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Electro-Optic Sampling of Single Electron Beam Bunches of Ultrashort Duration

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Abstract

The effect of ultrafast electron beam bunch dynamics on single shot electro-optic sampling detection schemes is examined. It is shown that ultrashort electron bunch fields of adequate magnitude can dynamically impose additional bandwidth on laser probe pulses. The significance of this effect is evaluated by comparing the dynamics of the laser probe to that of the nonradiative field of a single electron bunch for a given crystal material. Dynamic effects can be distinguished with ultrafast temporal resolution of the transmitted probe spectrum. Furthermore, velocity matching of probe and bunch fields in a co-propagation scheme is less restrictive. Such time resolved spectra then can noninvasively determine single bunch dynamics and represent a new type of electro-optic sampling.

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1. Introduction

Non-invasive single electron bunch diagnostics can be critical for applications of relativistic electron beams. In many cases (for example the Linac Coherent Light Source at SLAC), electron bunches are of ultrashort duration and ultrafast diagnostics are essential. The nonradiative field of a single relativistic electron bunch is predominantly radial and can be sampled noninvasively using known electro-optic techniques. This was first demonstrated using the field of 45 MeV electron bunches to induce phase retardation in a probing laser waveform via the Pockels effect in $LiNbO_3$ [1,2]. A chirped version of this technique, in which temporal features could be determined spectrally, has recently been demonstrated using a ZnTe crystal at the Felix facility with 46-MeV electrons of 1.7 picosecond duration [3]. In the latter work , the chirped laser probe pulse provided the time-frequency correlation needed to uniquely extract the time-dependent field for a single bunch and therefore also its longitudinal charge density distribution using spectral data. Conventional laser systems capable of ultrafast time resolution (i.e. of bandwidth consistent with the generation of ultrashort Fourier transform limited pulses) are commercially available for such work.

However, ultrafast beam dynamics can impose additional limitations on applications of the chirped probe method and on electro-optic beam diagnostics in general. A new type of electro-optic diagnostic is described in which the dynamics of refractive index variations are monitored with ultrafast temporal resolution ('ultrafast' and 'ultrashort' are reserved to describe subpicosecond behaviour). To provide specific detail, we consider $LiTaO_3$ and $LiNbO_3$ uniaxial crystals. Dynamic refractive index behaviour is well known in the study of laser-plasma interactions; especially in the ultrafast cases [4,5]. The abrupt ionization associated with laser-induced plasma formation imposes large 'blue' shifts on the spectra of transmitted laser waveforms. In fact, the laser waveform usually experiences only those effects attributed to the ionization phase (not the recombination phase) during which the refractive index is abruptly reduced. The crystal environment may be more interesting in that laser waveform probes can experience both increasing and decreasing refractive indices due to the rapid electro-optic response of the material.

2. Electro-Optic Background

The Pockels effect is considered for which an anisotropic crystal is placed close to the electron beamline such that the electron bunch field and laser probe pulse co-propagate through the crystal. The reduction of the ordinary, $n_0(E_3)$ and extraordinary, $n_e(E_3)$ indices depends linearly on the time-dependent electron bunch field, $E_3(t)$ according to:

$$\begin{aligned} n_0(E_3) &\approx n_0 - \frac{1}{2} n_0^3 r_{13} E_3(t); \\ n_e(E_3) &\approx n_e - \frac{1}{2} n_e^3 r_{33} E_3(t) \end{aligned} \qquad (1)$$

where $r_{ijk} \equiv \dfrac{\partial \eta_{ij}}{\partial E_k}$ and by symmetry $r_{113} \equiv r_{13}$ and $r_{333} \equiv r_{33}$. For simplicity let the unperturbed indices be $n_0(0) = n_0$ and $n_e(0) = n_e$. The electric impermeability tensor is η_{ij} and the bunch field is oriented parallel to the '3' axis of the crystal. The ordinary and extraordinary indices refer here to the '1' and '3' principal crystal axes respectively. For a central (vacuum) wavelength, λ_o the phase retardation accumulated along a distance, L (parallel to the '2' axis of the crystal) is the sum of the following static and dynamic terms:

$$\Gamma(E_3) \equiv \Gamma_i + \Gamma_d \qquad (2)$$

where $\Gamma_i \equiv \dfrac{2\pi L}{\lambda_0}(n_0(0) - n_e(0))$

and $\Gamma_d \equiv \dfrac{2\pi L}{2\lambda_0} E_3 \left(n_e^3 r_{33} - n_0^3 r_{13}\right)$

The dynamic term, Γ_d imposes a time-derivative on the two indices corresponding to the '1' and '3' principal directions. To describe the dynamics we approximate the bunch field, $E_3(t)$ to have a Gaussian temporal profile as follows:

$$E_3(t) \equiv f E_{3\pi} \exp\left(-\left(\frac{t - t_0}{\alpha_{1,2}} \right)^2 \right) \qquad (3)$$

where $\Gamma_d = \pi$ for

$$E_3 = E_{3\pi} \equiv \frac{\lambda_0}{L}\left(\frac{1}{n_e^3 r_{33} - n_0^3 r_{13}}\right) \text{ and } f < 1$$

The time constants are α_1 and α_2 for the leading and trailing edges respectively, where $\alpha_2 > \alpha_1$.

Dynamic spectral shifts imposed on the probe waveform are the longitudinal accumulation (along the '2' axis) of the time-derivative of the refractive indices according to [4,5]:

$$\frac{-\Delta\nu}{\nu_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{1}{c}\int \frac{\partial n}{\partial t} dz; \qquad (4)$$

It is assumed that the dynamics do not vary along the distance, L and the refractive index, n refers to either the ordinary or extraordinary case. In what follows the laser probe pulse is considered adequately chirped such that, inside the crystal, any electron microbunch overlaps in time with some portion of this probe.

3.Specific Case for $LiTaO_3$ and $LiNbO_3$

The fractional spectral shifts and their corresponding maximum values for the '1' and '3' axes are determined to be:

$$\left(\frac{\Delta\nu}{\nu_0}\right)_1 = \frac{-L n_0^3 r_{13}(t-t_0)E_3(t)}{c\alpha_{1,2}^2};$$
$$\left(\frac{\Delta\nu}{\nu_0}\right)_{1,\max} = \pm\frac{f\lambda_0}{\alpha_{1,2}c\sqrt{2e}}\frac{1}{\left(\left(\left(\frac{n_e}{n_0}\right)^3\frac{r_{33}}{r_{13}}\right)-1\right)} \qquad (5)$$

and

$$\left(\frac{\Delta\nu}{\nu_0}\right)_3 = \frac{-Ln_e^{\ 3} r_{33}(t-t_0)E_3(t)}{c\alpha_{1,2}^{\ 2}};$$

$$\left(\frac{\Delta\nu}{\nu_0}\right)_{3,\max} = \pm\frac{f\lambda_0}{\alpha_{1,2}c\sqrt{2e}}\frac{1}{\left(1-\left(\left(\frac{n_0}{n_e}\right)^3\frac{r_{13}}{r_{33}}\right)\right)} \quad (6)$$

The upper and lower signs refer to the leading and trailing edges respectively of the bunch field where α_1 applies to the leading edge and α_2 applies to the trailing edge, c is the speed of light in vacuum, and $e \approx 2.718$.

These spectral modifications can be larger than the intrinsic bandwidth of the incident laser probe pulse. The intrinsic fractional bandwidth is expressed as:

$$\left(\frac{\Delta\nu}{\nu_0}\right)_{\text{int}} = \frac{0.44}{\nu_0(\Delta t_p)} \quad (7)$$

The time duration, Δt_p is the minimum probe pulse FWHM duration (based on the incident bandwidth available) and in the case of a Gaussian temporal profile with time constant, α_p we have the usual relationship, $\Delta t_p = \alpha_p\sqrt{4\ln 2}$. Similarly for the electron bunch field the exponential time constants are related to a FWHM according to $\Delta t_e = \alpha_{1,2}\sqrt{4\ln 2}$ (in the asymmetric case it is the HWHM that is determined from each time constant). The ratios, $R_{i,\max}$ (for i = 1,3) then represent comparisons of the optimum dynamically induced spectral bandwidth to that of the intrinsic, incident probe pulse for each crystal axis and are defined as:

$$R_{i,\max} \equiv \frac{\left(\frac{\Delta\nu}{\nu_0}\right)_{i,\max}}{\left(\frac{\Delta\nu}{\nu_0}\right)_{\text{int}}} \quad (8)$$

For $LiTaO_3$ and $LiNbO_3$ the ratios are:

$$R_{1,\max} = \pm\left(\frac{1.63\ f}{\left(\frac{n_e}{n_0}\right)^3 \frac{r_{33}}{r_{13}} - 1}\right)\left(\frac{\Delta t_p}{\Delta t_e}\right); \qquad (9)$$

$$R_{3,\max} = \pm\left(\frac{1.63\ f}{1 - \left(\frac{n_0}{n_e}\right)^3 \frac{r_{13}}{r_{33}}}\right)\left(\frac{\Delta t_p}{\Delta t_e}\right)$$

For the first factor in brackets, the denominator depends only on crystal parameters and the numerator on the peak bunch field (at $t = t_0$). The second factor in brackets represents a comparison between the dynamics of the laser probe pulse and that of the electron bunch field. Substituting tabulated values for the crystal parameters gives (for f =1):

for $LiTaO_3$:

$$R_{1,\max} \approx \pm 0.48\left(\frac{\Delta t_p}{\Delta t_e}\right)$$

and

$$R_{3,\max} \approx \pm 2.12\left(\frac{\Delta t_p}{\Delta t_e}\right) \qquad (10)$$

and for $LiNbO_3$

$$R_{1,\max} \approx \pm 0.75\left(\frac{\Delta t_p}{\Delta t_e}\right)$$

and

$$R_{3,\max} \approx \pm 2.39\left(\frac{\Delta t_p}{\Delta t_e}\right) \qquad (11)$$

4.Discussion

Spectral effects are significantly stronger for the extraordinary case ('3' axis). Furthermore, the magnitude of the bandwidths imposed by bunch field dynamics can exceed that of the incident laser pulse. This additional

bandwidth will alter the unique time-frequency correlation established by the original chirp on the laser waveform. From equations (10) and (11) these comparative dynamics indicate that, under the conditions of reference [3] and for the $LiTaO_3$ and $LiNbO_3$ crystals, the R_i ratios are at the few percent level only and not of major concern (although they can still limit the temporal resolution of the chirped probe technique).

The additional bandwidth would be significant if both Δt_p and Δt_e were comparable and of order tens of femtoseconds. Similar results have been experimentally measured in ultrafast laser-plasma studies. This dynamic electro-optic effect is a useful diagnostic if the spectra are observed with ultrashort temporal resolution. The FROG (frequency resolved optical gating) diagnostic is well suited for this and can provide resolution at the femtosecond level [4]. FROG provides a single shot, time-dependent spectrum (a two-dimensional FROG trace or spectrogram). Furthermore, with ultrashort time resolution a difference between the propagation velocities of the laser probe and the bunch field can become beneficial. The time 'slippage' between these pulses can be measured and renders leading and trailing edge dynamic effects separable. Leading and trailing edges of the bunch field add 'blue' and 'red' spectral shifts, respectively, to the laser probe. During the time interval over which slippage occurs (between leading and trailing edges) each part of the laser probe experiences both 'blue' and 'red' shifts and the resultant spectrum is more complicated.

A comparison of maximum spectral 'excursions' from the intrinsic probe spectrum for leading and trailing edges can provide a single shot evaluation of the bunch field asymmetry. This is independent of any assumed functional form for the temporal envelope. The single edge effect on a FROG trace provides a time-dependent measurement of the bunch field time-derivative. According to equation (9) for the Gaussian case the maximum spectral excursion for a single edge provides a measure of the ratio, $\frac{f}{\alpha_{1,2}}$ (other functions can be examined). The spectra are sensitive to the time-derivative of the field, so ever-present wakefield effects may be isolated dynamically and temporally. Conventional birefringence can be cancelled by orienting the linear polarization of the laser probe parallel to one of the crystal axes. In this case the dynamic effects are isolated from phase retardation effects. In the general case, dynamics are combined with phase retardation. Interesting results can also be obtained for the case where the laser probe pulse and the electron bunch field counter-propagate through the crystal. Continued examination of these detailed effects (including other crystal examples) will be the subject of a subsequent report.

5. References

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