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Multi-bunch Longitudinal Instability due to the Electron Cloud

A. Novokhatski
SLAC, Stanford, CA 94025, USA

The mechanism of longitudinal interaction of high-energy positrons of a bunch train and secondary emitted particles of electron cloud is analyzed. We consider the case when electron cloud in solenoidal magnetic field is built mainly due to multipacting process. Positron bunches have to use some amount of their kinetic energy to give electron cloud some temperature. The field, which is responsible for the energy transformation, is longitudinal electric field. The amplitude of electric field depends on the density of electron cloud and varies with bunch spacing that leads to a resonant multi-bunch instability. Also longitudinal field acts as an oscillating force on the cloud electrons and gives, at the same time, an additional energy variation inside the positron bunches. The head of the positron bunch is accelerated and the tail is decelerated. This action of the longitudinal field is similar to the action of RF fields in a cavity. As a result the positron bunches will have different length and synchrotron frequency throughout the train.

1. INTRODUCTION

Electron cloud is usually associated with the transverse coupled bunch instability that arises in positron storage ring [1]. Many experimental studies and numerical simulations are devoted to this physical phenomenon. Variety of interesting results can be found in proceedings of Mini-Workshop on Electron-Cloud Simulations for Proton and Positron Beams (ECLOUD 02) [2]. Experimental study at PEP-II low energy positron ring showed that electron cloud is built up from multipacting electrons. Placing the solenoid magnetic fields successfully reduced multipacting and damped the electron cloud instability [3]. Simulating study showed that under some condition the multipacting process has a resonance character at larger solenoidal magnetic field [4]-[5]. However electron cloud interacts with a positron bunch not only in transverse direction. Figure1 shows the space distribution of secondary electrons emitted from the walls of a round vacuum chamber. Acceleration in transverse direction by the fields of a sequence of positron bunches produce longitudinal space variations of electron cloud. These variations of the charge density are the source for the electric field with transverse and longitudinal components. Longitudinal component of electric field decelerates positron bunch as a hole and gives additional energy spread in the bunch, as the head of a bunch is accelerated and the tail is decelerated. These effects may be the reasons for additional bunch lengthening and longitudinal multi-bunch resonance instability. In the steady state regime of periodical series of positron bunches the amplitude of the longitudinal electric field is established when the energy losses of positron bunches compensate the electron cloud energy spent for the bombarding the wall. In this way the vacuum chamber gets additional heating to synchrotron radiation heating from the positron beam.

In the PEP-II positron ring solenoids are wrapped around almost all components of the vacuum chamber. So we will study electron cloud in presence of solenoidal magnetic field. Like in references [4]-[5] we will use numerical solution of the Vlasov equations for electron cloud description and the model of ensemble [6] for computer simulation of the beam dynamics.

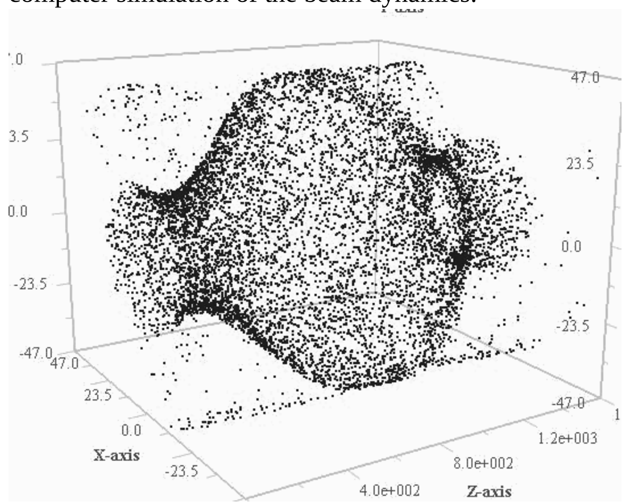

Figure 1: Space distribution of secondary electrons emitted from the walls of a round vacuum chamber and accelerated in radial direction by a positron bunch.

2. ELECTRON CLOUD AND LONGITUDINAL ELECTRIC FIELD

2.1. Multipacting and resonance conditions in solenoid field.

We consider electron cloud to be built mainly due to multipacting process and its density is limited by the space charge field. This requires that a positron bunch current is large enough to accelerate the secondary emitted electrons up to the energy when the secondary emission yield exceeds one. While studying the behavior of the electron cloud for different solenoidal fields we found a strong resonance [4]. This resonance happens when the time interval between the positron bunches is equal to the flight time of the secondary particles back to wall. The flight time is mainly determined by the solenoidal field H and partially by the cloud size and intensity. Naturally the presence of a resonance depends strongly upon the secondary emission function. The resonance is the boundary between completely different behavior of the electron cloud. Multipacting happens when the flight time is a

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little bit smaller than the positron time interval; when the solenoidal field is a little bit higher than the resonance field $H > H_{res}$. And there is no multipacting if $H < H_{res}$. A resonance can also happen if the flight time of the secondary particle is equal to an integer number of time intervals between positron bunches. Fig.2 shows the saturated values of the electron cloud as a function of the solenoidal field and growth/damping rates for different positron bunch patterns for parameters of PEP-II low energy ring, and secondary emission function presented in reference [4].

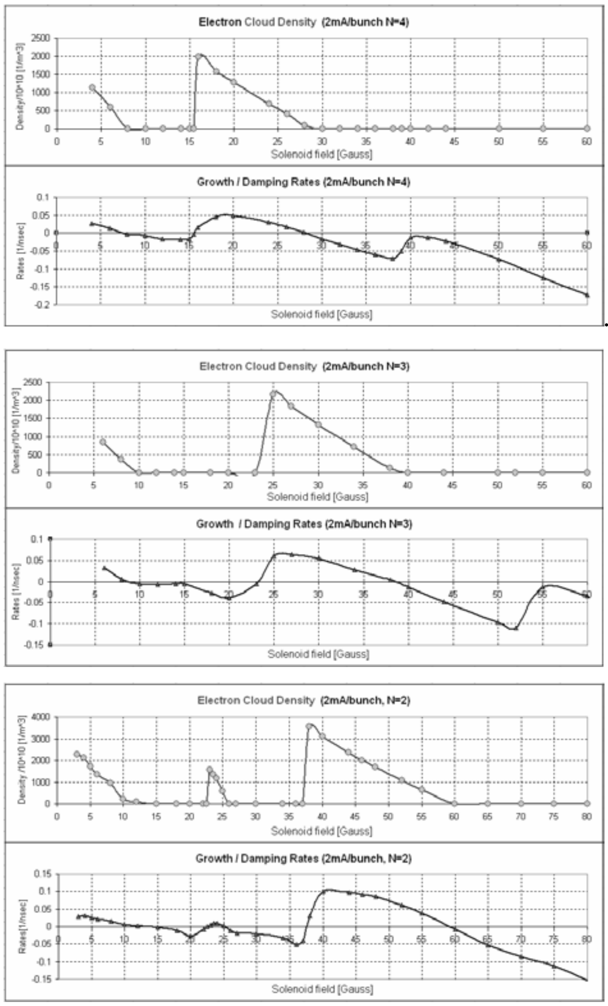

Figure 2: Saturated density of electron cloud and growth/damping rate for a positron train with a bunch pattern by 4, 3 and 2 RF buckets.

2.2. Electric field.

For the case of periodical series of positron bunches the formulas for electromagnetic fields can be easily derived from the Maxwell equations. In the round vacuum chamber longitudinal component of electric field $E_z(r,\tau)$ and radial current of electron cloud $j_r(\tau,r)$ are connected by very simple formula [4]

$$E_z(r,\tau) = Z_0 * \int_r^a j_r(r',\tau)dr'$$

where $\tau = t - z/c$, $Z_0 = 120\pi$ Ohm is the impedance of free space, a is the radius of the vacuum chamber. The sign of electric field is different when particles of electron cloud are moving to the center of vacuum chamber or moving back to the walls. This is demonstrated at Fig.3, where electric field (red line with circles) and average radial momentum (blue line) of electron cloud are shown. Also the positron bunch shape is presented there by light blue line. . It is worth noting that positron bunch not only loses energy but also gets additional energy spread. Thus we need two main parameters to describe the action of electric field on positron bunch. One is the bunch energy loss or mean value of electric field $< E >$ in the bunch. Another parameter is the gradient of electric field $< G >$ at bunch center, which determines the variation of energy in a positron bunch. We will use these parameters later in the beam dynamics simulations. In presented example we show the behavior of electron cloud at the main resonance (H =38 Gauss) for the bunch pattern spacing by 2 RF buckets.

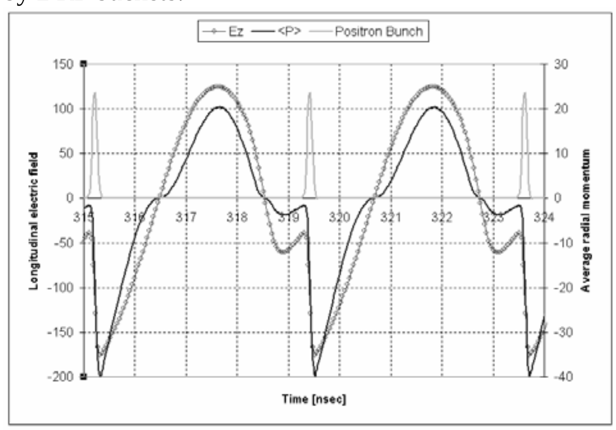

Figure 3: Longitudinal electric field, average radial momentums of electron cloud and positron bunch shape.

Another example, presented in Fig.4 - Fig.6 shows the case of second resonance condition also for bunch pattern spacing by 2. Average electron cloud density (green line) and positron bunch shape (blue line) are shown in the left upper plot. Electron cloud distribution on the phase plane of radius and radial momentum is shown in the left down plot. Particles travel from chamber axis to the wall in the upper half part of the plot and from the wall to the axis in the lower half part. Radial momentum is measured in equivalent of electron volts. The radial distribution is shown in the right down plot. Blue line shows momentum distribution in the right upper plot. Green line at this plot shows particle distribution at the wall. Fig.4 shows electron cloud distribution before a positron bunch to arrive. At this moment main part of electron cloud is returning to the wall (right peak in momentum distribution). High-energy particles produce secondary electrons, which have small energy (left peak in momentum distribution) and are located near the wall (right high peak in radial distribution). Electron cloud density achieves maximum value at this moment. Figure 5 shows the moment when a positron bunch has arrived. Positron bunch length at the PEP-II low-energy ring is very short (σ ~ 13 mm) in comparison with equivalent Larmor wavelength. Thus electrons do not move, but only get momentum kick from a positron bunch, as it can be seen in the figure 5.

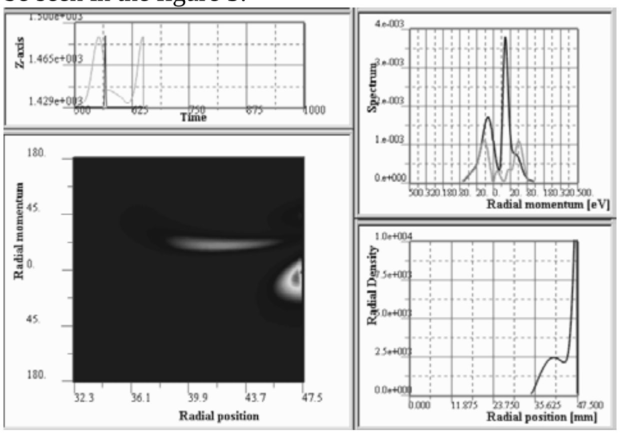

Figure 4: Electron cloud distribution before a positron bunch to arrive.

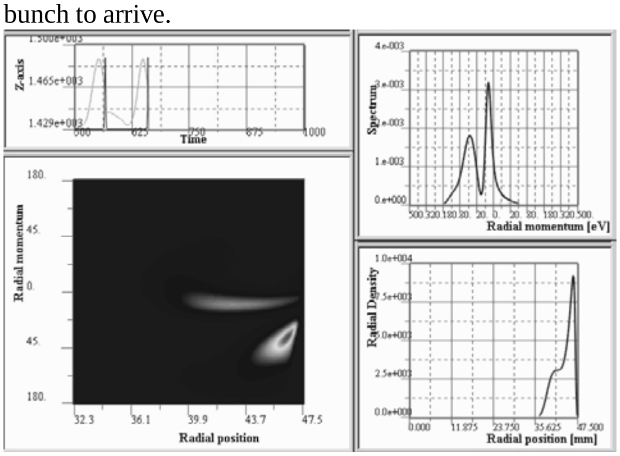

Figure 5: Positron bunch has just arrived and has given a radial kick to secondary electrons.

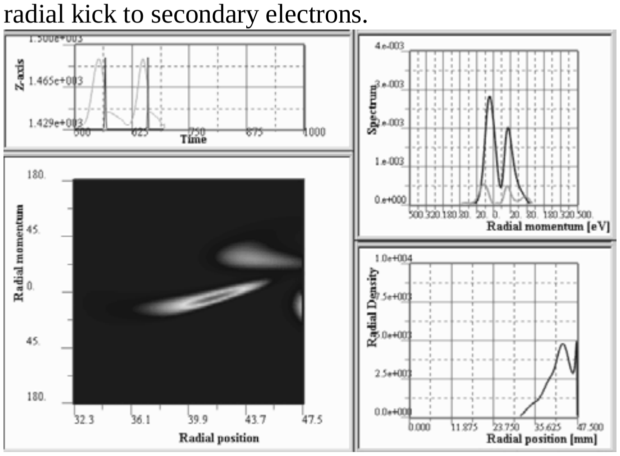

Figure 6: Positron bunch went away. Secondary electrons are rotated by magnetic field back to the wall.

Fig.6 shows the moment when positron bunch flew away and magnetic field rotates electrons back to the wall.

The action of longitudinal field to a positron bunch has the same behavior as in the case of the main resonance.

2.3. Green function.

Electric field, as well as its main parameters $<E>$ and $<G>$, strongly depend upon the time interval between positron bunches. Fig.7 shows the change of electric field with progressive surpasses of arrival of positron bunches. We choose here the conditions when magnetic field is higher than it is needed for the main resonance to demonstrate the effect. At the beginning the electric field increases, then gets maximum value near the main resonance and after goes down.

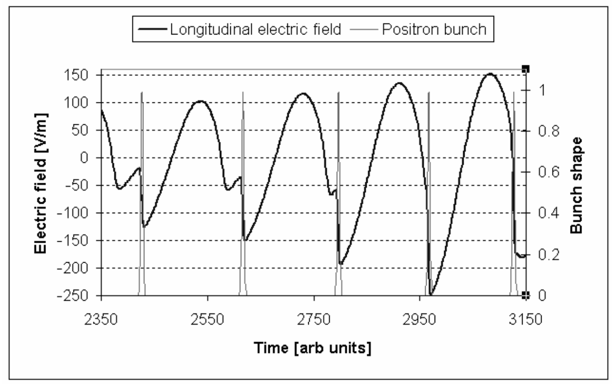

Figure 7: The change of electric field with progressive surpasses of arrival of positron bunches.

Main parameters of electric field $<E>$ and $<G>$ are shown in Fig.8 as triangle blue dots and circle red dots. For convenience, the gradient $<G>$ is multiplied by RF wavelength λ_{RF}. Distance between bunches is described by RF phase.

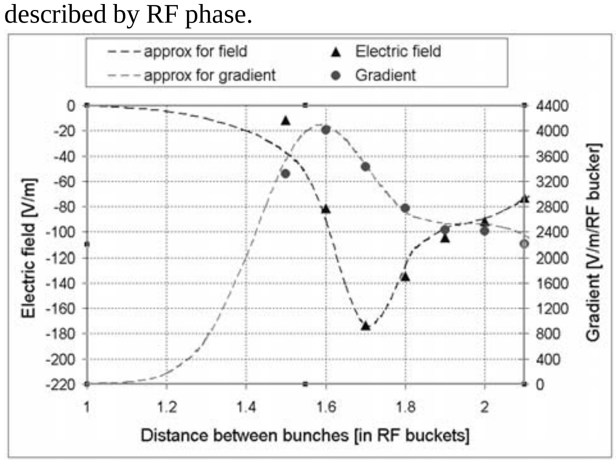

Figure 8: Mean value and gradient of electric field as functions of distance between two bunches (calculated results and approximations)

Arrival time of other previous bunches can also change the electric field because of changing the electron cloud density. In principle we can present main parameters for a bunch train over Green functions Ψ_k and Υ_k of RF phases of all previous bunches φ_{l-k}

$$< E >_l = \sum_{k=1,2,...} \Psi_k(\varphi_l - \varphi_{l-k})$$

$$< G >_l = \sum_{k=1,2,...} \Upsilon_k(\varphi_l - \varphi_{l-k})$$

However Fig. 8 shows that the main effect is coming from the first term, because small changes in RF phase produce tremendous change of electric field. So we will use only first term as Green function for beam dynamics simulations

$$< E >_l = \Psi_1(\varphi_l - \varphi_{l-1}) \quad < G >_l = \Upsilon_1(\varphi_l - \varphi_{l-1}) .$$

Based on calculated results we found good approximation by Gaussian functions with several numerical parameters:

$$\Psi_1(x) = \alpha_1 \,\mathrm{e}^{-\left(\frac{x-x_1}{\delta_1}\right)^2} + \alpha_2 \,\mathrm{e}^{-\left(\frac{x-x_2}{\delta_2}\right)^2}$$

$$\Upsilon_1(x) = \beta_1 \,\mathrm{e}^{-\left(\frac{x-\chi_1}{\zeta_1}\right)^2} + \beta_2 \,\mathrm{e}^{-\left(\frac{x-\chi_2}{\zeta_2}\right)^2}$$

These approximations are shown at Fig.8.

3. BEAM DYNAMICS

3.1. Gradient and bunch length

To see what the longitudinal effect of electron cloud field can be, we present gradient $< G >$ in units of equivalent RF voltage of wavelength λ_{RF} =63 cm

$$V_{cloud} =< G > \frac{\lambda_{RF}}{2\pi} * L$$

If we assume that electron cloud fills entire ring of circumference L =2.2k m, then equivalent voltage will be 875 kV for positron bunch pattern spacing by 2. We can also give an analytical estimation of equivalent voltage [3]

$$V_{cloud} = \frac{m_e c^2}{\pi * \mathrm{N}} * \frac{L}{\sigma} * \left(\frac{Z_0 I^{+}_{bunch}}{m_e c f_{rev} 2\pi a} \right)^2$$

where I^{+}_{bunch} is the positron current per one bunch, m_e is the mass of an electron. c is the speed of light, f_{rev} is the revolution frequency of particle, σ is the positron bunch length, N is the spacing number. These parameters for the PEP-II low energy ring are: f_{rev} =136kHz, λ_{RF} =63 cm, a =44.5mm, I^{+}_{bunch} =2mA, N=2. This formula gives 930 kV, which is in good agreement with computer result. Cavity voltage at low energy ring is currently 3.2 MV, so electron cloud can effectively increase the voltage by 28% and decrease the bunch length by 14%.

Because of the ion gap in the positron beam the action of electron cloud is different for the first and the last bunch. Fig. 9 shows results of the beam dynamics simulations for the length of the last bunch (pink curve). Starting with oscillations the bunch length comes to smaller value after approximately 2 damping times (one damping time is 4075 turns). For comparison the length of the first bunch is shown as well (blue line). Green curve shows the case when electron cloud fills only half of the ring.

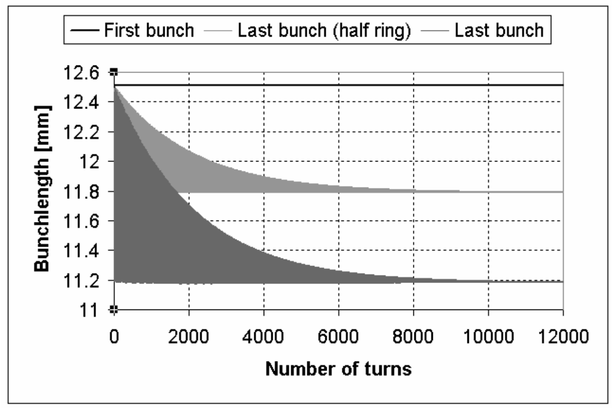

Figure 9: Dynamics of the bunch length due to the action of electron cloud.

When electron cloud changes the bunch length it changes the synchrotron frequency too. Thus, the positron beam with an ion gap will get synchrotron frequency spread.

3.2. Synchrotron oscillations and resonance instability

As it was shown in previous chapter electric field of electron cloud depends on the bunch phase difference. If a previous bunch has longitudinal oscillations then next bunch sees the oscillations of electric field. It means that the previous bunch acts as a resonance force for the next bunch, as they have the same synchrotron frequency. Such mechanism leads to a multi-bunch instability. According to Fig. 8 electric field as a function of bunch phases can have a negative and positive slope. Because of the resonance behavior, instability will happen in both cases.

Equations, which we used for the simulation of the longitudinal motion, are:

$$\frac{\Delta}{\Delta n}\varphi_l = -2\pi h \alpha \frac{p_l}{P_0}$$

$$\frac{\Delta}{\Delta n} p_l = V_{RF} + < E >_l L - \frac{2}{n_D} p_l$$

$$\frac{\Delta}{\Delta n}\sigma_l^2 = -2\frac{\alpha c}{f_{rev} P_0}\eta_l^2$$

$$\frac{\Delta}{\Delta n}\varepsilon_l^2 = 2(V'_{RF} + < G >_l L)\eta_n^2 - \frac{4}{n_D}\varepsilon_l^2$$

$$\frac{\Delta}{\Delta n}\eta_l^2 = (V'_{RF} + < G >_l L)\sigma_l^2 - \frac{\alpha c}{f_{rev} P_0}\varepsilon_l^2 - \frac{2}{n_D}\eta_l^2$$

In this equations positron bunch is described by its number in the bunch train l, RF phase φ_l, momentum p_l, bunch length σ_l, energy spread ε_l and energy-length correlation $\sqrt{\eta_l}$. Ring is described by nominal momentum P_0, momentum compaction α, RF voltage V_{RF}, harmonic number h and damping parameter n_D.

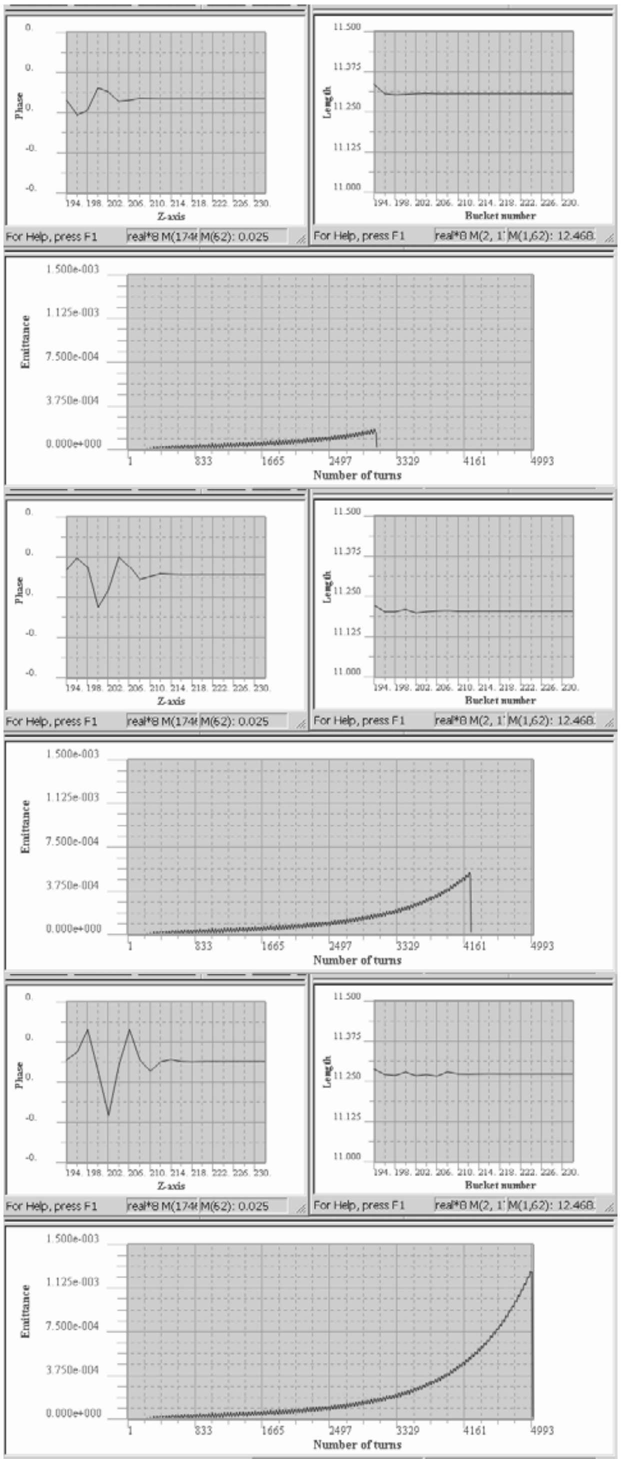

Figure 10: Bunch phases, lengths and longitudinal emittance for consecutive time periods.

Results of simulations are given in Fig. 10. RF phase along the bunch train is shown in the left upper plot, bunch length along the train is shown in the right upper plot and beam emittance as a function of time is shown in the down plot. Beam emittance growth time is 3.5msec. The result for the case when only half of the ring is filled by electron cloud is presented at Fig.11. Beam emittance growth time is 14 msec in this case.

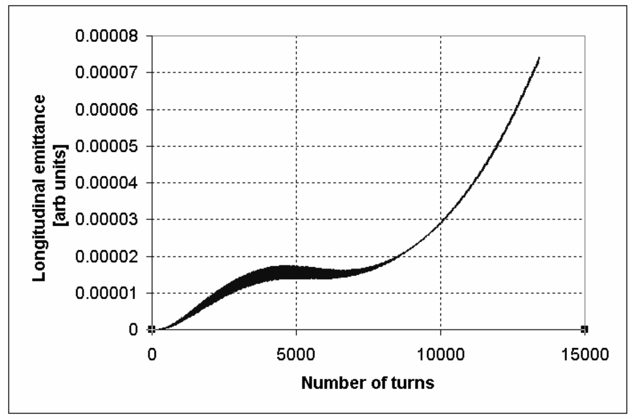

Figure 11: Beam emittance growth for the case when electron cloud fills only half of the ring.

4. CONCLUSIONS

- Positron bunches have to lose some of their kinetic energy in order to build the electron cloud.
- The field that is responsible for the energy transformation is the longitudinal electric field
- When the cloud is already built, this longitudinal field acts as an oscillating force on the cloud electrons and gives, at the same time, an additional energy variation inside the positron bunches. The head of the positron bunch is accelerated and the tail is decelerated. This action of the longitudinal field is similar to the action of RF fields in a cavity and it has the same sign. As a result the positron bunches will have different length and synchrotron frequency throughout the train.
- Electron cloud space-charge field acts as a resonance force for the longitudinal bunch motion that leads to the longitudinal multi-bunch instability with beam emittance growth.

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