# Probing the CKM Triangle at BABAR * 

G. EIGEN<br>FOR THE BABAR COLLABORATION

University of Bergen

We present recent BABAR measurements that constrain both the sides and the angles of the Unitarity Triangle.

PACS numbers: PACS numbers come here

## 1. Introduction

In the Standard Model (SM) CP violation results from Yukawa couplings of the Higgs field to the quark fields that are introduced to produce fermion masses. The quark mass matrix is diagonalized by four unitary transformations. In the charged-weak current the interactions with the $W$ boson to the quarks becomes flavor diagonal in the basis of weak eigenstates. The latter are related to mass eigenstates by the unitary Cabbibo-KobayashiMaskawa (CKM) matrix $V_{i j}[1]$ [2], where the first (second) index denotes an up-type (down-type) quark. A convenient representation of CKM matrix is the Wolfenstein parameterization [3], which to order $O\left(\lambda^{5}\right)$ is given by:

$$
V=\left(\begin{array}{ccc}
1-\frac{\lambda^{2}}{2}-\frac{\lambda^{4}}{8} & \lambda & A \lambda^{3}(\rho-i \eta)  \tag{1}\\
-\lambda+A^{2} \lambda^{5}\left(\frac{1}{2}-\rho-i \eta\right) & 1-\frac{\lambda^{2}}{2}-\frac{\lambda^{4}}{8}\left(1+4 A^{2}\right) & A \lambda^{2} \\
A \lambda^{3}(1-\bar{\rho}-i \bar{\eta}) & -A \lambda^{2}+A \lambda^{4}\left(\frac{1}{2}-\rho-i \eta\right) & 1-\frac{1}{2} A^{2} \lambda^{4}
\end{array}\right)+O\left(\lambda^{6}\right) .
$$

Of the four parameters $\lambda=0.2235 \pm 0.0033$, is the best measured, $A \simeq 0.82$ is known to $\sim \pm 5 \%$, while $\bar{\rho}, \bar{\eta}$ are poorly known. In the SM , $\bar{\eta}$ represents the $C P$-violating phase. Unitarity of CKM matrix yields six triangular relations, of which $V_{u d} V_{u b}^{*}+V_{c d} V_{c b}^{*}+V_{t d} V_{t b}^{*}=0$ is the most useful, since it specifies a triangle in the $\bar{\rho}-\bar{\eta}$ plane with apex $\bar{\rho}, \bar{\eta}$ and sides of similar length.

Work supported in part by the Department of Energy contract DE-AC03-76SF00515.

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309
*Presented at the 27th International Conference of Theoretical Physics: Matter to the Deepest:
Recent Developments in Physics of Fundamental Interactions (Ustron 03),
September 15-21, 2003, Ustron, Poland

The BABAR experiment has been running since May 1999 and by now has recorded an integrated luminosity of $\mathcal{L}_{\text {tot }}=131.3 \mathrm{fb}^{-1}$ on the $\Upsilon(4 S)$ peak and $\mathcal{L}_{\text {tot }}=12.5 \mathrm{fb}^{-1}$ in the continuum 30 MeV below the $\Upsilon(4 S)$ peak. This luminosity corresponds to a $B \bar{B}$ sample of $\sim 1.42 \times 10^{8}$ events. The performance of the BABAR detector is described elsewhere [4].

Since in the $\Upsilon(4 S)$ rest frame $B$ mesons are produced nearly at rest, we can take advantage of two kinematic variables, $m_{E S}=\sqrt{E_{\text {beam }}^{* 2}-p_{B}^{* 2}}$ and $\Delta E=E_{B}^{*}-E_{\text {beam }}^{*}$, where $E_{\text {beam }}^{*}$ and $E_{B}^{*}\left(p_{B}^{*}\right)$ respectively denote the beam energy and the energy (momentum) of the reconstructed $B$ meson in the $\Upsilon(4 S)$ rest frame. In the $\Delta E-m_{E S}$ plane signal events will cluster around $\Delta E=0$ and $m_{E S}=m_{B}$, while backgrounds from other $B$ decays and $q \bar{q}$ continuum typically show no peaking behavior in the signal region.

## 2. Measurement of $\left|V_{c b}\right|$

The CKM matrix element $\left|V_{c b}\right|$ is extracted from semileptonic decays involving charm quarks. The inclusive decay rate to order $O\left(1 / m_{B}^{2}\right)$ is predicted by the heavy quark expansion (HQE) [5]:

$$
\begin{align*}
\Gamma\left(B \rightarrow X_{c} \ell \nu\right) & =\frac{G_{F}^{2}\left|V_{u b}\right|^{2}}{192 \pi^{3}} \cdot 0.369 m_{B}^{5} \times\left[1-1.54 \frac{\alpha_{s}}{\pi}-1.65 \frac{\bar{\Lambda}}{m_{B}}\left(1-0.87 \frac{\alpha_{s}}{\pi}\right)\right. \\
& \left.-0.95 \frac{\bar{\Lambda}^{2}}{m_{B}^{2}}-3.18 \frac{\lambda_{1}}{m_{B}^{2}}+0.02 \frac{\lambda_{1}}{m_{B}^{2}}+O\left(\frac{\Lambda_{Q C D}}{m_{B}}\right)^{3}\right] \tag{2}
\end{align*}
$$

where $G_{F}$ is the Fermi constant, $\bar{\Lambda}$ denotes interactions of light degrees of freedom with the $b$-quark, $\lambda_{1}$ represents the kinetic energy squared of $b$ quark in the B meson, and $\lambda_{2}$ denotes the chromomagnetic coupling of the $b$-quark spin to the angular momentum of light degrees of freedom. The $B$ meson mass can be expanded in terms of the $b$ quark mass and the nonperturbative HQE parameters $\bar{\Lambda}, \lambda_{1}, \lambda_{2}, m_{B}=m_{b}+\bar{\Lambda}-\left(\lambda_{1}+3 \lambda_{2}\right) /\left(2 m_{b}\right)$. Similarly, hadronic mass moments and lepton energy moment can be expressed in terms of HQE parameters. In next order $\left(1 / m_{B}^{3}\right)$, however, six additional HQE parameters ( $\rho_{1}, \rho_{2}, T_{1}, T_{2}, T_{3}, T_{4}$ ) appear. Measuring the hadronic mass moment $\left\langle m_{X}^{2}\right\rangle$ we can determine $\bar{\Lambda}$ and $\lambda_{1}$ if we specify $\lambda_{2}$ and the next-order HQE parameters.

Following a method pioneered by CLEO [6], BABAR has measured the $m_{X}$ and $m_{X}^{2}$ moment distributions in semileptonic $B \rightarrow X_{c} \ell \nu$ decays using $8.9 \times 10^{7} B \bar{B}$ pairs [7]. In the recoil of fully reconstructed $B$ mesons in different hadronic final states, we select events that have exactly one lepton with momentum $\mathrm{p}^{*}>0.9 \mathrm{GeV}$ in the $B$ rest frame. The missing fourmomentum in the event serves as an estimator for the $\nu$ four-momentum $\left(p_{\text {miss }}=p_{\nu}\right)$ in a two-constraint kinematic fit, where we require equal masses
for the reconstructed and recoil $B$ mesons and zero missing-mass squared $\left(m_{\text {miss }}^{2}=0\right)$. For $m_{E S}>5.27 \mathrm{GeV} 7114$ candidates are retained above a background of 2102 events. The resulting $m_{X}^{2}$ distribution (for $\mathrm{p}^{*}>$ 0.9 GeV ) corrected for combinatorial backgrounds is shown in Figure 1a.


Fig. 1. a) The measured $M_{X}^{2}$ distribution for $\mathrm{p}_{\min }^{*}=0.9 \mathrm{GeV}$ after subtracting combinatorial backgrounds. The histogram shows the dominant $D$ and $D^{*}$ contributions and the shaded area indicates residual background. b) Constraints on HEQ parameters from the measured semileptonic decay rate and the $m_{X}^{2}$ distribution projected into the $V_{c b}-m_{b}$ plane. For comparison, combined measurements for hadronic mass moments from BABAR , CLEO and DELPHI and lepton energy moments from CLEO and DELPHI are also shown.

To extract $\bar{\Lambda}$ and $\lambda_{1}$ we perform a $\chi^{2}$ minimization of the $\left\langle m_{X}^{2}\right\rangle$ moment distributions for seven different lepton threshold momenta, where we fixed $\lambda_{2}=0.128 \pm 0.01 \mathrm{GeV}^{2}, T_{i}=0 \mathrm{GeV}^{3}, \rho_{1}=1 / 2(0.5)^{3} \mathrm{GeV}^{3}$ and expressed $\rho_{2}$ in terms of $T_{2}, T_{4}$, and the $D^{*}-D$ and $B^{*}-B$ mass splittings [8]. Using the $\overline{M S}$ scheme [5] we extract $\overline{\Lambda^{M S}}=\left(0.53 \pm 0.09_{\text {exp }}\right) \mathrm{GeV}$ and $\lambda_{1}^{\overline{M S}}=\left(0.36 \pm 0.09_{\text {exp }}\right) \mathrm{GeV}^{2}$. A combined fit to the hadronic mass moments and the semileptonic width $\Gamma_{s l}=(4.37 \pm 0.18) \times 10^{-11} \mathrm{MeV}$ in the $\Upsilon(1 S)$ mass scheme [9] yields $m_{b}(1 S)=4.638 \pm 0.094_{\text {exp }} \pm 0.090_{\text {th }} \mathrm{GeV}, \lambda_{1}=$ $\left(0.26 \pm 0.06_{\text {exp }} \pm 0.06_{t h}\right) \mathrm{GeV}^{2}$, and $\left|V_{c b}\right|=\left(42.10 \pm 1.04_{\text {exp }} \pm 0.72_{t h}\right) \times 10^{-3}$. While the first error results from all experimental uncertainties, the second error denotes uncertainties from perturbative effects [10], dimensional analysis and $1 / m_{b}^{3}$ corrections summed in quadrature. The $V_{c b}-m_{b}$ contour is
plotted in Figure 1b showing good agreement with results of other experiments [6] [12] [11]. The world average for inclusive measurements yields $\left|V_{c b}\right|=\left(42.0 \pm 0.5_{\text {exp }} \pm 0.9_{t h}\right) \times 10^{-3}$ [13].

In exclusive $B \rightarrow D^{*} \ell \nu$ decays, $\left|V_{c b}\right|$ is extracted from the phase-space corrected decay rate at zero recoil. Heavy quark effective theory (HQET) [14] predicts the value at zero recoil to be $\left|V_{c b}\right| \mathcal{F}_{D^{*}}(1)[15]$, where $\mathcal{F}_{D^{*}}(1)=$ $0.91 \pm 0.04$ is the universal form factor for finite $b$-quark mass at zero recoil [16] [17]. BABAR measures $\left|V_{c b}\right| \mathcal{F}_{D^{*}}(1)=(34.1 \pm 1.3) \times 10^{-3}$ yielding $\left|V_{c b}\right|=$ $\left(37.5 \pm 1.4_{\text {exp }} \pm 1.6_{\text {th }}\right) \times 10^{-3}$ [18], which is consistent with the world average for exclusive measurements of $\left|V_{c b}\right|=\left(40.2 \pm 0.8_{\text {exp }} \pm 1.8_{t h}\right) \times 10^{-3}$ [13]. Within errors exclusive and inclusive $\left|V_{c b}\right|$ measurements are consistent.

## 3. Measurement of $\left|V_{u b}\right|$

The CKM matrix element $\left|V_{u b}\right|$ is measured in charmless semileptonic $B$ decays. In HQE [19], $V_{u b}$ is related to the branching fraction by:
$\left|V_{u b}\right|=0.00445 \cdot\left(\frac{\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)}{0.002} \cdot \frac{1.55 \mathrm{ps}}{\tau_{b}}\right)^{1 / 2} \times\left(1.0 \pm 0.02_{\text {pert }} \pm 0.052_{1 / m_{b}^{3}}\right)$.
Since the branching fraction $\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)$ is only $2 \%$ of $\mathcal{B}\left(B \rightarrow X_{c} \ell \nu\right)$ [20], we need to look at kinematic regions that are depleted in the $B \rightarrow X_{c} \ell \nu$ background. Besides the lepton endpoint spectrum [21] and $B \rightarrow \rho \ell \nu$ [22], $B A B A R$ has explored the hadronic mass $m_{X}$ below the $D$ meson [21]. Using the fully reconstructed hadronic $B$ meson sample selected from $8.9 \times 10^{7} B \bar{B}$ decays we look for events that contain an identified lepton ( $p^{*}>1.0 \mathrm{GeV}$ ) in the recoil and perform $\nu$ reconstruction. To minimize the $B \rightarrow X_{c} \ell \nu$ background, events with a $K^{ \pm}$or a $K_{S}^{0}$ are rejected. We measure the ratio of branching fractions $R_{u}=\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right) / \mathcal{B}(B \rightarrow X \ell \nu)$ to reduce experimental systematic effects. The total sample of semileptonic candidates comprises 29982 events. Figure 2a shows the observed hadronic mass spectrum. A $\chi^{2}$ fit with the $B \rightarrow X_{u} \ell \nu$ signal shape, a $B \rightarrow X_{c} \ell \nu$ background shape and other background contributions (hadrons misidentified as leptons, secondary $\tau$ decays or charm decays) yields $175 \pm 21$ signal candidates for $m_{X}<1.55 \mathrm{GeV}$. The background-subtracted spectrum is displayed in Figure 2b.

For $m_{X}<1.55 \mathrm{GeV}$ we measure $R_{u}=\left(2.06 \pm 0.25_{\text {stat }} \pm 0.26_{\text {sys }} \pm\right.$ $\left.0.36_{t h}\right) \%$, where the theory error is obtained by varying the non-perturbative parameters $\bar{\Lambda}$ and $\lambda_{1}$ within their uncertainties including a -0.8 correlation between them [24] [6]. We further measure a charmless inclusive branching fraction of $\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)=\left(2.24 \pm 0.27_{\text {stat }} \pm 0.26_{\text {sys }} \pm 0.39_{\text {th }}\right) \times 10^{-3}$, that yields $\left|V_{u b}\right|=\left(4.62 \pm 0.28_{\text {stat }} \pm 0.27_{\text {sys }} \pm 0.49_{t h}\right) \times 10^{-3}$. Our result agrees with other inclusive $V_{u b}$ measurements as shown in Figure 3 [13]. My weighted


Fig. 2. The $m_{X}$ distribution for $\bar{B} \rightarrow X \ell^{-} \bar{\nu}$ candidates; a) data (points) and fit components and b) data after subtraction of $b \rightarrow c \ell \nu$ and the other backgrounds.
average of inclusive measurements is $\left|V_{u b}\right|=\left(4.20 \pm 0.12_{\text {exp }} \pm 0.60_{t h}\right) \times 10^{-3}$, where the theoretical uncertainty is obtained by taking the difference of weighted means with the theoretical uncertainty added linearly and the weighted $V_{u b}$ average, $\Delta_{V_{u b}}^{t h}=\left\langle V_{u b}+\Delta_{V_{u b}}\right\rangle-\left\langle V_{u b}\right\rangle$.

## 4. Measurement of $\left|V_{t d}\right|$

The CKM matrix element $V_{t d}$ is obtained from the $B_{d}^{0} \bar{B}_{d}^{0}$ oscillation frequency $\Delta m_{B_{d}}$, which has been measured in several experiments with different methods [13]. BABAR has measured $\Delta m_{B_{d}}$ using dileptons [25], a $B \rightarrow D^{*} \ell \nu$ sample [26] and a fully reconstructed $B$-meson sample [27]. For the same luminosity, the dilepton sample yields the most precise result for the oscillation frequency $\Delta m_{B_{d}}$, which is extracted from the timedependent asymmetry of opposite-sign and same-sign dileptons originating from unmixed and mixed $B \bar{B}$ events, respectively:

$$
\begin{equation*}
A(\Delta t)=\frac{N_{\ell^{+} \ell^{-}}(\Delta t)-N_{\ell^{ \pm} \ell^{\mp}}(\Delta t)}{N_{\ell^{+} \ell^{-}}(\Delta t)+N_{\ell^{ \pm} \ell^{\mp}}(\Delta t)} \propto \cos \left(\Delta m_{B_{d}} \Delta t\right) . \tag{4}
\end{equation*}
$$

The time difference between the two $B$ decays, $\Delta t$, is computed from the nominal boost and the spatial separation of the two $B$ decay vertices, $\Delta z=z_{2}-z_{1}$. The decay vertices are determined from the positions of closest approach to the primary vertex in the transverse plane, which is obtained for each event from a vertex fit of the two lepton tracks and the beam spot constraint. To parameterize the measured asymmetry we have to convolve $N_{\ell \ell}$ with the time resolution function and include all backgrounds. The largest background comes from $B^{+} B^{-}$decays ( $50 \%$ ), which have no


Fig. 3. Comparison of inclusive $\left|V_{u b}\right|$ measurements (compiled by HFAG).
oscillatory term and mainly contribute to $N_{\ell^{+} \ell^{-}}$. Another large source are $b \rightarrow c \rightarrow \ell$ cascade decays, while contributions from $J / \psi$ decays, $q \bar{q}$ continuum and hadrons misidentified as leptons are found to be small.

In a sample of $2.3 \times 10^{7} B \bar{B}$ events, $B A B A R[25]$ has selected 99010 dilepton events after significantly reducing background of leptons from cascade decays with the help of a neural network. The sample has a purity of $87 \%$ and a efficiency of $9 \%$. The time-dependent asymmetry is plotted in Figure 4. The oscillation frequency is extracted from a binned maximum likelihood fit to the data sample with the requirement $|\Delta t|<12 \mathrm{ps}$, yielding $\Delta m_{B_{d}}=\left(0.493 \pm 0.012_{\text {stat }} \pm 0.009_{\text {sys }}\right) \hbar \mathrm{hs}^{-1}$. The dominant systematic errors result from the $B$ lifetimes and the time-dependence of the resolution.

Using 14,000 reconstructed $B^{0} \rightarrow D^{*-} \ell^{+} \nu$ events, BABAR [26] measured $\Delta m_{B_{d}}=\left(0.492 \pm 0.018_{\text {stat }} \pm 0.013_{\text {sys }}\right) \hbar \mathrm{hs}^{-1}$. Both results are in good agreement with the world average of $\Delta m_{B_{d}}=(0.502 \pm 0.006) ~ \hbar \mathrm{ps}^{-1}$ [13].


Fig. 4. Time-dependent asymmetry between opposite-sign and same-sign dileptons.

## 5. Measurement of $\sin 2 \beta$

Theoretically, the cleanest $C P$ violation measurements in the $B^{0} \bar{B}^{0}$ system are obtained for $C P$ eigenstates $f$ that have a single amplitude. Here, $C P$ violation is caused by the interference of the direct decay $B^{0} \rightarrow f$ with the decay after mixing $B^{0} \rightarrow \bar{B}^{0} \rightarrow f$. The relevant parameter is [28]:

$$
\begin{equation*}
\lambda_{f}=\eta_{f} \cdot \frac{q}{p} \cdot \frac{\bar{A}_{\bar{f}}}{A_{f}} \tag{5}
\end{equation*}
$$

where $\eta_{f}$ is the $C P$ eigenvalue of $f, q / p \approx e^{-2 i \beta}$ is the mixing phase and $\bar{A}_{\bar{f}} / A_{f}$ is the amplitude ratio. $C P$ violation occurs, if $\left|\lambda_{f}\right| \neq 1$ (direct $C P$ violation) or if $\operatorname{Im} \lambda \neq 0$ ( $C P$ violation in the interference between decays with and without mixing), which are both sufficient and necessary conditions for $\Gamma\left(\bar{B}^{0}(\Delta t) \rightarrow f_{C P}\right)$ and $\Gamma\left(B^{0}(\Delta t) \rightarrow f\right)$ to differ. Generally, the time-dependent $C P$ asymmetry is expressed by [28]:

$$
\begin{align*}
& a_{C P}(\Delta t)=\frac{\Gamma\left(\bar{B}^{0}(\Delta t) \rightarrow f\right)-\Gamma\left(B^{0}(\Delta t) \rightarrow f\right)}{\Gamma\left(\bar{B}^{0}(\Delta t) \rightarrow f\right)+\Gamma\left(B^{0}(\Delta t) \rightarrow f\right)} \\
& =-C_{f} \cdot \cos \left(\Delta m_{B_{d}} \Delta t\right)+S_{f} \cdot \sin \left(\Delta m_{B_{d}} \Delta t\right) \tag{6}
\end{align*}
$$

where $C_{f}=\left(1-\left|\lambda_{f}\right|^{2}\right) /\left(1+\left|\lambda_{f}\right|^{2}\right)$ and $S_{f}=2 \mathcal{I} m \lambda_{f} /\left(1+\left|\lambda_{f}\right|^{2}\right)$. In case of a single amplitude with $\left|\lambda_{f}\right|=1$ the first term vanishes and the time dependence is solely governed by the sine term. It is important to measure the time dependence of the $C P$ asymmetry, since the time-integrated $C P$ asymmetry vanishes. The experimental separation between $\Gamma\left(\bar{B}^{0}(\Delta t) \rightarrow f\right)$ and $\Gamma\left(B^{0}(\Delta t) \rightarrow f\right)$ depends upon, how well the $b$ flavor can be tagged at production and upon the shape of the time resolution function. For
example, the mistag rate $w$ and the resolution function $\mathcal{R}(\Delta t)$ can be measured independently in $B \bar{B}$ mixing analyses [25] [26]. Experimentally, a time-dependent $C P$-violation measurement basically involves three steps: i) kinematic reconstruction of one $B$ in a $C P$ eigenstate, ii) flavor tagging of the other $B$ at production and iii) measurement of the time difference $\Delta t$ between the two decay vertices [29]. Since the $B$ mesons are boosted in $\operatorname{BABAR}(\beta \gamma=0.55)$, we need to determine the decay vertex separation along the boost direction $(z)$.

The golden mode for measuring $\sin 2 \beta$ is $B^{0} \rightarrow J / \psi K_{S}^{0}$. Since the tree amplitude dominates, we encounter a single weak phase ( $\beta$ ) expecting negligible direct $C P$ violation $\left(\left|\lambda_{\psi_{K_{S}}}\right|=1+\mathcal{O}\left(10^{-3}\right)[28]\right.$ and the $C P$ asymmetry is simply given by $\sin 2 \beta \sin \left(\Delta m_{B_{d}} t\right)$. Experimentally, $B \rightarrow J / \psi K_{S}^{0}$ has a large branching fraction and can be cleanly reconstructed with high efficiency. Other charmonium $K_{S}^{0}\left(K_{L}^{0}\right)$ eigenstates are as well rather suited.

In a sample of $8.8 \times 10^{7} B \bar{B}$ pairs $B A B A R[30]$ has measured the $C P$ asymmetry in charmonium $K_{S}^{0}\left(K_{L}^{0}\right)$ modes. We reconstruct the $C P$-odd eigenstates $B^{0} \rightarrow J / \psi K_{S}^{0}, B^{0} \rightarrow \psi(2 S) K_{S}^{0}, B^{0} \rightarrow \chi_{c 1} K_{S}^{0}$, and $B^{0} \rightarrow \eta_{c} K_{S}^{0}$ with $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}, K_{S}^{0} \rightarrow \pi^{0} \pi^{0}$ (only for $J / \psi$ final states), $\psi(2 S) \rightarrow e^{+} e^{-}, \mu^{+} \mu^{-}$ or $J / \psi \pi^{+} \pi^{-}, \chi_{c 1} \rightarrow \gamma J / \psi, J / \psi \rightarrow e^{+} e^{-}$or $\mu^{+} \mu^{-}$, and $\eta_{c} \rightarrow K \bar{K} \pi$. We reconstruct the $C P$-even eigenstate $J / \psi K_{L}^{0}$ [31]. After vertex requirements and flavor tagging we retract $1506 C P$-odd signal candidates with a purity of $94 \%$ and $988 C P$-even signal candidates with a purity of $55 \%$. In addition, we detect 147 events in $B^{0} \rightarrow J / \psi K^{* 0}$ with $K^{* 0} \rightarrow K_{S}^{0} \pi^{0}$ [33]. The $J / \psi K^{* 0}$ decay is a vector vector mode that has mixed symmetry requiring a transversity angle analysis [32] to separate the $C P$-even and $C P$-odd eiegenstates. The $C P$-odd component was measured to be $R_{\perp}=0.16 \pm 0.035$, yielding $\eta_{f}=0.65 \pm 0.07$ after corrections.
$B$-flavor tagging at production utilizes correlations of the charge of a primary lepton or a kaon and the $b$ quark flavor. For example, a primary that is $e^{+}\left(\mu^{+}\right)$produced in $B^{0} \rightarrow D^{*-} \ell^{+} \nu$ or similarly a $K^{+}$that is produced in the cascade $B^{0} \rightarrow \bar{D} X, \bar{D} \rightarrow K^{+} Y$ originates from a $\bar{b}$ quark. In addition, the charge of the slow $\pi^{-}$is anticorrelated with the $\bar{b}$ quark flavor.

For each event we combine the information on number of leptons, kaons and slow pions along with tracking, particle identification, and kinematics information into a neural network. We classify each event into four mutually exclusive tagging categories: lepton, kaon I, kaon II and inclusive category [29]. The relevant parameter is the effective tagging efficiency, $Q=\epsilon(1-$ $2 w$ ), which combines the reconstruction efficiency with the mistag rate, because the error on $\sin 2 \beta$ is affected by $Q$ as $\sigma(\sin 2 \beta) \sim 1 / \sqrt{Q}$. We measure $Q=7.9 \pm 0.3$ (lepton), $Q=10.7 \pm 0.4$ (kaon I), $Q=6.7 \pm 0.4$ (kaon II) and $Q=2.7 \pm 0.3$ (inclusive), yielding total tagging efficiency of $Q=28.1 \pm 0.7$ [29]. While the lepton category has the highest purity, the
kaon categories have the highest efficiency.
For a boost of $\beta \gamma=0.55$, the average vertex separation is $\langle\Delta z\rangle=254 \mu \mathrm{~m}$ [29], while the typical resolution for measuring $\Delta z$ is $\sigma=180 \mu \mathrm{~m}$. The time distributions and $C P$ asymmetry of charmonium $K_{S}^{0}$ and $J / \psi K_{L}^{0} C P$ eigenstates are displayed in Figure 5. Performing a 34 -parameter unbinned maximum likelihood fit to the time distribution of the full $C P$ sample (2461 events) and a sample of fully reconstructed $B$ hadronic final states (25375 events), BABAR measures $\sin (2 \beta)=0.741 \pm 0.067_{\text {stat }} \pm 0.033_{\text {syst }}$ [30]. Our result is in perfect agreement with a recent Belle measurement [34].


Fig. 5. Comparison of the measured $\bar{B}^{0}$ and $B^{0}$ decay time distributions for (a) $\eta_{f}=-1$ and (c) $\eta_{f}=+1 C P$ eigenstates of charmonium $K^{0}$ eigenstates and the corresponding time-dependent $C P$ asymmetries (b,d). The solid lines represent projections of the maximum likelihood fits and the shaded regions represent backgrounds.

In the SM, $b \rightarrow c \bar{c} d$ processes, such as $B \rightarrow J / \psi \pi^{0}$ or $B \rightarrow D^{(*) \pm} D^{(*) \mp}$, the weak phase is the same as that for $B \rightarrow J / \psi K_{S}^{0}$. The measurements, however, may differ from $\sin 2 \beta$, because of additional phases from non negligible penguin amplitudes. The $C P$ asymmetry is parameterized with both the sine and cosine terms, where $S_{f}=\sin 2 \beta_{e f f}$. We have measured $C P$ asymmetries in $B \rightarrow J / \psi \pi^{0}[35], B \rightarrow D^{* \pm} D^{\mp}[36]$, and $B \rightarrow D^{* \pm} D^{* \mp}$ [37]. The latter mode requires a transversity angle analysis to separate
$C P$-even and $C P$-odd eigenstates. The results are summarized in Table 1. Presently, the errors are too large to draw definite conslusions on direct $C P$ violation. In $B \rightarrow D^{*+} D^{*-}, S_{D^{*} D^{*}}$ is $2.5 \sigma$ below the SM expectation.

In the $\mathrm{SM}, C P$ eigenstates that originate from $b \rightarrow s \bar{s} s$ processes also measure $\sin 2 \beta$. Theoretically, $B \rightarrow \phi K_{S}^{0}$ is the cleanest mode, since both the leading and sub-dominant mode are penguin loops with the $t$ quark dominating as shown in Figure 6 ( $u$ quark contribution is 0.02) [38]. Rescattering contributions are expected to be small [39]. In the SM, one expects at most a $5 \%$ deviation for $S_{\phi K_{S}^{0}}$ from $\sin 2 \beta . B \rightarrow \phi K_{S}^{0}$ is rather sensitive to new physics, as additional diagrams with new heavy particles in the penguin loop may produce deviations from the SM prediction [40]. Experimentally, we need a high luminosity to be sensitive to deviations, since the branching fraction is very small. $B A B A R$ has selected $70 \pm 9 \phi K_{S}^{0}$ signal events in a sample of $1.24 \times 10^{8} B \bar{B}$ pairs. With a selection efficiency of $7.4 \%$ this yields a branching fraction of $\mathcal{B}\left(B \rightarrow \phi K_{S}^{0}=\left(7.6_{-1.2}^{+1.3} \pm 0.5\right) \times 10^{-6}[41]\right.$.

Figure 7 shows the time dependence of the $\bar{B}^{0}$ and $B^{0}$ decay rates and the resulting $C P$ asymmetry. A maximum likelihood fit yields $S_{\phi K_{S}^{0}}=$ $0.45 \pm 0.43_{\text {stat }} \pm 0.07_{\text {sys }}$, and $C_{\phi K_{S}^{0}}=-0.38 \pm 0.37_{\text {stat }} \pm 0.12_{\text {sys }}$, which is consistent with the SM prediction. Using $1.5 \times 10^{8} B \bar{B}$ pairs Belle [42], however, measured $S_{\phi K_{S}^{0}}=-0.96 \pm 0.50$ stat ${ }_{-0.11}^{+0.09}$ sys , which deviates by $3.5 \sigma$ from the $\sin 2 \beta$ world average. Averaging both experiments yields $S_{\phi K_{S}^{0}}=-0.14 \pm 0.33$, which is $2.6 \sigma$ below the present $\sin 2 b e t a$ world average and thus is still consistent with the SM.


Fig. 6. SM penguin diagrams and rescattering diagram for $B \rightarrow \phi K_{S}^{0}$.

The $C P$ eigenstate $B \rightarrow \eta^{\prime} K_{S}^{0}$ has an order of magnitude larger branching fraction than $\mathcal{B}\left(B \rightarrow \phi K_{S}^{0}\right)$. In a sample of $8.8 \times 10^{7} B \bar{B}$ events BABAR measured $\mathcal{B}\left(B \rightarrow \eta^{\prime} K_{S}^{0}=(55.4 \pm 5.2 \pm 4.0) \times 10^{-6}\right.$ [44]. Here, however, the sub-dominant processes include a $b \rightarrow u$ tree diagram, which is estimated to be of the order of $|T / P| \approx(8 \pm 3) \%$ [43]. Table 1 summarizes all BABAR measurements of $S_{f}$ and $C_{f}$ or $\left|\lambda_{f}\right|$. Figure 8 summarizes the world average of $\sin 2 \beta_{\text {eff }}$ measurements compiled by the heavy flavor averaging group (HFAG). At the present level of precision all $S_{f}$ measurements


Fig. 7. Measured time dependence of $\bar{B}^{0}$ and $B^{0}$ decays and the $C P$ asymmetry for $B \rightarrow \phi K_{S}^{0}$. The dotted curves represent the backgound contribution.

Table 1. Summary of $B A B A R$ measurements for parameters of the $C P$ asymmetry $S_{f}$ and $C_{f}$ or $\left|\lambda_{f}\right|$ (eqn. 6) obtained in different $C P$ eigenstates originating from $b \rightarrow c \bar{c} s, b \rightarrow s \bar{s} s$, and $b \rightarrow c \bar{c} d$ processes. In addition, the order of magnitude of branching fractions is shown as well as the dominant decay amplitude in bold face.

| Mode | $\mathcal{B}\left[10^{-6}\right]$ | $\mathcal{A}$ | Yields | $S_{f}$ | $C_{f}$ or $\left(\left\|\lambda_{f}\right\|\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $J / \psi K_{S}^{0}$ | 440 | $\mathbf{T}$ | 2461 | $0.741 \pm 0.067 \pm 0.033$ | $(0.95 \pm 0.05 \pm 0.03)$ |
| $\eta^{\prime} K_{S}^{0}$ | 29 | $\mathbf{P}+T$ | $203 \pm 19$ | $0.02 \pm 0.34 \pm 0.03$ | $0.10 \pm 0.22 \pm 0.04$ |
| $\phi K_{S}^{0}$ | 4 | $\mathbf{P}$ | $70 \pm 9$ | $0.45 \pm 0.43 \pm 0.07$ | $-0.38 \pm 0.37 \pm 0.12$ |
| $J / \psi \pi^{0}$ | 20 | $\mathbf{T}+P$ | $40 \pm 7$ | $0.05 \pm 0.49 \pm 0.16$ | $0.38 \pm 0.41 \pm 0.08$ |
| $D^{*+} D^{-}$ | 900 | $\mathbf{T}+P$ | $113 \pm 13$ | $-0.82 \pm 0.75 \pm 0.14$ | $-0.47 \pm 0.40 \pm 0.12$ |
| $D^{+} D^{*-}$ | 900 | $\mathbf{T}+P$ |  | $-0.24 \pm 0.69 \pm 0.12$ | $-0.22 \pm 0.37 \pm 0.10$ |
| $D^{*+} D^{*-}$ | 1000 | $\mathbf{T}+P$ | $156 \pm 14$ | $0.05 \pm 0.29 \pm 0.10$ | $(0.75 \pm 0.19 \pm 0.02)$ |

are consistent with the SM.

## 6. Measurement of $\sin 2 \alpha$

In order to determine the angle $\alpha$, we need to measure time-dependent $C P$ asymmetries of $b \rightarrow u \bar{u} d$ processes, such as $B \rightarrow \pi^{+} \pi^{-}, B \rightarrow \rho \pi$, $B \rightarrow \rho \rho[28]$. Penguin pollution, however, generally complicates the extraction of $\alpha[45]$. In $B \rightarrow \pi^{+} \pi^{-}$we expect $P / T \sim 30 \%$. Leading-order and


Fig. 8. Compilation of $\sin 2 \beta_{\text {eff }}$ measurements by HFAG for different $C P$ eigenstates averaged over BABAR and Belle.
subdominant Feynman diagrams for $B \rightarrow \pi \pi$ are depicted in Figure 9. The $C P$ asymmetry $a_{\pi \pi}$ contains both the cosine and sine terms and $S_{\pi \pi}$ measures $\sin 2 \alpha_{e f f}$, where $2 \alpha_{e f f}=2 \alpha+\Delta \phi$. Using the Gronau-London method [45], $\Delta \phi$ can be determined by a $B \rightarrow \pi \pi$ isospin analysis [46] that requires to measure six branching fractions: $B^{0} \rightarrow \pi^{+} \pi^{-}, B^{0} \rightarrow \pi^{0} \pi^{0}, B^{+} \rightarrow \pi^{+} \pi^{0}$, $\bar{B}^{0} \rightarrow \pi^{+} \pi^{-}, \bar{B}^{0} \rightarrow \pi^{0} \pi^{0}$ and $B^{-} \rightarrow \pi^{-} \pi^{0}$. Via isospin relations the amplitudes of $B$ and $\bar{B}$ decays form two triangles as shown in Figure 10. Since the amplitudes of the charged $B$ decays are solely determined by the tree diagram, their absolute values are equal, providing a common basis of the two triangles. The angle between the two triangles defines $\Delta \phi$. Inclusion of electroweak penguin processes modifies the extraction of $\alpha$ slightly [47].

The decays $B^{0} \rightarrow \pi^{+} \pi^{-}$and $B^{+} \rightarrow \pi^{+} \pi^{0}$ have been measured by CLEO [58], BABAR [48] [51] and Belle [57]. For example, BABAR measures branching fractions of $\mathcal{B}\left(B^{0} \rightarrow \pi^{+} \pi^{-}\right)=(4.7 \pm 0.6 \pm 0.2) \times 10^{-6}$ [48] and $\mathcal{B}\left(B^{+} \rightarrow \pi^{+} \pi^{0}\right)=\left(5.5_{-0.9}^{+1.0} \pm 0.5\right) \times 10^{-6}[51]$ for the $\pi^{+} \pi^{-}$and $\pi^{+} \pi^{0}$ modes, respectively. Recently, also evidence for $B \rightarrow \pi^{0} \pi^{0}$ was found by BABAR [49] and Belle [50]. Analyzing $1.24 \times 10^{8} B \bar{B}$ pairs, BABAR selected $46 \pm 13 \pm 3 \pi^{0} \pi^{0}$ candidates (a $4.2 \sigma$ significance) yielding a branching fraction
of $\mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=(2.1 \pm 0.6 \pm 0.3) \times 10^{-6}$ that is rather large.
For a sample of $1.24 \times 10^{8} B \bar{B}$ pairs $B A B A R$ measured the time-dependent $\bar{B}^{0}$ and $B^{0}$ decay rates and the $C P$ asymmetry for $B \rightarrow \pi^{+} \pi^{-}$as shown in Figure 12. A maximum likelihood fit to the $C P$ asymmetry yields $C_{\pi \pi}=$ $-0.19 \pm 0.19 \pm 0.05$ and $S_{\pi \pi}=-0.40 \pm 0.22 \pm 0.03$ [55]. Without an isospin analysis we can presently place an upper bound on $\Delta \phi$ of $\left|\alpha_{e f f}-\alpha\right|<48^{0}$. In order to extract $\alpha$ from $\alpha_{\text {eff }}$ with a precision of $\sigma(\Delta \phi)=5^{0}$ a sample of the order of $10^{10} B \bar{B}$ events is required. [56] Our results differ from the Belle measurements [54] as shown in Figure 12. The present world average is consistent with the expected value for $\alpha$ obtained from sides of the unitarity triangle and $\sin 2 \beta$. In the near future a Dalitz plot analysis in $B \rightarrow \rho \pi$ may provide additional information on $\alpha$.

$T\left(T^{\prime}\right)$


C( $C^{\prime}$ )

$\boldsymbol{P}\left(\mathbf{P}^{\prime}\right)$

Fig. 9. Leading-order and subdominant Feynman diagrams for $B \rightarrow \pi \pi$ decays.


Fig. 10. Isospin relation for $B \rightarrow \pi \pi$ and $\bar{B} \rightarrow \pi \pi$ decays.

## 7. Measurement of $\gamma$

The ratios of branching fractions involving $B \rightarrow K \pi$ or $B \rightarrow \pi \pi$ decays bear sensitivity to the angle $\gamma$. As an example Figure 13 shows the predictions of QCD factorization [53] for $R_{k}=\mathcal{B}\left(B^{0} \rightarrow K^{ \pm} \pi^{\mp}\right) /\left(2 \cdot \mathcal{B}\left(B^{0} \rightarrow\right.\right.$


Fig. 11. $B A B A R$ measurement of the $\Delta t$ distribution for $\bar{B}^{0}$ and $B^{0}$ decays to $\pi^{+} \pi^{-}$ final state and the resulting $C P$ asymmetry. The solid curve shows the projection of the fit and the dotted curves represent background contributions.


Fig. 12. Comparison of $S_{\pi \pi}$ and $C_{\pi \pi}$ measurements from BABAR and Belle.
$\left.\left.K^{0} \pi^{0}\right)\right), R_{c}=\mathcal{B}\left(B^{0} \rightarrow \pi^{ \pm} \pi^{\mp}\right) /\left(2 \cdot \mathcal{B}\left(B^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}\right)\right) \times \tau_{B^{ \pm}} / \tau_{B^{0}}$, and $R_{n}=$ $\mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right) / \mathcal{B}\left(B^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}\right) \times \tau_{B^{ \pm}} / \tau_{B^{0}}$. Using average values for the $K \pi$ and $\pi \pi$ branching fractions listed in Table 2, we determine $R_{k}=0.79 \pm 0.1$, $R_{c}=0.48 \pm 0.08$ and $R_{n}=0.47 \pm 0.12$. While the measurement of $R_{k}$ prefers small values of $\gamma\left(<50^{0}\right)$, the measurement of $R_{c}$ prefers $\gamma$ values

Table 2. Branching fraction measurements of charmless $B \rightarrow K \pi$ and $B \rightarrow \pi \pi$ decays from BABAR [52], Belle [57], and CLEO [58]

| Mode | BABAR <br> $\mathcal{B}\left[10^{-6}\right]$ | Belle <br> $\mathcal{B}\left[10^{-6}\right]$ | CLEO <br> $\mathcal{B}\left[10^{-6}\right]$ | Average <br> $\mathcal{B}\left[10^{-6}\right]$ |
| :--- | :---: | :---: | :---: | :---: |
| $K^{ \pm} \pi^{\mp}$ | $17.9 \pm 0.9 \pm 0.7$ | $18.5 \pm 1.0 \pm 0.7$ | $18.0_{-2.1-0.9}^{+2.3+1.2}$ | $17.9 \pm 1.0$ |
| $K^{0} \pi^{0}$ | $10.4 \pm 1.5 \pm 0.8$ | $12.6 \pm 2.4 \pm 1.4$ | $12.8_{-3.3-1.4}^{+4.0+1.7}$ | $11.3 \pm 1.3$ |
| $K^{ \pm} \pi^{0}$ | $12.8_{-1.0}^{+1.2} \pm 1.0$ | $12.8 \pm 1.4_{-1.0}^{+1.4}$ | $12.9_{-2.2-1.1}^{+2.4+1.2}$ | $12.8 \pm 1.1$ |
| $K^{0} \pi^{\mp}$ | $17.5_{-1.7}^{+1.8} \pm 1.3$ | $22.0 \pm 1.9 \pm 1.1$ | $18.8_{-3.3-1.8}^{+3.7+2.1}$ | $19.7 \pm 1.5$ |
| $\pi^{ \pm} \pi^{\mp}$ | $4.7 \pm 0.6 \pm 0.2$ | $4.4 \pm 0.6 \pm 0.3$ | $4.5_{-1.2-0.4}^{+1.4+0.5}$ | $4.6 \pm 0.4$ |
| $\pi^{0} \pi^{0}$ | $2.1 \pm 0.6 \pm 0.3$ | $1.7 \pm 0.6 \pm 0.3$ | $<4.4$ | $2.0 \pm 0.5$ |
| $\pi^{ \pm} \pi^{0}$ | $5.5_{-0.9}^{+1.0} \pm 0.5$ | $5.3 \pm 1.3 \pm 0.5$ | $4.6_{-1.6-0.6}^{+1.8+0.7}$ | $5.2 \pm 0.8$ |

above $70^{\circ}$. The measurement of $R_{n}$ excludes all values of $\gamma$ at the $1.67 \sigma$ level (see Figure 13).


Fig. 13. Predictions from QCD factorization [53] for $R_{k}, R_{c}$ and $R_{n}$ in comparison to recent measurements (black boxes).

Another method for measuring $\gamma$ is based upon $D^{0} \bar{D}^{0}$ mixing between a color-allowed Cabbibo-suppressed $b \rightarrow c \bar{u} s$ transition and a color-suppressed Cabbibo-allowed $b \rightarrow u \bar{c} s$ transition that both are order $O\left(\lambda^{3}\right)$ processes. This idea for example is utilized in $B^{ \pm} \rightarrow D K^{ \pm}$transitions depicted in Figure 14 [59]. The amplitudes of the decays $B^{-} \rightarrow D^{0} K^{-}, B^{-} \rightarrow \bar{D}^{0} K^{-}$ and $B^{-} \rightarrow D_{C P}^{0} K^{-}$form a triangle, where $D_{C P}^{0}$ denotes a $C P$ eigenstate. For example, we can reconstruct $C P$-even eigenstates ( $D_{+}^{0}$ ) in decays such as $D_{+}^{0} \rightarrow \pi^{+} \pi^{-}$or $D_{+}^{0} \rightarrow K^{+} K^{-}$. Another triangle is obtained for the corresponding $B^{+}$decays. Since the color-allowed $b \rightarrow c$ transition involves no weak phase, the absolute values of the amplitudes $\left|\mathcal{A}\left(B^{-} \rightarrow D^{0} K^{-}\right)\right|$ and $\left|\mathcal{A}\left(B^{+} \rightarrow D^{0} K^{+}\right)\right|$are identical. Using the amplitude $\mathcal{A}\left(B^{-} \rightarrow D^{0} K^{-}\right)$ as a common basis for both triangles, the opening angle between them is $2 \gamma$, as shown in Figure 15.


Fig. 14. Leading-order Feynman diagrams for $B^{+} \rightarrow \bar{D}^{0} K^{+}$and $B^{+} \rightarrow D^{0} K^{+}$.


Fig. 15. Triangular relations for $B^{ \pm} \rightarrow D K^{ \pm}$amplitudes.

To determine $\gamma$ we measure the following ratio of branching fractions

$$
\begin{equation*}
R_{C P}=\frac{\mathcal{B}\left(B^{-} \rightarrow D_{C P}^{0} K^{-}\right)+\mathcal{B}\left(B^{+} \rightarrow D_{C P}^{0} K^{+}\right)}{\mathcal{B}\left(B^{-} \rightarrow D^{0} K^{-}\right)+\mathcal{B}\left(B^{+} \rightarrow D^{0} K^{+}\right)}=1+r_{D K}^{2} \pm 2 r_{D K} \cos \delta \cos \gamma, \tag{7}
\end{equation*}
$$

and the $C P$ asymmetry

$$
\begin{equation*}
A_{C P}=\frac{\mathcal{B}\left(B^{-} \rightarrow D_{C P}^{0} K^{-}\right)-\mathcal{B}\left(B^{+} \rightarrow D_{C P}^{0} K^{+}\right)}{\mathcal{B}\left(B^{-} \rightarrow D_{C P}^{0} K^{-}\right)+\mathcal{B}\left(B^{+} \rightarrow D_{C P}^{0} K^{+}\right)}=\frac{2 r_{D K} \sin \delta \sin \gamma}{R_{C P}} \tag{8}
\end{equation*}
$$

where we denote the strong phase by $\delta$ and the ratio of color-suppressed amplitudes by $r_{D K}=\left|\mathcal{A}\left(B^{-} \rightarrow \bar{D}^{0} K^{-}\right)\right| /\left|\mathcal{A}\left(B^{+} \rightarrow D^{0} K^{+}\right)\right|$. In addition to $\gamma$ both $\delta$ and $r_{D K}(\simeq 0.1-0.3)$ are also unknown. BABAR has measured these quantities for the $C P$-even $D^{0}$ state, yielding $R_{+}=0.89 \pm 0.21 \pm 0.08$ and $A_{+}=0.17 \pm 0.23_{-0.07}^{+0.09}[55]$. The ratio of Cabbibo-suppressed to Cabbiboallowed decays amount to $\mathcal{B}\left(B^{ \pm} \rightarrow D_{C P}^{0} K^{ \pm}\right) / \mathcal{B}\left(B^{ \pm} \rightarrow D_{C P}^{0} \pi^{ \pm}\right)=(7.4 \pm$ $1.7 \pm 0.6) \%$ and $\left[\mathcal{B}\left(B^{+} \rightarrow \bar{D}^{0} K^{+}\right)+\mathcal{B}\left(B^{-} \rightarrow D^{0} K^{-}\right)\right] /\left[\mathcal{B}\left(B^{+} \rightarrow \bar{D}^{0} \pi^{+}\right)+\right.$ $\left.\mathcal{B}\left(B^{-} \rightarrow D^{0} \pi^{-}\right)\right]=(8.31 \pm 0.35 \pm 0.2) \%$. In order to determine the three
unknown parameters $\gamma, \delta$ and $r_{D K}, R_{C P}$ and $A_{C P}$ also need to be measured for $C P$-odd $D^{0}$, e.g. $K_{S}^{0} \pi^{0}$ or $K_{S}^{0} \phi$ states. Assuming $r_{D K}=0.3$ the errors are presently too large to constrain $\gamma$. For a luminosity of $\mathcal{L}=0.5$ (2.0) $\mathrm{ab}^{-1}$ we expect a precision of $\sigma\left(\sin ^{2} \gamma\right)=0.32(0.2)$ for $r_{D K}=0.3$ [63].

The angle $\gamma$ can be also extracted by measuring the time-dependent decays rates of $\bar{B}^{0} \rightarrow D^{(*)+} \pi^{-}[61]$. Here, the color-allowed process $\left(O\left(\lambda^{2}\right)\right)$ interferes with the color-suppressed process $\left(O\left(\lambda^{4}\right)\right)$ after $B \bar{B}$ mixing. The $B^{0}$ and $\bar{B}^{0}$ time-dependent decays rates are given by:
$\Gamma\left(B^{0} \rightarrow D^{\mp} \pi^{ \pm}, \Delta t\right)=N e^{-|\Delta t| / \tau_{B^{0}}}\left[1 \pm C \cos \left(\Delta m_{B_{d}^{0}} \Delta t\right)+S^{\mp} \sin \left(\Delta m_{B_{d}^{0}} \Delta t\right)\right]$
$\Gamma\left(\bar{B}^{0} \rightarrow D^{\mp} \pi^{ \pm}, \Delta t\right)=N e^{-|\Delta t| / \tau_{B^{0}}}\left[1 \mp C \cos \left(\Delta m_{B_{d}^{0}} \Delta t\right)-S^{\mp} \sin \left(\Delta m_{B_{d}^{0}} \Delta t\right)\right]$,
where $C=\left(1-r_{D \pi}^{2}\right) /\left(1+r_{D \pi}^{2}\right), S^{\mp}=\left(2 r_{D \pi}\right) /\left(1+r_{D \pi}^{2}\right) \cdot \sin (2 \beta+\gamma \pm \delta)$ and $r_{D \pi}=\mathcal{A}\left(\bar{B}^{0} \rightarrow D^{-} \pi^{+}\right) / \mathcal{A}\left(B^{0} \rightarrow D^{-} \pi^{+}\right) \approx 0.02$. Using $8.8 \times 10^{7} B^{0} \bar{B}^{0}$ decays, $B A B A R$ has studied both $B^{0} \rightarrow D^{\mp} \pi^{ \pm}$and $B^{0} \rightarrow D^{* \mp} \pi^{ \pm}$decays. Using a maximum likelihood fit to the $\Delta t$ distributions of $\bar{B}^{0}$ and $B^{0}$ decays to $D^{(*) \mp} \pi^{ \pm}$final states, $B A B A R$ measures the following combinations [60]

$$
\begin{gather*}
2 r_{D^{*} \pi} \sin (2 \beta+\gamma) \cos \delta_{D^{*} \pi}=-0.068 \pm 0.038 \pm 0.021 \\
2 r_{D^{*} \pi} \sin (2 \beta+\gamma) \sin \delta_{D^{*} \pi}=-0.031 \pm 0.070 \pm 0.035 \\
2 r_{D \pi} \sin (2 \beta+\gamma) \cos \delta_{D \pi}=-0.022 \pm 0.038 \pm 0.021 \\
2 r_{D \pi} \sin (2 \beta+\gamma) \sin \delta_{D \pi}=-0.025 \pm 0.068 \pm 0.035 \tag{10}
\end{gather*}
$$

From these results we determine $\sin (2 \beta+\gamma)>0.69 @ 68.3 \%$ confidence level (C.L.).

## 8. Present Status of the Unitarity Triangle

Figure 16 shows the present status of the unitarity triangle, where we have included world average measurements of $\mathcal{B}\left(B \rightarrow X_{c} \ell \nu\right)$, $\mathcal{B}\left(B \rightarrow D^{*} \ell \nu\right)$, $\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right), \mathcal{B}\left(B^{0} \rightarrow \rho^{\mp} \ell^{ \pm} \nu\right), \Delta m_{B_{d}}, \epsilon_{K}$ and $\sin 2 \beta=0.736 \pm 0.049$ measured in charmonium $K_{S}^{0}\left(K_{L}^{0}\right) C P$ eigenstates [20] [64]. We further include the information of $B_{s}^{0} \bar{B}_{s}^{0}$ mixing, where we use a new parameterization based on the significance of observing $\Delta m_{B_{s}}$ in the present LEP and SLD amplitude measurements [13]. The method is discussed in detail in reference [64]. A $\chi^{2}$ minimization is used to determine the parameters $\bar{\rho}, \bar{\eta}$, $A$ of the CKM matrix. The prediction of observables in terms of Wolfenstein parameters involves theoretical parameters, such as reduced rates for the branching fractions, $f_{B} \sqrt{B_{B}}, B_{K}$ and $\xi[28]$, which are affected by
non-gaussian uncertainties. We include terms that take care of correlations among the observables and the gaussian errors in the theoretical parameters. The explicit values of the observables and theoretical parameters are listed in reference [64]. We perform a particular $\chi^{2}$ fit with fixing the theoretical parameters to specific values within their allowed, which we call a "model". We perform $\chi^{2}$ fits for many different models scanning over the entire non-gaussian range for each of the theoretical parameters. We accept only models that have a fit probability of $\left.P\left(\chi^{2}\right)>5 \%\right)$. The light-colored ellipse in Figure 16 represents a $95 \%$ C.L. contour of a typical fit. The black region shows the most-probable (central) value for $\bar{\rho}, \bar{\eta}$ for each model accepted, while the dark-shaded region represents the overlay of the $95 \%$ C.L. contours of all accepted models. In order to guide the eye we also show individual $95 \%$ C.L. bounds for $\left|V_{u b} / V_{c b}\right|, \Delta m_{B_{d}}, \Delta m_{B_{s}}, \epsilon_{K}$, and $\sin 2 \beta$.


Fig. 16. Present status of the unitarity triangle. The black points represent a $\bar{\rho}, \bar{\eta}$ central value obtained from an individual $\chi^{2}$ fit, the dark-shaded region shows an overlay of the corresponding $95 \%$ C.L. contours, while the light ellipse indicates a typical shape of a $95 \%$ C.L. contour. Also shown are the $95 \%$ bounds of the observables.

This procedure allows us to place non-gaussian ranges and experimental errors on the Wolfenstein parameters and angles of the unitarity triangle, yielding $0.103_{-0.067} \leq \bar{\rho} \leq 0.337^{+0.026}, 0^{0.280_{-0.020} \leq \bar{\eta} \leq 0.409^{+0.034}}$, $0.80_{-0.024} \leq A \leq 0.85^{+0.027},\left(83.1_{16.6}\right)^{0} \leq \alpha \leq\left(130.0^{+5.4}\right)^{0}$, and $\left(40.4_{-3.2}\right)^{0} \leq$ $\gamma \leq\left(74.5^{+8.3}\right)^{0}$. Additional measurements of $\sin 2 \alpha$ and $\gamma$ in the future will further overconstrain the unitarity triangle. Adding information of $C P$ asymmetries of $B \rightarrow \phi K_{S}^{0}$ will provides a crucial test of the SM, once the
precision on $S_{\phi K_{S}^{0}}$ is improved. The ultimate test of the SM, however, will be to check if $\alpha+\beta+\gamma=\pi$. For this test $30-50 \mathrm{ab}^{-1}$ will be needed.

## 9. Conclusion

About three years after the start of the $B$ factories, $\sin 2 \beta$ measured in charmonium $K_{S}^{0}\left(K_{L}^{0}\right)$ eigenstates is becoming a precision measurement. $\sin 2 \beta_{e f f}$ measured in $B \rightarrow D^{*+} D^{*-}$ decays may indicate the presence of penguin processes. The present reults of $\sin 2 \beta$ measured in $B \rightarrow \phi K_{S}^{0}$ look interesting, but the errors are too large to draw conclusions concerning new physics phases. The determination of $\sin 2 \alpha$ is complicated due to the presence of penguin amplitudes. In order to separate $\sin 2 \alpha$ from $\sin 2 \alpha_{e f f}$ measured in $C P$ asymmetry $a_{\pi^{+} \pi^{-}}$an integrated luminosity of tens of ab ${ }^{-1}$ is needed. The determination of $\gamma$ in $B^{ \pm} \rightarrow D_{C P}^{0} K^{ \pm}$final states or from time-dependent $C P$ asymmetries of $B^{0} \rightarrow D^{(*)}-\pi^{+}$modes is even more difficult. The $B \rightarrow K \pi$ and $B \rightarrow \pi \pi$ branching fractions may already indicate a problem with QCD factorization in the BBNS model [53] and require further investigation with increased luminosities.

Inclusive analysis methods for $B \rightarrow X_{u} \ell \nu$ and $B \rightarrow X_{c} \ell \nu$ allow the extraction of $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$ with a reduced model dependence. The extraction of $\left|V_{t d}\right|$ will depend on improvements of the non-gaussian uncertainties in $f_{B_{d}} \sqrt{B_{B_{d}}}$. In the future also the ratio of branching fractions $\mathcal{B}(B \rightarrow \rho \gamma) / \mathcal{B}\left(B \rightarrow K^{*} \gamma\right)$ may provide additional constraints on $\left|V_{t d}\right| /\left|V_{t s}\right|$ [65]. Presently, the precision of the $\bar{\rho}-\bar{\eta}$ plane is determined by theoretical uncertainties. In order to reduce the allowed region in the $\bar{\rho}-\bar{\eta}$ plane, we need both precision measurements and reduced non-gaussian theoretical uncertainties. In the future measurements of $\sin 2 \alpha$ and $\gamma$ will provides further constraints on the unitarity triangle. The ultimate test, however, will consist of checking the relation $\alpha+\beta+\gamma=\pi$. With increasing luminosity the uncertainties of $C P$ asymmetries will decrease. While for charmonium $K_{S}^{0}\left(K_{L}^{0}\right) C P$ eigenstates the uncertainty at $\mathcal{L}=50 \mathrm{ab}^{-1}$ is expected to level off around $\sigma(\sin 2 \beta) \sim 1.5 \%$ due to dominating systematic errors, we expect to reach a precision for $B \rightarrow \phi K_{S}^{0}$ of $\sigma(\sin 2 \beta)<3 \%$ (for $\sin 2 \beta=0.736$ ) [66]. The precision for $a_{\pi \pi}$ is expected to be $\sigma(\alpha) \sim f e w^{0}$, while that of the $C P$ violating asymmetry in $B \rightarrow D^{(*)} \pi$ will be of the order of $3 \%$ [56].

## 10. Acknowledgment

I would like to thank the organizers of the conference for their hospitality and the wonderful conference arrangements. The BABAR collaboration I would like to thank for their support in arranging this talk. In particluar I would like to thank Bob Cahn, David Hitlin, Gautier Hamel de Monchenault
and Antimo Palano for fruitful discussions. This work has been supported by the Norwegian Research Council.

## REFERENCES

[1] N. Cabbibo, Phys. Rev. Lett. 10, 531 (1963).
[2] M. Kobayashi, T. Maskawa, Prog.Theor.Phys. 49652 (1973).
[3] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
[4] BABAR Collaboration (B. Aubert et al.), Nucl.Instrum.Meth.A479, 1 (2002).
[5] A. F. Falk and M. E. Luke, Phys. Rev. D57, 424 (1998).
[6] CLEO Collaboration (D. Cronin-Hennessy et al.), Phys. Rev. Lett. 87, 251808 (2001).
[7] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-10067, hep-ex/0307046 (2003).
[8] M. Gremm and A. Kapustin, Phys. Rev. D55, 6924 (1997).
[9] C. W. Bauer, Z. Ligeti, M. Luke, and A. V. Manohar, Phys. Rev. D67, 054012 (2003).
[10] S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, Phys. Rev. D28, 228 (1983).
[11] DELPHI Collaboration (M. Calvi et al.), hep-ex/0210046 (2002).
[12] CLEO Collaboration (R. A. Briere et al.), hep-ex/0209024 (2002).
[13] http://www.slac.stanford.edu/xorg/hfag/.
[14] N. Isgur and M. Wise, Phys. Lett. 237, 527 (1990); A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. B 343, 1 (1990); M. Neubert, Phys. Lett. B 264, 455 (1991); M. Neubert, Phys. Lett. B 338, 84 (1994).
[15] A. F. Falk, M. Neubert, Phys. Rev. D 47, 2965 and 2982 (1993); T. Mannel, Phys. Rev. D 50, 428 (1994); M. A. Shifman, N. G. Uraltsev and M. B. Voloshin, Phys. Rev. D 51, 2217 (1995).
[16] A. Czarnecki, Phys. Rev. Lett. 76, 4126 (1996).
[17] S. Hashimoto et al., Phys. Rev. D 66, 014503 (2002).
[18] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-10105, 22pp, hepex/0308027 (2003).
[19] V. D. Barger, C. S. Kim, and R. J. Phillips, Phys. Lett. B 251, 629 (1990); A. F. Falk, Z. Ligeti, and M. B. Wise, Phys. Lett. B 406, 225 (1997); R. D. Dikeman and N. G. Uraltsev, Nucl. Phys. B 509, 378 (1998); I. I. Bigi,R. D. Dikeman, and N. Uraltsev, Eur. Phys. J. C 4, 453 (1998).
[20] Particle Data Group, K. Hagiwara et al.,Phys.Rev. D66, 010001 (2002).
[21] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-9282, 17pp, hepex/0207081 (2002).
[22] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 90, 181801 (2003).
[23] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-10045, 7pp, hepex/0307062 (2003).
[24] F. De Fazio and M. Neubert, JHEP 9906, 017 (1999).
[25] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 88, 231801 (2003).
[26] BABAR Collaboration (B. Aubert et al.), Phys.Rev.D67, 072002 (2003).
[27] BABAR collaboration (B. Aubert et al.), Phys.Rev.Lett. 88, 221802 (2002).
[28] $B A B A R$ collaboration (P.F. Harrison and H.R. Quinn, eds.) SLAC-R-0504 (1998).
[29] BABAR Collaboration (B. Aubert et al.), Phys.Rev.D66, 032003 (2002).
[30] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 89, 201802 (2002).
[31] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-9170, hep-ex/0203040 (2002).
[32] I. Dunietz et al., Phys. Rev. D 43, 2193 (1991), A.S. Dighe et al., Phys. Lett. B 369, 144 (1996).
[33] BABAR Collaboration (B. Aubert et al.), Phys. Rev. Lett. 87, 241801 (2001).
[34] Belle Collaboration (K. Abe et al.), Belle-conf-0353 9 pp, hep-ex/0308037 (2003).
[35] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 91, 061802 (2003).
[36] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 90, 221801 (2003).
[37] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 91131801 (2003).
[38] N.G. Deshpande and J. Trampetic, Phys. Rev. D 41, 895 (1990); N.G. Deshpande and X.-G. He, Phys. Lett. B 336, 471 (1994); R. Fleischer, Z. Phys. C 62, 81 (1994).
[39] Y. Grossman, G. Isidori and M. P. Worah, Phys. Rev. D 58, 057504 (1998).
[40] Y. Grossman and M.P. Worah, Phys. Lett. B 395, 241 (1997); R. Fleischer, Int. J. Mod.Phys. A12, 2459 (1997).
[41] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-9684, hep-ex/0303029 (2003).
[42] Belle Collaboration (K. Abe et al.), KEK-PREPRINT-2003-47, 10 pp , hepex/0308035 (2003).
[43] M. Beneke and M. Neubert, Nucl.Phys B651, 225 (2003).
[44] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 91161801 (2003).
[45] M. Gronau and D. London, Phys.Rev.Lett. 653381 (1990).
[46] M. Gronau, Phys.Lett. B 265, 389 (1991); L. Lavoura Mod.Phys.Lett. A7, 1553 (1992).
[47] M. Gronau et al., Phys.Rev. D 52, 6374 (1995).
[48] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 89281802 (2002).
[49] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-9303, hep-ex/0308012 (2003).
[50] Belle Collaboration (K. Abe et al.), BELLE-CONF-0354, 9pp (2003).
[51] BABAR Collaboration (B. Aubert et al.), Phys.Rev.Lett. 91021801 (2003).
[52] H. Jawahery, invited talk at $21^{s} t$ int. conf. on Lepton and Photon Int. at HE, Batavia 11-16 Aug (2003).
[53] M. Beneke et al., Nuc.Phys. B 606, 245 (2001).
[54] By Belle Collaboration (K. Abe et al.), Phys.Rev.Lett.89, 071801 (2002); Phys.Rev.D68, 012001 (2003).
[55] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-9304, hep-ex/0207065 (2002).
[56] P. Burchat et al., snowmass summer study, eConf C010630, E214 (2001).
[57] BELLE Collaboration (K. Abe et al.), Phys.Rev.D66 092002, (2002); Belle-conf-0311, LP03-ID 264, 13 pp (2003).
[58] CLEO Collaboration (A. Bornheim et al.), Phys. Rev.D 68, 052002 (2003).
[59] M. Gronau and D, Wyler, Phys.Lett. B 265, 172 (1991).
[60] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-10220, hep-ex/0311032 (2003).
[61] I. Dunietz, Phys. Lett. B 427, 179 (1998); I. Dunietz, R.G. Sachs, Phys. Rev. D 37, 3186 (1988).
[62] BABAR Collaboration (B. Aubert et al.), SLAC-PUB-10155, hep-ex/0309017 (2003).
[63] D.G. Hitlin, invited talk at $B$ physics day at SLAC, March 20 (2003).
[64] G.P. Dubois-Felsmann, D.G. Hitlin, F. C. Porter and G. Eigen, hepph/0308262 (2003); G. Eigen et al., Proc. of Int. Europhys. conf. in Aachen, July (2003).
[65] A. Ali and E. Lunghi, Eur.Phys.J.C26, 195 (2002).
[66] D. MacFarlane, invited talk at SLAC colloquium, May 29 (2003).

