure 4 shows that the 36-cell variational bend TME lattice can obtain satisfactory dynamic aperture, larger than 15 times injected beam size, which is comparable to that in the conventional lattice.

Thus far, we have discussed simply replacing cells in a conventional lattice with the variational bending cells. As another option, one could reduce the number of cells while keeping the emittance constant. Calculations show that we can replace the 36-cell conventional TME lattice with 28-cell variational bend lattice having the same cell

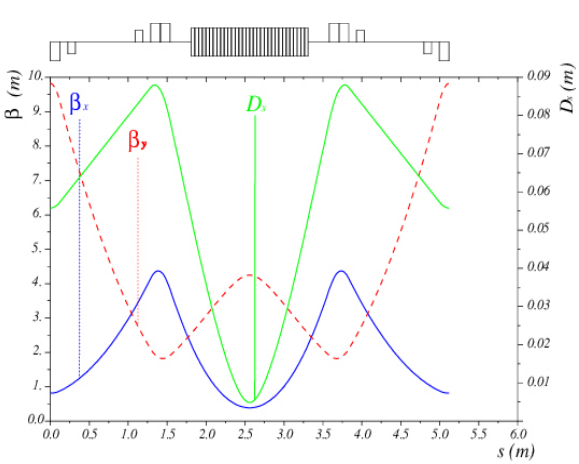


Fig. 3 Variational combined function bend TME lattice with parameters shown in Table 3

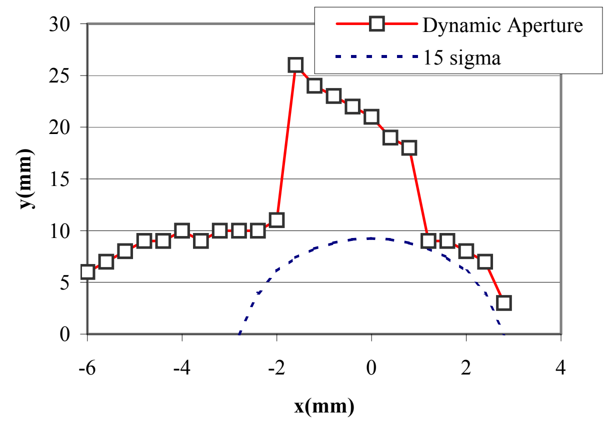


Fig. 4 Dynamic aperture of variational bend TME lattice with parameters shown in Table 3 (Dashed lines are 15 times the injected beam size)

tunes. It is likely that the ring with fewer cells will have a larger dynamic aperture, larger momentum compaction factor, and shorter circumference. The larger momentum compaction is desirable since it leads to a longer bunch length and eases the requirements on the longitudinal impedance. By pushing the horizontal tune to 0.8, we can further reduce the cell number to 25 without significant changes in the dynamic aperture or momentum compaction.

## 4 CONCLUSIONS

From the above discussion, we may make the following conclusions:

- Using the same cell length and tunes as in a conventional lattice and assuming a practical maximum field, the variational bend TME lattice is able to reduce the storage ring emittance by a factor of 2-3, without a significant change in the dynamic aperture.
- To get the same emittance, the variational bend TME lattice needs 25% to 30% fewer cells, which might lead to a larger dynamic aperture, a larger momentum compaction, and shorter circumference.

## 5 REFERENCES

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