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Using untagged $B^0 \to DK_S$ to determine γ

M. Gronau,[1] Y. Grossman,[1,2,3] N. Shuhmaher,[1] A. Soffer,[4] and J. Zupan[1,5]

[1] *Department of Physics, Technion–Israel Institute of Technology, Technion City, 32000 Haifa, Israel*

[2] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309*

[3] *Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, CA 95064*

[4] *Department of Physics, Colorado State University, Fort Collins, CO 80523*

[5] *J. Stefan Institute, Jamova 39, P.O. Box 3000,1001 Ljubljana, Slovenia*

Abstract

It is shown that the weak phase $\gamma \equiv \arg(-V_{ud}V_{ub}^* V_{cb}V_{cd}^*)$ can be determined using only untagged decays $B^0/\bar{B}^0 \to DK_S$. In order to reduce the uncertainty in γ, we suggest combining information from $B^\pm \to DK^\pm$ and from untagged B^0 decays, where the D meson is observed in common decay modes. Theoretical assumptions, which may further reduce the statistical error, are also discussed.

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I. INTRODUCTION

CP violation measured in $B \to J/\psi K_S$ [1] is interpreted in terms of the phase $\beta \equiv \arg(-V_{tb}V_{td}^*V_{cd}V_{cb}^*)$ in a way which is practically free of theoretical uncertainties, providing an important test of the Kobayashi-Maskawa mechanism [2]. On the other hand, the current interpretation of CP asymmetry measurements in $B \to \pi^+\pi^-$ [3] in terms of the phase $\alpha \equiv \arg(-V_{td}V_{tb}^*V_{ub}V_{ud}^*)$ involves an uncertainty in the ratio of penguin-to-tree amplitudes [4]. A theoretically clean method [5, 6] for measuring the phase $\gamma \equiv \arg(-V_{ud}V_{ub}^*V_{cb}V_{cd}^*)$ involves interference between tree amplitudes $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$, governing $B \to \bar{D}^0 X_s$ and $B \to D^0 X_s$, where the $\bar{D}^0$ and D^0 decay to a common hadronic state, and $X_s = K, K^*, K\pi, \ldots$ is a strangeness one state.

Originally, this idea for measuring γ was proposed for charged B decays [5] and for time-dependent neutral B decays [6] of the type $B \to D_{\rm CP}K$. Here positive (negative) CP eigenstates, such as K^+K^- ($K_S\pi^0$), identify equal admixtures of D^0 and $\bar{D}^0$ states with equal (opposite) signs. A variety of other D decay final states can be used as well, leading to several variants of the original method [7, 8]. Every hadronic state accessible at tree level to D^0 decay is also accessible at tree level to $\bar{D}^0$ decay with varying levels of Cabibbo suppression. Flavor states, $K^-\pi^+$ and $K^{*-}\pi^+$, which are Cabibbo-favored in D^0 decays, are doubly Cabibbo-suppressed in $\bar{D}^0$ decays. Flavorless states, such as $K^{*+}K^-$ and K^+K^{*-}, are produced in singly Cabibbo-suppressed decays of both D^0 and $\bar{D}^0$. Recently, it was shown that a model-independent extraction of γ is also possible by considering $B^\pm \to DK^\pm$ with subsequent multibody D decay, such as $D \to K_S\pi^+\pi^-$ [9, 10]. Other variants make use of the decays $B^\pm \to D^*K^\pm, D^* \to D\pi^0$, the self-tagged decay mode $B^0 \to DK^{*0}, K^{*0} \to K^+\pi^-$ [11], multibody B decays of the type $B \to DK\pi$ [12] and combinations of these processes [13].

First results for $B^\pm \to D_{\rm CP}K^\pm$ were presented recently by the Belle [14] and BaBar [15] collaborations. These studies were based on several tens of events in each experiment and demonstrate the potential of a larger data sample in providing useful constraints on γ [16]. The Belle collaboration has also presented a preliminary analysis of about one hundred events of the type $B^\pm \to DK^\pm, D \to K_S\pi^+\pi^-$ [17], from which constraints on γ were obtained. The main difficulty of each of these methods is that each decay mode by itself has a very low rate. Reaching high sensitivity in near future measurements of γ requires combining several relevant B and D decay modes.

Measurements relevant to studying γ have so far focused on charged B decays, $B^\pm \to DK^\pm$ and $B^\pm \to DK^{*\pm}$ [18]. At first glance, neutral B decays seem to be less promising for two reasons: they have much smaller rates and require B^0 flavor tagging. Let us discuss these two points one at a time:

- The processes $B^0 \to DK^0$ and $B^0 \to DK^{*0}$ are expected to be color-suppressed [11], implying that their rates are about an order of magnitude below the rates of corresponding charged B decays. However, the crucial factor determining the sensitivity of a measurement of γ is not the decay rate itself. Rather, the sensitivity is governed by the magnitude of the smaller of the two interfering amplitudes in a given process. Since the smaller amplitudes in B^+ and B^0 decays are both color-suppressed, they are expected to be of comparable magnitudes. Therefore, this by itself is not a limiting factor for neutral B decays.

- Only time-integrated rates have so far been measured in $B^0 \to DK^0$, combining rates for $B^0 \to \bar{D}^0 K^0$ and $\bar{B}^0 \to \bar{D}^0 \bar{K}^0$ [19]. It was shown in [6, 20, 21] that a determination of γ is possible from time-dependent measurements, for which one must tag the flavor of the initial B^0. The effective flavor tagging efficiency at B factories is about 30% (and much smaller at hadron machines), resulting in a doubling of the statistical error relative to the perfect-tag case. As we show below, γ can be determined using untagged data alone. This makes use of events that cannot be tagged or even events that are mis-tagged, regaining a significant part of the sensitivity lost due to the low effective tagging efficiency.

In the present paper we investigate what can be learned from untagged decays $B^0/\bar{B}^0 \to DK_S$, where the D meson is observed in several decay modes. A potential lower bound on $|\cos\gamma|$ from untagged decays, in which D mesons are observed in CP-eigenstates, was noted by Fleischer [22]. We will go beyond this bound by showing that γ can actually be completely determined in the range $0 < \gamma < \pi$, using only untagged decays. In practice, it is useful to combine information from untagged neutral B decays with information from charged B decays, since the observables related to D decays are common to both cases. This provides an overconstrained information, permitting a more accurate determination of γ than when using B^+ decays alone.

The plan of the paper is as follows. In Section II we introduce parallel notations for charged and neutral $B \to DK$ decays, discussing briefly relative magnitudes of decay amplitudes in these processes. Section III studies two-body and quasi two-body D decays, distinguishing between several classes of decay modes. We show that γ can be determined from untagged neutral B decays alone and derive an explicit expression for $\tan^2\gamma$ in terms of measurable rates. Multibody D decays in $B^0 \to DK_S$ are studied in Section IV. In Section V we discuss a way of reducing the number of hadronic parameters by assuming isospin symmetry and by neglecting an annihilation contribution in $B \to DK$. Finally, Section VI concludes. We also add an appendix studying time-dependence in $B^0(t) \to (K_S\pi^+\pi^-)_D K_S$.

II. AMPLITUDES IN $B \to DK$ DECAYS

We define decay amplitudes of $B \to DK$ for charged B mesons,

$$\begin{aligned} A(B^+ \to \bar{D}^0 K^+) &\equiv A_c \ , \\ A(B^+ \to D^0 K^+) &\equiv A_c r_c e^{i(\delta_c + \gamma)} \ , \end{aligned} \tag{1}$$

and for neutral B mesons,

$$\begin{aligned} A(B^0 \to \bar{D}^0 K_S) &\equiv A_n \ , \\ A(B^0 \to D^0 K_S) &\equiv A_n r_n e^{i(\delta_n + \gamma)} \ . \end{aligned} \tag{2}$$

By convention, $A_i \geq 0, r_i \geq 0$, and $0 \leq \delta_i \leq 2\pi$ $(i = c, n)$. Amplitudes for the CP conjugated decays have the same expressions, but the phase γ occurs with an opposite sign.

Let us discuss briefly the relevant ratios of amplitudes. The amplitude A_c is a combination of color-allowed and color-suppressed contributions, while the amplitude A_n is purely color-suppressed. The ratio A_n/A_c may be estimated in two ways leading to comparable values. We mention in each case the required approximation:

- Measurements of $B^+ \to \bar{D}^0 K^+$ [14, 15, 23] and $B^0 \to \bar{D}^0 K^0$ [19] imply

$$\frac{A_n}{A_c} \simeq \sqrt{\frac{\Gamma(B^0/\bar{B}^0 \to \bar{D}^0 K_S)}{\Gamma(B^+ \to \bar{D}^0 K^+)}} = 0.25 \pm 0.07 \ . \tag{3}$$

  In addition to a term A_n^2, the untagged rate in the numerator includes also a smaller term $A_n^2 r_n^2$ which we neglect.

- Using flavor SU(3) [24], one may relate A_n/A_c to a corresponding ratio measured in $B \to \bar{D}\pi$ [26],

$$\frac{A_n}{A_c} \simeq \sqrt{\frac{\Gamma(B^0 \to \bar{D}^0 \pi^0)}{\Gamma(B^+ \to \bar{D}^0 \pi^+)}} = 0.24 \pm 0.01 \ . \tag{4}$$

  This relation is affected by SU(3) breaking corrections and by a small exchange amplitude in $B^0 \to \bar{D}^0 \pi^0$ [25] which we neglect. SU(3) breaking effects, which are common in $A(B^+ \to \bar{D}^0 K^+)/A(B^+ \to \bar{D}^0 \pi^+)$ and $A(B^0 \to \bar{D}^0 K_S)/A(B^0 \to \bar{D}^0 \pi^0)$, cancel and do not affect this estimate.

It is more difficult to obtain reliable estimates for r_c and r_n. The two parameters are expected to be smaller than one since they contain a CKM factor $|V_{ub}V_{cs}/V_{cb}V_{us}| \simeq 0.4$. The parameter r_c involves also an unknown color-suppression factor in $\bar{b} \to \bar{u}c\bar{s}$, while r_n involves the ratio of color-suppression factors in $\bar{b} \to \bar{u}c\bar{s}$ and $\bar{b} \to \bar{c}u\bar{s}$. Since the dynamics of $B \to \bar{D}K$ decays (caused by $\bar{b} \to \bar{c}u\bar{s}$) and $B \to DK$ (caused by $\bar{b} \to \bar{u}c\bar{s}$) are different, color-suppression may be different in the two cases. This introduces large uncertainties in r_c and r_n.

It is easier to justify an approximate relation between the magnitudes of the two color-suppressed amplitudes $A_c r_c$ and $\sqrt{2} A_n r_n$. Noting that the two processes $B^+ \to D^0 K^+$ and $B^0 \to D^0 K^0$ differ only by the flavor of the spectator quark, one expects

$$A_c r_c \simeq \sqrt{2} A_n r_n \ . \tag{5}$$

In Section V we will discuss the approximation involved in this relation and a way of testing it experimentally. This approximate equality implies that the sensitivity to γ is comparable in charged and neutral B decays, since the sensitivity in each case is governed by the smaller of the two interfering amplitudes. This point provides a major motivation for our study.

III. TWO-BODY AND QUASI TWO-BODY D DECAYS

Considering decays of $\bar{D}^0$ and D^0 into a generic two-body or quasi two-body hadronic state f_D and its CP conjugate $\bar{f}_D$, we denote the corresponding amplitudes by

$$\begin{aligned} A(\bar{D}^0 \to f_D) &= A(D^0 \to \bar{f}_D) \equiv A_f \ , \\ A(D^0 \to f_D) &= A(\bar{D}^0 \to \bar{f}_D) \equiv A_f r_f e^{i\delta_f} \ , \end{aligned} \tag{6}$$

where by convention $A_f \geq 0$, $r_f \geq 0$, and $0 \leq \delta_f \leq 2\pi$. Here and below we set the weak phase in D decays to zero and neglect $D^0 - \bar{D}^0$ mixing. The effects of $D^0 - \bar{D}^0$ mixing can be included as in [27], but are not further discussed here.

Using these notations, one finds expressions for decay rates in charged B decays,

$$\begin{aligned} \Gamma(B^+ \to f_D K^+) &= A_c^2 A_f^2 \left[1 + r_c^2 r_f^2 + 2 r_c r_f \cos(\delta_c + \delta_f + \gamma)\right] \ , \\ \Gamma(B^- \to \bar{f}_D K^-) &= A_c^2 A_f^2 \left[1 + r_c^2 r_f^2 + 2 r_c r_f \cos(\delta_c + \delta_f - \gamma)\right] \ , \\ \Gamma(B^+ \to \bar{f}_D K^+) &= A_c^2 A_f^2 \left[r_c^2 + r_f^2 + 2 r_c r_f \cos(\delta_c - \delta_f + \gamma)\right] \ , \\ \Gamma(B^- \to f_D K^-) &= A_c^2 A_f^2 \left[r_c^2 + r_f^2 + 2 r_c r_f \cos(\delta_c - \delta_f - \gamma)\right] \ . \end{aligned} \tag{7}$$

Combining B^+ and B^- decay rates for states involving a common D decay mode, f_D or $\bar{f}_D$, one finds

$$\begin{aligned} \langle\Gamma(B \to f_D K_c)\rangle &\equiv \Gamma(B^+ \to f_D K^+) + \Gamma(B^- \to f_D K^-) \\ &= A_c^2 A_f^2 \left[(1 + r_c^2)(1 + r_f^2) + 4 r_c r_f \cos(\delta_f + \gamma) \cos\delta_c\right] \ , \\ \langle\Gamma(B \to \bar{f}_D K_c)\rangle &\equiv \Gamma(B^+ \to \bar{f}_D K^+) + \Gamma(B^- \to \bar{f}_D K^-) \\ &= A_c^2 A_f^2 \left[(1 + r_c^2)(1 + r_f^2) + 4 r_c r_f \cos(\delta_f - \gamma) \cos\delta_c\right] \ . \end{aligned} \tag{8}$$

Studying neutral B decays, one finds similar expressions for untagged decay rates [20]:

$$\begin{aligned}
\langle\Gamma(B \to f_D K_n)\rangle &\equiv \Gamma(B^0 \to f_D K_S) + \Gamma(\bar{B}^0 \to f_D K_S) \\
&= A_n^2 A_f^2 \left[(1+r_n^2)(1+r_f^2) + 4 r_n r_f \cos(\delta_f + \gamma)\cos\delta_n\right] \ , \\
\langle\Gamma(B \to \bar{f}_D K_n)\rangle &\equiv \Gamma(B^0 \to \bar{f}_D K_S) + \Gamma(\bar{B}^0 \to \bar{f}_D K_S) \\
&= A_n^2 A_f^2 \left[(1+r_n^2)(1+r_f^2) + 4 r_n r_f \cos(\delta_f - \gamma)\cos\delta_n\right] \ . \qquad (9)
\end{aligned}$$

Individual time-dependent decay rates for $B^0(t) \to f_D K_S$, $\bar{B}^0(t) \to f_D K_S$ and their CP-conjugates are given in [20], and include more information than the untagged rates. These, however, will not be needed in the following.

The decay rates in Eqs. (8) and (9) display a dependence on two types of quantities. Amplitudes and strong phases in $B \to DK$, $(A_i, r_i, \delta_i;\ i = c, n)$, which in general obtain different values in charged and neutral B decays, and the corresponding quantities in $D^0/\bar{D}^0 \to f_D$, (A_f, r_f, δ_f), which are common to both B^+ and B^0 decays. We will refer to these quantities as B and D decay parameters, respectively. In the following we will assume that the D decay quantities A_f and r_f have been measured and are known. They can be obtained through branching ratio measurements in an independent sample of neutral D mesons, flavor-tagged through their production in the decay $D^{*+} \to D^0\pi^+$ [28]. In quasi two-body D decays the three parameters A_f, r_f and δ_f can be determined simultaneously through a complete Dalitz plot analysis. Although the phases δ_f can in principle be measured [10, 29, 30, 31], we will treat them as unknown, unless indicated otherwise.

Note that the combined $B^\pm$ rates in (8) and those in the untagged B^0 decays (9) depend in each case only on two combinations of B decay parameters,

$$X_i \equiv A_i^2(1 + r_i^2) \ , \qquad Y_i \equiv 2A_i^2 r_i \cos\delta_i \ , \qquad i = c, n \ , \qquad (10)$$

which obey

$$|Y_i| \leq X_i \ . \qquad (11)$$

We see that the individual branching ratios for $B^0 \to \bar{D}^0 K_S$ and $B^0 \to D^0 K_S$, proportional to A_n^2 and $A_n^2 r_n^2$, respectively, cannot be measured from untagged decays alone.

The three D decay parameters A_f, r_f and δ_f in (6) depend, of course, on the final state f_D. One may distinguish between three cases for which we give examples:

1. $f_D =$ CP-state (*e.g.*, $f_{\rm CP+} = K^+K^-$, $f_{\rm CP-} = K_S\pi^0$), for which $r_{\rm CP\pm} = 1, \cos\delta_{\rm CP\pm} = \pm 1$,

2. $f_D =$ flavorless (*e.g.*, $K^{*+}K^-$), for which $r_f = \mathcal{O}(1)$ but generally $r_f \neq 1, \delta_f =$ unknown,

3. f_D = flavor state (*e.g.*, $K^+\pi^-$), for which $r_f \simeq \tan^2\theta_C$ [28], δ_f = unknown, where θ_C is the Cabibbo angle.

Using Eqs. (9) and the observation (10), it is simple to show that γ may be determined solely from untagged B^0 decays. Consider N different non-CP neutral D decay modes f_D^k ($k = 1, ..., N$) together with their CP conjugates $\bar{f}_D^k$, as the final states in the $B^0 \to DK_S$ decay chain. The unknown variables are γ, X_n, Y_n and N strong phases δ_f^k. Eqs. (9), which provide $2N$ measurables for $N+3$ unknowns, are solvable for $N \geq 3$. That is, γ may be determined from untagged $B^0 \to DK_S$ decay rates, where D^0 is observed in at least three different non-CP decay modes and their CP conjugates. This argument may be generalized to include other untagged B^0 decays, such as $B^0 \to D^*K_S$ ($D^* \to D^0\pi^0$). Assuming M different B^0 decay modes of this kind, each of which introduces a pair of unknowns X_n^j and Y_n^j ($j = 1, ..., M$), one has $2MN$ measurables for $2M+N+1$ unknowns. For $M \geq 2$ this set of equations is solvable for $N \geq 2$. Namely, two non-CP decay modes of D^0 are sufficient for determining γ from untagged $B^0 \to DK_S$ and $B^0 \to D^*K_S$.

For a CP-eigenstate the strong phase $\delta_{f(\mathrm{CP})}$ is either 0 or π. In this case the two equations in (9) become identical and provide a single measurable. Choosing the decay modes f_D^k to be (i) an even-CP state, (ii) an odd-CP state, and (iii) a single non-CP eigenstate and its CP conjugate (involving an unknown phase δ_f), one can solve the four equations for γ, X_n, Y_n and δ_f. For this case we now derive an explicit expression for $\tan^2\gamma$ in terms of measurable rates. The derivation holds for both charged and neutral B decays.

Using Eqs. (8) and (9), one has

$$\langle\Gamma(B \to f_{\mathrm{CP}\pm}K_i)\rangle = 2A^2_{\mathrm{CP}\pm}\left[X_i \pm Y_i\cos\gamma\right] , \qquad (i = c, n) , \tag{12}$$

where the two signs on the right-hand-side correspond to positive and negative CP-eigenstates. Adding and subtracting rates for even-CP and odd-CP eigenmodes, one finds

$$\Sigma^i_{\mathrm{CP}} \equiv \frac{\langle\Gamma(B \to f_{\mathrm{CP}+}K_i)\rangle}{2A^2_{\mathrm{CP}+}} + \frac{\langle\Gamma(B \to f_{\mathrm{CP}-}K_i)\rangle}{2A^2_{\mathrm{CP}-}} = 2X_i , \tag{13}$$

$$\Delta^i_{\mathrm{CP}} \equiv \frac{\langle\Gamma(B \to f_{\mathrm{CP}+}K_i)\rangle}{2A^2_{\mathrm{CP}+}} - \frac{\langle\Gamma(B \to f_{\mathrm{CP}-}K_i)\rangle}{2A^2_{\mathrm{CP}-}} = 2Y_i\cos\gamma , \qquad (i = c, n) . \tag{14}$$

These definitions apply in practice to sums over individual CP states. Eq. (13) provides the most direct way to determine X_c and X_n. We define CP-*conserving* rate asymmetries between even and odd CP-states,

$$\mathcal{A}^i_{\mathrm{CP}} \equiv \frac{\Delta^i_{\mathrm{CP}}}{\Sigma^i_{\mathrm{CP}}} , \qquad (i = c, n) . \tag{15}$$

Using Eq. (11), one obtains two potential inequalities for $\cos\gamma$ in terms of these ratios,

$$|\cos\gamma| > |\mathcal{A}^i_{\mathrm{CP}}| , \qquad (i = c, n) . \tag{16}$$

This inequality holds separately for charged and neutral B decays. The inequality for neutral B decays was noted in [22].

As mentioned above, in order to determine γ from untagged neutral B decays, one needs in addition to the two rates for CP eigenstates given in (12) ($i = n$) two rate measurements for a non-CP state f_D and its CP-conjugate $\bar{f}_D$. These rates are given by the two equations in (9). We denote the sum and difference of these rates and of the corresponding rates in charged B decays by

$$\Sigma^i_f \equiv \frac{\langle\Gamma(B \to \bar{f}_D K_i)\rangle + \langle\Gamma(B \to f_D K_i)\rangle}{A^2_f(1 + r^2_f)} ~,$$
$$\Delta^i_f \equiv \frac{\langle\Gamma(B \to \bar{f}_D K_i)\rangle - \langle\Gamma(B \to f_D K_i)\rangle}{A^2_f(1 + r^2_f)} ~, \qquad (i = c, n) ~, \tag{17}$$

which imply CP-violating asymmetries,

$$\mathcal{A}^i_f \equiv \Delta^i_f / \Sigma^i_f ~, \qquad (i = c, n) ~. \tag{18}$$

It is then straightforward to show that $\tan^2\gamma$ is proportional to $(\mathcal{A}^i_f)^2$ and is given by

$$\tan^2\gamma = \frac{(\Delta^i_f)^2}{\rho^2_f(\Delta^i_{\rm CP})^2 - (\Sigma^i_{\rm CP} - \Sigma^i_f)^2} = \frac{(\mathcal{A}^i_f)^2(\Sigma^i_f/\Sigma^i_{\rm CP})^2}{\rho^2_f(\mathcal{A}^i_{\rm CP})^2 - (1 - \Sigma^i_f/\Sigma^i_{\rm CP})^2} ~, \qquad (i = c, n) ~, \tag{19}$$

where

$$\rho_f \equiv \frac{2r_f}{1 + r^2_f} ~. \tag{20}$$

This result applies to both charged and neutral B mesons. We stress that when determining γ we do not rely on separating the two terms A^2_i and $A^2_i r^2_i$ contributing to X_i.

While the possibility of measuring γ from untagged B^0 decays alone is interesting, the most efficient way to determine the weak phase would be to combine information from decays of charged B decays and untagged neutral B decays. The derivation we have just presented applies also to charged B decays alone. One needs to measure only the combined $B^\pm$ rates given in Eqs. (8), without needing to separate the small $A^2_c r^2_c$ term. Thus, Eq. (19) also gives $\tan^2\gamma$ in terms of decay rates for combined $B^\pm \to DK^\pm$ events, where D mesons are observed in decays into an even-CP, an odd-CP eigenstates and a non-CP flavorless state or a flavor state. We see that, in principle, a determination of γ does not require measuring CP asymmetries in $B^\pm \to f_{\rm CP}K^\pm$. These asymmetry measurements, for even-CP and odd-CP states [14, 15], provide extra useful information. Using all these measurements, and rate measurements for untagged neutral B decays, will lead to a more accurate determination of γ than using only charged B mesons.

Eq. (19) displays an explicit dependence of γ on rate measurements defined in Eqs. (13), (14) and (17). We see that $\tan\gamma$ is proportional to the CP asymmetries

$\mathcal{A}_f^i \sim r_f \cos\delta_i \sin\delta_f \sin\gamma$ $(i = c, n)$, indicating that the sensitivity for measuring γ increases with r_f. The sensitivity depends also on the value of δ_f. In the extreme case that δ_f vanishes the two rates in (8) (and in (9)) become equal and γ cannot be extracted. For the two-body flavor state $f = K^+\pi^-$, the phase δ_f vanishes in the SU(3) symmetry limit, however SU(3) breaking effects are known to be large in D decays. Consequently, sizable values of δ_f have been calculated for this final state in several models [32]. This phase can be measured at a charm factory [29, 30]. There are experimental indications for small phases in two cases of quasi two-body states, $f = K^{*+}\pi^-$ where $\delta_f = (12 \pm 3)^\circ (\mathrm{mod}\ \pi)$ was measured [17], and $f = \rho^+\pi^-$ where $\delta_f = (4 \pm 3 \pm 4)^\circ$ was measured [33]. These (preliminary) results were obtained by studying the Dalitz plot of $D^0 \to K_S\pi^+\pi^-$ and $D^0 \to \pi^+\pi^-\pi^0$, respectively. No measurement exists for δ_f in $D^0 \to K^{*+}K^-$, which can be measured by studying the Dalitz plot of $D^0 \to K^+K^-\pi^0$ [31]. As we note in the next section, a complete Dalitz plot analysis of three body D^0 decays involves other strong phases which are large in regions where two resonances overlap. These phases will be shown to be useful when studying γ in $B \to DK$, where the D meson is observed in a three body final state.

IV. $B \to DK_S$ OBSERVED IN MULTI-BODY D DECAYS

The study of untagged $B^0 \to DK_S$ presented in the previous section for D mesons decaying in two-body modes may be extended to multibody decays. To be specific, we focus on the case of the three-body D decay,

$$D \to K_S\pi^-\pi^+, \tag{21}$$

following the discussion of $B^\pm \to DK^\pm$ in [9]. In order to make our point, we start with a model-independent approach. We also explain how modeling the amplitude for $D^0 \to K_S\pi^-\pi^+$ in terms of a sum of a given set of intermediate resonances [34], as done recently in [17], may help in reducing the experimental error in γ.

We denote the amplitude for $D^0 \to K_S\pi^-\pi^+$ at a given point in the Dalitz plot by

$$A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) \equiv A(s_{12}, s_{13})e^{i\delta(s_{12},s_{13})}\ , \tag{22}$$

where $s_{ij} \equiv (p_i + p_j)^2$. As in two body decays, we use the convention $A(s_{12}, s_{13}) \geq 0$ and $0 \leq \delta(s_{12}, s_{13}) \leq 2\pi$. Assuming that CP is conserved in this decay, one has

$$A(\bar{D}^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) = A(D^0 \to K_S(p_1)\pi^-(p_3)\pi^+(p_2)) \equiv A(s_{13}, s_{12})\, e^{i\delta(s_{13},s_{12})}\ . \tag{23}$$

That is, the (complex) decay amplitude for $\bar{D}^0$ at a given point (s_{12}, s_{13}) in the Dalitz plot equals the decay amplitude for D^0 at a point (s_{13}, s_{12}) obtained by reflection across a symmetry axis corresponding to exchanging the momenta of the two pions.

The density of events in the D decay Dalitz plot for untagged $B \to (K_S\pi^-\pi^+)_D K_S$ is obtained using Eqs. (2), (22) and (23) (or from the time-dependence in Appendix A),

$$\frac{d^2\Gamma}{ds_{12}ds_{13}}(B^0/\bar{B}^0 \to [K_S(p_1)\pi^-(p_2)\pi^+(p_3)]_D K_S) = A_n^2 \left[\left(A^2(s_{12}, s_{13}) + A^2(s_{13}, s_{12})\right)(1 + r_n^2) \right.$$
$$\left. + 4r_n A(s_{12}, s_{13})A(s_{13}, s_{12})\cos(\delta(s_{12}, s_{23}) - \delta(s_{13}, s_{12}) + \gamma)\cos\delta_n\right] \ . \quad (24)$$

This is in complete analogy with the first of Eqs. (9). The second equation, describing the density at the point of reflection across the symmetry axis, involves an opposite sign for γ. Integrating (24) over an area (a bin) i lying below the symmetry axis and over a corresponding symmetry-reflected area $\bar{i}$ lying above the symmetry axis, one has[1]

$$\Gamma_i \equiv \int_i d\Gamma(B^0/\bar{B}^0 \to [K_S\pi^-\pi^+]_D K_S) = X_n(T_i + T_{\bar{i}}) + 2Y_n[c_i\cos\gamma - s_i\sin\gamma] \ ,$$
$$\Gamma_{\bar{i}} \equiv \int_{\bar{i}} d\Gamma(B^0/\bar{B}^0 \to [K_S\pi^-\pi^+]_D K_S) = X_n(T_i + T_{\bar{i}}) + 2Y_n[c_i\cos\gamma + s_i\sin\gamma] \ , \quad (25)$$

where we define

$$T_i \equiv \int_i ds_{12}ds_{13}A^2(s_{13}, s_{23}) \ ,$$
$$c_i \equiv \int_i ds_{12}ds_{13}A(s_{12}, s_{13})A(s_{13}, s_{12})\cos(\delta(s_{12}, s_{23}) - \delta(s_{13}, s_{12})) \ ,$$
$$s_i \equiv \int_i ds_{12}ds_{13}A(s_{12}, s_{13})A(s_{13}, s_{12})\sin(\delta(s_{12}, s_{23}) - \delta(s_{13}, s_{12})) \ . \quad (26)$$

The partial rates T_i in D decays may be measured using flavor-tagged D^0 decays and are assumed to be known. The other D decay variables, c_i and s_i, which in principle can be measured model-independently at a charm factory (up to a sign ambiguity in s_i), will nonetheless be taken as unknown. Consider k different bins i lying below the symmetry axis, each contributing two unknowns c_i and s_i. Together with X_n, Y_n and γ, there are $2k + 3$ unknowns. Eqs. (25), which provide $2k$ measurables (Γ_i and $\Gamma_{\bar{i}}$), are therefore unsolvable.

The situation changes when one measures another neutral B decay of this type, e.g. the sequence $B^0 \to D^*K_S$, $D^* \to D^0\pi^0$, $D^0 \to K_S\pi^+\pi^-$, which introduces a pair of new variables analogous to X_n, Y_n. In this case one has $4k$ measurables for $2k + 5$ unknowns, a solution for which requires $k \geq 3$. That is, γ may be determined by measuring partial rates in the two untagged neutral B decay modes, $B \to DK_S$ and $B \to D^*K_S$, for at least three pairs of Dalitz plot bins in $D \to K_S\pi^+\pi^-$.

[1] The method outlined here applies to any multibody D^0 decay with the set of equations (25) unchanged. If the i-th bin is in the phase space of a final state f_D, then the $\bar{i}$-th bin is a CP transformed bin in the phase space of the corresponding $\bar{f}_D$ state.

A more powerful approach is to combine information from all the untagged neutral B decays and charged B decays, using both multibody and two-body D decays. For instance, in analogy with Eqs. (7), the decays $B^\pm \to (K_S\pi^+\pi^-)_D K^\pm$ provide four measurables for each bin [9], instead of the two in (25). Combining charged and untagged neutral $B \to DK$ decays, where $D \to K_S\pi^+\pi^-$, yields $6k$ measurables for $2k+6$ unknowns, $c_i, s_i, A_c, r_c, \delta_c, X_n, Y_n$ and γ. Therefore, two pairs of bins provide an overconstrained system of equations for determining γ.

The two equations (25) are not mutually independent when $s_i = 0$, in analogy with the singularity noted in Eq. (19) when $\delta_f = 0$. However, the relevant strong phase differences which determine s_i are large at least in some regions of the Dalitz plot of $D^0 \to K_S\pi^+\pi^-$. Consider, for instance, the two overlapping regions of a vertical band describing the Cabbibbo-allowed mode $K^{*-}\pi^+$ and a horizontal band describing the doubly Cabibbo-suppressed mode $K^{*+}\pi^-$ with a diagonal band representing the Cabibbo-allowed mode $\rho^0\bar{K}^0$. The local strong phases which determine s_i for these two regions are the phase differences between amplitudes describing the sum of the $K^{*-}\pi^+$ and $\rho^0\bar{K}^0$ contributions and the sum of the $K^{*+}\pi^-$ and $\rho^0\bar{K}^0$ contributions. These phases are large and vary a lot over the overlapping regions because of the two largely different $K^*\pi$ contributions in the two amplitudes.

One can reduce the number of unknowns appearing in the determination of γ, if the unknowns coming from the D decay, c_i and s_i, appearing in (25), are determined independently. This can be done by assuming a Breit-Wigner (BW) form for the intermediate resonances contributing to this decay [9]. The parameters of the model describing the D decay amplitude can then be fitted to data of tagged D decays, which are abundant at B-factories. The observables in (25) now depend only on three unknowns, X_n, Y_n and γ.

It is hard to quantify the theoretical error introduced by assuming a BW form. One way to proceed is to change the number of resonances and see how the sensitivity changes. This is only a partial determination of the error. Another source of error is the accuracy of the BW assumption. This can be determined by the goodness of the fit to the tagged D decays, or by using a different model for resonances, such as a K-matrix model for wide resonances [35]. A rough estimate of the theoretical error caused by assuming a superposition of BW amplitudes is about 10° [17]. Further studies are required in order to evaluate possible contributions of non-BW terms in the D decay amplitude and their effect on determining γ. As mentioned, this model-dependence can be avoided by measuring the parameters c_i and s_i at a charm factory.

V. USING ISOSPIN AND NEGLECTING AN ANNIHILATION AMPLITUDE

As already mentioned, the most powerful approach for improving the determination of γ is to combine charged B decays with the information from untagged neutral B decays. Adding untagged neutral B decays to a sample of charged B decays with the same D decay final states introduces only two unknown parameters, X_n and Y_n. Here we discuss an approximation which may be used to reduce the number of parameters further. This introduces a theoretical error in γ. Yet, it is worthwhile considering such an approximation as long as this error is smaller than the statistical error.

We recall two isospin relations [36], one for $\bar{b} \to \bar{c}u\bar{s}$ transitions,

$$A(B^0 \to D^- K^+) = A(B^+ \to \bar{D}^0 K^+) - A(B^0 \to \bar{D}^0 K^0) \ , \tag{27}$$

and another for $\bar{b} \to \bar{u}c\bar{s}$ transitions,

$$A(B^0 \to D^0 K^0) = A(B^+ \to D^0 K^+) + A(B^+ \to D^+ K^0) \ . \tag{28}$$

The amplitude of $B^+ \to D^+ K^0$ is pure annihilation, and is expected to be smaller than the other two amplitudes in the last relation [24]. The absence of rescattering effects, which may enhance this amplitude to a level comparable to the other two amplitudes in this relation, can be tested [37] by setting very stringent experimental bounds on the branching ratio for $B^+ \to D^+ K^0$. Neglecting $A(B^+ \to D^+ K^0)$, Eq. (28) reduces to [36]

$$A(B^0 \to D^0 K^0) = A(B^+ \to D^0 K^+) \ . \tag{29}$$

Namely, the two color-suppressed amplitudes have equal magnitudes and equal strong phases. Note that the error due to isospin violation, at most a few percent, is likely to be much smaller than that involved in neglecting the annihilation amplitude.

Equations (27) and (29) may be used to simplify the determination of γ when combining $B^\pm$ decays and untagged B^0 decays. The two equations imply

$$\begin{aligned} A_c^2 + A_n^2/2 - \sqrt{2} A_c A_n \cos(\delta_n - \delta_c) &= \Gamma(B^0 \to D^- K^+) \ , \\ \sqrt{2} A_n r_n &= A_c r_c \ . \end{aligned} \tag{30}$$

The right-hand side of the first equation, which involves a color-allowed process, has already been measured [38] and will be assumed to be given. Eqs. (30) reduce the six parameters describing charged and neutral $B \to DK$ decays, $A_i, r_i, \delta_i (i = c, n)$, to four independent ones. The measurable parameters in untagged $B^0 \to D K_S$ decays, $X_n \equiv A_n^2(1 + r_n^2)$ and $Y_n \equiv 2A_n^2 r_n \cos\delta_n$, can now be expressed in terms of the three B^+ decay parameters and a single B^0 decay parameter (δ_n, for instance). That is, under the above assumption, adding information from untagged neutral B^0 decays to studies of $B^\pm$ decays involves a single new unknown parameter instead of two parameters. This is expected to reduce the statistical error in determining γ.

VI. CONCLUDING REMARKS

Before concluding let us make several general comments:

- Since the data set of $B^+ \to DK^+$ is larger than that of $B^0 \to DK_S$, hadronic parameters such as X_c are easier to measure than X_n. As noted, the approximate relation $\sqrt{2}A_n r_n \simeq A_c r_c$ implies comparable sensitivities to γ in charged and neutral B decays. The sensitivity in B^0 decays is smaller by a factor $\sqrt{2}$ since only K_S mesons are experimentally useful. Moreover, the detection efficiency for K_S is about a factor of two smaller than that for charged kaons. This is expected to reduce somewhat the effect of neutral B decays on determining γ.

- Our study focused on $B \to DK$ decays. It can be extended to multibody B decays, including $B^+ \to D^*K^+$ and $B^0 \to D^*K_S$, where $D^* \to D\pi^0$, as well as to the self-tagged decays $B^+ \to D^{(*)}K^{*+}$ and $B^0 \to D^{(*)}K^{*0}$. This would add to the statistical power of the analysis since the parameters related to D decays are common to these processes and to $B \to DK$.

- The method we discussed for B^0 decays can be applied also to B_s decays, replacing the K_S by ϕ, η', η. In that case the advantage of being able to use untagged data is greater, because the hadronic environment where B_s decays will be studied makes flavor tagging less efficient. We have neglected the width difference between the two neutral B meson states, which is a very good approximation for nonstrange B mesons. In the case of B_s, the width difference is expected to be nonnegligible and may be taken into account in a straightforward manner.

- In our discussion we assumed that strong phases in D decays are unknown and need to be determined from the analysis simultaneously with γ. As we did already mention, the strong phases in two-body and quasi two-body D decays can be determined independently [10, 29, 30, 31]. The strong phases in three-body decays may also be determined by assuming that the D decay amplitude is given as a sum of Breit-Wigner amplitudes. Knowledge of strong phases would imply, for instance, that fewer D decay modes are needed in order to determine γ from untagged decays alone. In practice, this implies that a combined fit of the data to fewer hadronic parameters will result in a smaller error in γ.

- The extraction of γ from $B^\pm \to DK^\pm$ involves a number of discrete ambiguities [5, 39]. The ambiguities in untagged B^0 decays may be identified in Eqs. (9) which are invariant under

$$
\begin{aligned}
P_{\rm ex} &\equiv \{\gamma \to \delta_f\,,\ \delta_f \to \gamma\}\ , \\
P_{-} &\equiv \{\gamma \to -\gamma\,,\ \delta_f \to -\delta_f\}\ , \\
P_{\pi} &\equiv \{\gamma \to \gamma + \pi\,,\ \delta_f \to \delta_f + \pi \quad \text{or} \quad \delta_n \to \delta_n + \pi\}\ .
\end{aligned} \tag{31}
$$

Once several two-body D decay modes are combined, or once a multibody D decay mode is used, the first two ambiguities may be resolved. The ambiguity $P_{\rm ex}$ is lifted, since δ_f is not expected to be the same for all two-body D decay modes, and is known to change over the Dalitz plot in three-body decays. Measuring the sign of s_i in (25) through a fit to a sum of Breit-Wigner resonance functions would resolve the P_- ambiguity. This introduced essentially no model-dependence, since one needs only the sign of s_i, which is easily determined in the vicinity of a BW resonance. Resolving P_- avoids the ambiguity $\gamma \to \pi - \gamma$, a combination of P_- and P_π, which is particularly problematic in view of the proximity of γ to $\pi/2$ [39]. The only remaining ambiguity is $\gamma \to \gamma + \pi$. This ambiguity is the least problematic, since the two corresponding values of γ are maximally separated.

To conclude, we have studied the information obtained from untagged neutral B decays of the type $B \to DK_S$ involving several D decay modes. We have shown that these measurements alone can, in principle, determine γ. Of course, B^0 tagging information, while limited, can only improve this determination. By combining information from untagged B^0 decays with that obtained in corresponding charged B decays one gains statistics, thereby permitting a more accurate determination of γ. While statistics are limited, one may neglect an annihilation amplitude in $B \to DK$, reducing by one the number of hadronic parameters and resulting in a smaller experimental error in γ. This introduces a theoretical error in γ which must be further studied.

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APPENDIX A: TIME-DEPENDENT $B \to DK_S$ WITH MULTIBODY D DECAYS

In this appendix we provide a formalism allowing the extraction of γ from time-dependent rates in $B^0 \to f_D K_S$ where f_D is a multibody final state. As we show, this does not only serve the purpose of determining γ, but also helps resolve the current two-fold ambiguity, $\beta \to \pi/2 - \beta$. For simplicity we take f_D to be the three-body final state $K_S\pi^+\pi^-$ studied in Section IV.

Time-dependent partial rates, integrated over a bin i in the Dalitz plot of $D \to K_S\pi^+\pi^-$ lying below the symmetry axis, and over a corresponding symmetry-reflected bin $\bar{i}$ above the axis, are readily calculated for initial B^0 and $\bar{B}^0$ states (a positive B meson bag parameter is assumed [40]):

$$\Gamma_i \equiv \int_i d\Gamma\left(B^0(t) \to (K_s\pi^-\pi^+)_D K_s)\right) = e^{-\Gamma_B t}\, A_n^2 \times \left\{ I_i^+ \cos^2\left(\frac{\Delta m_B\, t}{2}\right) + I_{\bar{i}}^- \sin^2\left(\frac{\Delta m_B\, t}{2}\right) + S_i \sin(\Delta m_B t) \right\} , \tag{A1}$$

$$\bar{\Gamma}_i \equiv \int_i d\Gamma\left(\bar{B}^0(t) \to (K_s\pi^-\pi^+)_D K_s)\right) = e^{-\Gamma_B t}\, A_n^2 \times \left\{ I_{\bar{i}}^- \cos^2\left(\frac{\Delta m_B\, t}{2}\right) + I_i^+ \sin^2\left(\frac{\Delta m_B\, t}{2}\right) - S_i \sin(\Delta m_B t) \right\} , \tag{A2}$$

$$\Gamma_{\bar{i}} \equiv \int_{\bar{i}} d\Gamma\left(B^0(t) \to (K_s\pi^-\pi^+)_D K_s)\right) = e^{-\Gamma_B t}\, A_n^2 \times \left\{ I_{\bar{i}}^+ \cos^2\left(\frac{\Delta m_B\, t}{2}\right) + I_i^- \sin^2\left(\frac{\Delta m_B\, t}{2}\right) + S_{\bar{i}} \sin(\Delta m_B t) \right\} , \tag{A3}$$

$$\bar{\Gamma}_{\bar{i}} \equiv \int_{\bar{i}} d\Gamma\left(\bar{B}^0(t) \to (K_s\pi^-\pi^+)_D K_s)\right) = e^{-\Gamma_B t}\, A_n^2 \times \left\{ I_i^- \cos^2\left(\frac{\Delta m_B\, t}{2}\right) + I_{\bar{i}}^+ \sin^2\left(\frac{\Delta m_B\, t}{2}\right) - S_{\bar{i}} \sin(\Delta m_B t) \right\} . \tag{A4}$$

The six observables determined from the time-dependence, $I_i^\pm$, $I_{\bar{i}}^\pm$, S_i and $S_{\bar{i}}$, are defined in terms of the quantities in Eqs. (26),

$$I_i^\pm \equiv T_{\bar{i}} + r_n^2 T_i + 2r_n\left[\cos(\gamma \pm \delta_n)c_i \mp \sin(\gamma \pm \delta_n)s_i\right] , \tag{A5}$$

$$S_i \equiv r_n T_{\bar{i}} \sin(2\beta + \gamma - \delta_n) + [\sin(2\beta)c_i - \cos(2\beta)s_i] + r_n^2[\sin(2\beta + 2\gamma)c_i + \cos(2\beta + 2\gamma)s_i] + r_n T_i \sin(2\beta + \gamma + \delta_n) . \tag{A6}$$

Expressions for the observables $I_{\bar{i}}^\pm$ and $S_{\bar{i}}$ are obtained from (A5) and (A6) by replacing $T_i \leftrightarrow T_{\bar{i}},\ s_i \to -s_i$. Note that $I_{i,\bar{i}}^\pm$ correspond directly to the partial decay widths $\Gamma_{i,\bar{i}}^\pm$ for $B^\pm \to DK^\pm$ defined in [9].

Dividing the Dalitz plot into k pairs of bins, i and $\bar{i}$, the $6k$ observables permit an extraction of γ. There are $2k + 4$ unknowns, $c_i, s_i, A_n, r_n, \delta_n, \gamma$ ($\sin(2\beta)$ is assumed to be known), so that the system is solvable for $k \geq 1$. Namely, in order to determine γ from time-dependent decay rates into $(K_S\pi^+\pi^-)_D K_S$, it is sufficient to divide the D decay Dalitz plot into two bins, symmetric with respect to the symmetry axis.

The solution for γ involves a four-fold discrete ambiguity. Equations (A1)–(A4) are invariant under the following four independent discrete transformations:

$$\begin{aligned}
&P^\gamma_\pi \equiv \{\gamma \to \gamma+\pi, \delta_n \to \delta_n + \pi\} \ , \\
&P'_\pi \equiv \{\gamma \to \gamma+\pi, \beta \to \beta + \pi/2, c_i \to -c_i, s_i \to -s_i\} \ , \\
&P^\beta_\pi \equiv \{\beta \to \beta+\pi\} \ , \\
&P_- \equiv \{\gamma \to -\gamma, \beta \to \pi/2-\beta, \delta_n \to -\delta_n, s_i \to -s_i\} \ .
\end{aligned} \tag{A7}$$

The P'_π ambiguity can be resolved model-independently, either by using the sign of $\sin 2\beta$ or by measuring the sign of c_i at a $\Psi(3770)$ charm factory [9]. The P_- ambiguity can be resolved if one determines the sign of s_i by fitting the Dalitz plot to a sum of Breit-Wigner forms. (See Section VI.) Note that resolving the P_- ambiguity in this way leads to the determination of the sign of $\cos(2\beta)$ in an essentially model-independent way. It also determines the sign of γ or equivalently the sign of $\cos(2\alpha)$. Fixing the sign of $\cos(2\beta)$ is a consequence of knowing the sign of s_i in multibody decays. This is impossible in two-body D decays, where the sign of $\sin\delta_f$ cannot be determined. The remaining two ambiguities, P^γ_π and P^β_π, cannot be resolved without further theoretical input.

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