SLAC-PUB-10269
December 19, 2003

Quantized Cosmology II: de Sitter Space*

Marvin Weinstein and Ratin Akhoury†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Dated: December 19, 2003)

This work applies the formalism developed in our earlier paper[1] to de Sitter space. After exactly solving the relevant Heisenberg equations of motion we give a detailed discussion of the subtleties associated with defining physical states and the emergence of the classical theory. This computation provides the striking result that quantum corrections to this long wavelength limit of gravity eliminate the problem of the *big crunch*. We also show that the same corrections lead to possibly measureable effects on the CMB radiation. Finally, for the sake of completeness we discuss the special case, $\Lambda = 0$, and its relation to Minkowski space.

PACS numbers: F06.60.Ds, 98.80.Hw, 98.80.Cq

I. INTRODUCTION

In our previous paper[1] we presented a quantum mechanical formalism for the part of the computation of the anisotropy in the CMB radiation[2] which is usually treated purely classically. In this paper we apply this formalism to the case of de Sitter space. There are two reasons for doing this: first, the problem is interesting in its own right; second, it is exactly solvable and the solution clarifies subtle features of the discussion given in our first paper.

Our earlier discussion began by assuming the usual Friedmann-Robertson-Walker metric in homogeneous isotropic coordinates,

$$ds^2 = -dt^2 + a(t)^2 d\vec{x} \cdot d\vec{x}, \tag{1}$$

and an action of the form

$$\mathcal{S} = \mathbf{V}\left[-\frac{3}{\kappa^2}a(t)\left(\frac{da(t)}{dt}\right)^2 + \frac{1}{2}a(t)^3\left(\frac{d\Phi(t)}{dt}\right)^2 - a(t)^3 V(\Phi(t))\right]. \tag{2}$$

We then introduced the change of variables $u(t)^2 = a(t)^3$ and rewrote the action in the simpler form

$$\mathcal{S} = \mathbf{V}\left[-\frac{4}{3\kappa^2}\left(\frac{du(t)}{dt}\right)^2 + \frac{1}{2}u(t)^2\left(\frac{d\Phi(t)}{dt}\right)^2 - u(t)^2 V(\Phi(t))\right]. \tag{3}$$

The rest of our discussion followed from canonically quantizing this theory and seeing how much of the Einstein equations could be recovered at the level of the Heisenberg equations of motion.

This paper follows the same steps, but for an action in which $V(\Phi)$ is replaced by a cosmological constant Λ; i.e.,

$$\mathcal{S} = \mathbf{V}\left[-\frac{4}{3\kappa^2}\left(\frac{du(t)}{dt}\right)^2 - u(t)^2\Lambda\right]. \tag{4}$$

II. SOLVING THE HEISENBERG EQUATIONS OF MOTION

Direct commutation of the Hamiltonian with the operators $u(t)$ and $p_u(t)$ yields the Heisenberg equations of motion

$$\mathbf{H} = -\frac{3\kappa^2}{16\mathbf{V}}p_u^2 + \mathbf{V}u^2\Lambda \tag{5}$$

---

* This work was supported by the U. S. DOE, Contract No. DE-AC03-76SF00515.
† On sabbatical leave from Dept. of Physics, University of Michigan, Ann Arbor, MI 48109-1120

and the Hamilton equations of motion for $u(t)$ and $p_u(t)$ are

$$\frac{du(t)}{dt} = -\frac{3\kappa^2}{8\mathbf{V}} p_u \,;\; \frac{d^2u(t)}{dt^2} = \frac{3\kappa^2\Lambda}{4} u. \tag{6}$$

The exact solutions to these equations, written in terms of the operators $u(t=0) = u$ and $p_u(t=0) = p_u$ are

$$\begin{aligned} u(t) &= \cosh(\omega t) u - \frac{3\kappa^2}{8\mathbf{V}\omega} \sinh(\omega t) p_u \\ p_u(t) &= \cosh(\omega t) p_u - \frac{8\mathbf{V}\omega}{3\kappa^2} \sinh(\omega t) u, \end{aligned} \tag{7}$$

where we have defined

$$\omega = \sqrt{\frac{3\kappa^2\Lambda}{4}}. \tag{8}$$

It is convenient to rewrite Eq.7 in terms of exponentials; i.e.,

$$u(t) = \frac{e^{\omega t}}{2}\left(u - \frac{3\kappa^2}{8\mathbf{V}\omega} p_u\right) + \frac{3\kappa^2 e^{-\omega t}}{16\mathbf{V}\omega}\left(p_u + \frac{8\mathbf{V}\omega}{3\kappa^2} u\right) \tag{9}$$

and to introduce the canonically conjugate asymptotic operators

$$u_\infty = \frac{1}{\sqrt{2}}\left(u - \frac{3\kappa^2}{8\mathbf{V}\omega} p_u\right) \,;\; p_\infty = \frac{1}{\sqrt{2}}\left(p_u + \frac{8\mathbf{V}\omega}{3\kappa^2} u\right). \tag{10}$$

In terms of these operators the solution for the operator $u(t)$ and the Hamiltonian take the simple forms

$$u(t) = \frac{1}{\sqrt{2}} e^{\omega t} u_\infty + \frac{1}{\sqrt{2}} \frac{3\kappa^2}{8\mathbf{V}\omega} e^{-\omega t} p_\infty, \tag{11}$$

and

$$\mathbf{H} = \frac{\sqrt{3\Lambda}\kappa}{4}\left(u_\infty p_\infty + p_\infty u_\infty\right). \tag{12}$$

From this point on all of the technical work is finished, the only chore which remains is to extract the physical significance of these results.

III. THE MISSING FRIEDMANN EQUATION: DEFINING PHYSICAL STATES

Before discussing the content of this solution, we must spend a few moments defining the space of physical states. This question comes up because, as we pointed out in our previous paper, as a consequence of working in a fixed coordinate system, we don't obtain all of the Einstein equations as Heisenberg equations of motion. We showed that in the classical theory, the missing equations could be imposed as constraints; since, as a consequence of the equations of motion which we do have, it is possible to prove that if they are satisfied at any one time, then they are always satisfied. Next, we showed that in the quantum theory we could parallel the classical discussion and define a one parameter family of operators, $\mathbf{G}_\alpha$, each of which satisfies an equation of the form

$$\frac{3u}{4}\left(\frac{1}{\mathbf{A}_\alpha}\frac{d\mathbf{G}_\alpha}{dt} + 3\mathbf{G}_\alpha\right) = 0. \tag{13}$$

for some non-vanishing operator $\mathbf{A}_\alpha$ (where $0 \le \alpha \le 1$). Finally, we argued that, in contrast to the classical situation, it doesn't make any sense to define the space of physical states by the strong condition $G_\alpha(t)|\Psi\rangle = 0$, for some value of α. Instead, we stated that the correct condition is that $|\Psi\rangle$ is physical if and only if

$$\lim_{u(t)^2\to\infty} \mathbf{G}(t)|\Psi\rangle = 0. \tag{14}$$

The nice thing about the example with just a cosmological constant is that we can easily understand why we say that Eq.14 is the best one can do.

Adopting the same definition of the Hubble parameter obtained in our earlier paper

$$\mathcal{H} = -\frac{\kappa^2}{8\mathbf{V}}\left(p_u \frac{1}{u} + \frac{1}{u^3} p_u u(t)^2\right) \tag{15}$$

and defining the operator $\mathbf{Q}$ to be

$$\mathbf{Q} = -\frac{3\kappa^4}{64\mathbf{V}^2 u^4}, \tag{16}$$

it is a straightforward exercise in taking commutators to show that the one parameter family of operators $\mathbf{G}_\alpha$ can be written as

$$\begin{aligned} \mathbf{G}_\alpha &= \mathcal{H}^2 + \alpha\mathbf{Q} - \frac{\kappa^2}{3}\Lambda \\ &= \frac{\kappa^4}{16\mathbf{V}^2 u^2} p_u^2 + (1-\alpha)\frac{3\kappa^4}{64\mathbf{V}^2 u^4} - \frac{\kappa^2}{3}\Lambda \\ &= -\frac{\kappa^2}{3\mathbf{V}u^2}\mathbf{H} + (1-\alpha)\frac{3\kappa^4}{64\mathbf{V}^2 u^4}. \end{aligned} \tag{17}$$

Noting that the Hamiltonian, $\mathbf{H}$, is time independent we have

$$\mathbf{G}_\alpha(t) = -\frac{\kappa^2}{3\mathbf{V}u(t)^2}\left[\mathbf{H} - (1-\alpha)\frac{9\kappa^2}{64\mathbf{V}u(t)^2}\right] \tag{18}$$

Thus, we see that for the case $\alpha = 1$, defining the space of physical states by the condition $\mathbf{G}_1|\Psi\rangle = 0$ is equivalent to the Wheeler-Dewitt equation; i.e., $\mathbf{H}|\Psi\rangle = 0$. Unfortunately, the statement that the Hamiltonian is zero on this subspace of states means they don't evolve. However, this is in direct conflict with the Heisenberg equations of motion (Eq.7), which is, of course, unacceptable.

On the other hand, if we choose another value of α, to avoid an immediate contradiction, we still run into trouble. This is because we can explicitly solve for such states by using the explicit form of $\mathbf{G}_\alpha$ in Eq.17, to rewrite the $\mathbf{G}_\alpha\Psi(u) = 0$ as a differential equation in u. The result of this computation is that the equation has no square integrable solutions. It therefore follows that there are no satisfactory candidates for physical states which satisfy this strong form of the constraint. In contrast, given the exact solution for $u(t)$, we see that any state for which $\mathbf{H}|\Psi\rangle$ has a finite norm will, for sufficiently large $|t|$, satisfy Eq.14 to arbitrary accuracy. More precisely, any state $|\Psi\rangle$ such that

$$\langle\Psi|\mathbf{H}^2|\Psi\rangle < \infty \tag{19}$$

will satisfy the asymptotic condition

$$\lim_{t\to\pm\infty} \mathbf{G}(t)|\Psi\rangle = 0 \tag{20}$$

This means that any Gaussian wave packet in u_∞ will be a physical state. It also means that for large times all the physics measured in such a state will be compatible with the full set of Einstein equations. In the next section we explicitly demonstrate this fact.

IV. SEEING THE CLASSICAL THEORY EMERGE

Now that we have defined the space of physical states, we turn to a discussion of the only two physical observables in this theory; the expansion rate and the volume of the universe. In what follows we call an allowed quantum state a *quantum observer*. What we wish to ascertain is to what degree the value of each of these observables depends upon the *quantum observer*. Obviously, the exact solution given in Eq.11 shows that at large times the expansion rate is attached to the scale factor and is totally independent of the observer, however this is not true of the volume. Thus, in the remainder of this section we will discuss the degree to which the measured properties of the volume operator differ from quantum observer to quantum observer.

Since we started off quantizing in a volume with coordinate size $\mathbf{V}$, the volume of the universe at any time is given by

$$V(t) = \mathbf{V}u(t)^2$$

$$= \frac{\mathbf{V}}{2}\left[e^{2\omega t}u_\infty^2 + \left(\frac{3\kappa^2}{8\mathbf{V}\omega}\right)^2 e^{-2\omega t}p_\infty^2 + \frac{3\kappa^2}{8\mathbf{V}\omega}\left(u_\infty p_\infty + p_\infty u_\infty\right)\right]. \tag{21}$$

A surprising feature of this formula is that for large times in the past and future the volume operator $V(t)$ behaves classically. By the phrase $V(t)$ *behaves classically*, we mean that if one measures $V(t)$ at some early or late time t_1 and obtain a definite value, then we will be able to predict the value we will obtain if we measure $V(t)$ at some later time t_2. A cursory examination of Eq.21 shows that for very large positive times $V(t)$ is, to arbitrarily high accuracy, proportional to the single operator u_∞^2 (at large negative times it is proportional to p_∞^2). Thus, for example, we see that a measurement of $V(t_1)$, for sufficiently large t_1, corresponds to a measurement of u_∞^2, which means that we know $V(t)$ for all times $t_2 > t_1$.

From the fact that u_∞ and p_∞ are canonically conjugate variables we see that if we were to try and identify a quantum observer with an eigenstate of p_∞, then the volume operator would be well determined in the past, but completely undetermined in the future. Conversely, eigenstates of u_∞ correspond to states for which the volume operator is completely well determined in the future, but completely undetermined in the past. Fortunately, the condition that physical states must be normalizeable states for which Eq.19 holds, tells us that we cannot identify such states with quantum observers. States which can be identified with quantum observers are Gaussian packets,

$$|\Psi\rangle = e^{-\frac{\gamma}{2}u_\infty^2} \tag{22}$$

and the coherent states, $|u_0, p_0, \gamma\rangle$, obtained from them. These coherent states are defined by

$$|u_0, p_0, \gamma\rangle = e^{ip_0 u_\infty}\, e^{-iu_0 p_\infty}|\Psi\rangle, \tag{23}$$

and the expectation values of u_∞ and p_∞ in these states are given by

$$\langle u_0, p_0, \gamma|u_\infty|u_0, p_0, \gamma\rangle = u_0,\ \langle u_0, p_0, \gamma|p_\infty|u_0, p_0, \gamma\rangle = p_0. \tag{24}$$

Moreover, the relevant products of these operators have the values

$$\begin{aligned}
\langle u_0, p_0, \gamma|u_\infty^2|u_0, p_0, \gamma\rangle &= u_0^2 + \frac{1}{2\gamma},\\
\langle u_0, p_0, \gamma|p_\infty^2|u_0, p_0, \gamma\rangle &= p_0^2 + \frac{\gamma}{2},\\
\langle u_0, p_0, \gamma|u_\infty p_\infty + p_\infty u_\infty|u_0, p_0, \gamma\rangle &= 2\Re(\langle u_\infty\, p_\infty\rangle) = 2\, u_0 p_0.
\end{aligned}$$

The nice thing about such coherent states is that they are the kind of states we would expect to obtain if, in the past, we make a measurement which determines $V(-t)$ to have a central value $\frac{\mathbf{V}}{2}\, e^{\omega|t|}p_0^2$, with a width parameterized by γ. For this same packet, measurements of $V(t)$ in the distant future will produce results centered about the value $\frac{\mathbf{V}}{2}\, e^{\omega|t|}u_0^2$, with a width parameterized by $1/\gamma$.

V. EQUIVALENCE CLASSES OF OBSERVERS

From this point on we will restrict the term quantum observer to mean a coherent state of the form defined above. What we wish to discuss next is the fact that many of these observers are equivalent to one another in a way which we will make precise. Begin by considering

$$\langle V(t)\rangle = \langle u_0, p_0, \gamma|V(t)|u_0, p_0, \gamma\rangle = \frac{\mathbf{V}}{2}\left[e^{2\omega t}\left\langle u_\infty^2\right\rangle + \left(\frac{3\kappa^2}{8\mathbf{V}\omega}\right)^2 e^{-2\omega t}\left\langle p_\infty^2\right\rangle + \frac{3\kappa^2}{8\mathbf{V}\omega}\left(2\Re(\langle u_\infty p_\infty\rangle)\right)\right]. \tag{25}$$

It is obvious from Eq.25 that at large times the volume behaves as a single exponential, as expected from the solution of the classical Einstein equations. More interesting, however, is the fact that letting $t \to t + t_0$, where t_0 is defined by the condition

$$e^{2\omega t_0} = \frac{3\kappa^2}{8\mathbf{V}\omega}\sqrt{\frac{\langle p_\infty^2\rangle}{\langle u_\infty^2\rangle}}, \tag{26}$$

allows us to rewrite Eq.25 as

$$\begin{aligned}
\langle V(t)\rangle &= \frac{3\kappa^2\sqrt{\langle u_\infty^2\rangle\,\langle p_\infty^2\rangle}}{8\omega}\left[\cosh(\omega t) + \frac{\Re(\langle u_\infty p_\infty\rangle)}{\sqrt{\langle u_\infty^2\rangle\,\langle p_\infty^2\rangle}}\right]\\
&= \frac{\kappa^2\sqrt{\langle u_\infty^2\rangle\,\langle p_\infty^2\rangle}}{4\mathcal{H}}\left[\cosh(\omega t) + \frac{\Re(\langle u_\infty p_\infty\rangle)}{\sqrt{\langle u_\infty^2\rangle\,\langle p_\infty^2\rangle}}\right]
\end{aligned} \tag{27}$$

Thus, we see $\langle V(t)\rangle$ corresponds to a system which is contracting at large times in the past and which then bounces and begins to re-expand in the future. During most of this history the system satisfies the Friedmann equation to high accuracy and expands (or contracts) with a Hubble constant equal to

$$\mathcal{H} = \frac{2}{3}\omega = \sqrt{\frac{\kappa^2\Lambda}{3}}. \tag{28}$$

However, there is a period in time where the quantum corrections to the Friedmann equation dominate the behavior; namely, at times $t \approx 1/\omega$. Assuming, for the sake of argument, that were to set $1/\kappa\mathcal{H} \approx 10^3$, as it is in many models of slow roll inflation, and assuming $\sqrt{\langle u_\infty^2\rangle\langle p_\infty^2\rangle}$ to be of order unity, then the minimum volume of the universe at the time of the bounce is on the order of 10^3 Hubble volumes; i.e., on the order to 10 Planck-lengths in each dimension. This sets the order of magnitude of the scale at which the quantum corrections become important. It is gratifying that these quantum corrections keep the system from contracting forever and ending in a *big crunch*.

Another very interesting feature of Eq.27 is that it is characterized by only two numbers, $\sqrt{\langle u_\infty^2\rangle\langle p_\infty^2\rangle}$ and $\Re\langle u_\infty p_\infty\rangle/\sqrt{\langle u_\infty^2\rangle\langle p_\infty^2\rangle}$. The first number is unrestricted in magnitude and roughly determines the physical volume of the universe at the time of the bounce. The second number, is constrained by the Schwarz inequality to lie between plus and minus one, and parameterizes the degree to which the behavior of the system during the time of the bounce deviates from a pure hyperbolic cosine. If the time over which the deviation takes place is characterized by $1/\omega \approx 1/\mathcal{H}$, then the minimum size to which the system contracts is characterized by the ratio of the energy density in the state to the cosmological constant. This statement follows from taking the expectation value of the Hamiltonian as written in Eq.12, which implies

$$\Re(\langle u_\infty p_\infty\rangle) = \frac{2}{\kappa\sqrt{3\Lambda}}\langle \mathbf{H}\rangle. \tag{29}$$

Note, it would appear from the Schwarz inequality that in principle one could have an observer for whom the universe actually shrinks to zero size before it bounces. Fortunately it is easy to see that this can only occur if u_0 or p_0 diverges, which violates the condition on allowable physical states, since such states would have infinite values for $\langle \mathbf{H}^2\rangle$.

Finally, Eq.27 shows that any two quantum observers which give the same values for $\sqrt{\langle u_\infty^2\rangle\langle p_\infty^2\rangle}$ and $\Re(\langle u_\infty p_\infty\rangle)\sqrt{\langle u_\infty^2\rangle\langle p_\infty^2\rangle}$, see the same physics. They only differ by the time they see the bounce occur. For Gaussian packets we see that this will be true for observers which are related by the transformation

$$u_0 \to \lambda u_0,\ p_0 \to \frac{p_0}{\lambda},\ \text{and}\ \gamma \to \lambda^2\gamma. \tag{30}$$

It is easy to check that this can be implemented by a unitary transformation. The values of u_0 and p_0 can be changed by means of the shift operators used to define the coherent states in the first place. The width of the Gaussian can be changed by application of a unitary *squeezing operator* of the form

$$e^{\left(\alpha(\gamma)\, a^{\dagger 2} - \alpha(\gamma)^*\, a^2\right)}, \tag{31}$$

where the creation and annihilation operators are defined such that

$$u_\infty = \frac{1}{\sqrt{2\gamma}}\left(a^\dagger + a\right) \quad \text{and} \quad p_\infty = -i\sqrt{\frac{\gamma}{2}}\left(a^\dagger - a\right). \tag{32}$$

VI. REMARKS CONCERNING THE COMPUTATION OF CMB ANISOTROPY

While we have not yet done any detailed computations, it is clear that the fact that the quantum system deviates from pure exponential growth at a finite time in the past could have implications for the usual derivation of CMB fluctuations. It is entirely possible that the delay in the time at which the long wavelength modes of the scalar field exit the horizon relative to the shorter wavelength modes might produce visible effects in the predicted measurement of $\delta\rho/\rho$. If this is so then one should be able to put an experimental limit on how far back in time one can push the start of the usual computation.

VII. MINKOWSKI SPACE – $\Lambda = 0$

Finally, we would like to discuss what happens when we take $\Lambda = 0$, because, in this case, things work quite a bit differently. The $\Lambda = 0$ Hamiltonian is

$$\mathbf{H} = -\frac{3\kappa^2}{16\mathbf{V}} p_u^2 \tag{33}$$

and the Heisenberg equations of motion take the form

$$\frac{du}{dt}(t) = -\frac{3\kappa^2}{16\mathbf{V}} p_u \quad ; \quad \frac{dp_u}{dt}(t) = 0. \tag{34}$$

The exact solution to these equations is

$$u(t) = u - \frac{3\kappa^2}{16\mathbf{V}}\, p_u\, t \quad ; \quad p_u(t) = p_u \tag{35}$$

Taking the square of $u(t)$ we obtain the volume operator

$$V(t) = \mathbf{V} u^2(t) = \mathbf{V}\left[u^2 - \frac{3\kappa^2}{16\mathbf{V}}\ (u\, p_u + p_u\, u)\ t + \left(\frac{3\kappa^2}{16\mathbf{V}}\right)^2\ p_u^2\, t^2.\right] \tag{36}$$

It follows once again that, as in the de Sitter case, the volume operator becomes classical at large times in the past and the future. In this case however there is a state which, while non-normalizeable, satisfies the condition $G(t)|\Psi\rangle = 0$ for all times; namely, the eigenstate of p_u with eigenvalue 0. Now, however, this condition is consistent with the Heisenberg equations of motion, because in this eigenstate $u(t) = u$ and is independent of time. Moreover, this state satisfies the requirement that $\left\langle \mathbf{H}^2 \right\rangle$ is finite. Obviously, this state is the limit of sequence Gaussian packets in p_u of smaller and smaller width. If we choose this quantum observer then, after we absorb the scale factor into $\vec{x}$, we find that this observer sees a time-independent Minkowski space.

It is interesting to ask what other, less special, observers see. Let us assume we are working with an arbitrary coherent state of the form discussed in the previous section. Then, the expectation value of the volume operator is

$$\langle V(t)\rangle = \mathbf{V}\left[\left\langle u^2\right\rangle - 2\frac{3\kappa^2}{16\mathbf{V}}\,\Re(\langle u\, p_u\rangle)\, t + \left(\frac{3\kappa^2}{16\mathbf{V}}\right)^2\ \left\langle p_u^2\right\rangle\ t^2\right], \tag{37}$$

which can be rewritten in the form

$$\langle V(t)\rangle = \mathbf{V}\left[\frac{\left\langle u^2\right\rangle \left\langle p_u^2\right\rangle - \Re(\langle u\, p_u\rangle)^2}{\langle p_u^2\rangle} + \left(\frac{3\kappa^2}{16\mathbf{V}}\right)^2\ \left\langle p_u^2\right\rangle \left(t - \frac{16\mathbf{V}\Re(\langle u\, p_u\rangle)}{3\kappa^2\ \langle p_u^2\rangle}\right)^2\right]. \tag{38}$$

Thus, we see that for the generic observer, the case of zero cosmological constant actually corresponds to a universe for which the volume factor is expanding like t^2, or for which the scale factor $a(t)$ is growing like $t^{2/3}$. Surprisingly this corresponds to a universe dominated by non-relativistic matter. In other words, a non-vanishing energy density present in the quantum excitations of the scale factor produce the same effect as cold matter.

A final point worth mentioning is that, as in the case of de Sitter space, the Schwarz inequality guarantees that the volume never shrinks to zero for any allowable physical observer; i.e., we never are in the situation of a *big crunch.* It is interesting to note that in this formalism the big crunch is averted due to the quantum physics of the long wavelength modes of the gravitational field and not short distance physics.

VIII. SUMMARY

In our first paper we outlined a general formalism for setting up a fully quantum calculation of the CMB fluctuations, including back reaction. We also suggested a pixelization scheme which should allow us to extend this computation to include non-linear quantum effects for a finite number of long-wavelength modes of both the Newtonian potential and matter fields. In this paper we applied the general formalism to the case of de Sitter and Minkowski space in order to show in an explicit, exactly solvable, case how the formalism works in detail and why we are generically forced to choose to impose the classical constraint condition as an asymptotic condition on allowable quantum states. The most important result of our discussion is that in the case of de Sitter space the system deviates from the expected pure exponential expansion at a finite time in the past. One possible consequence of this fact, is that one might be able to experimentally measure the effects of the quantum corrections to the pure Einstein equations as deviations from the conventionally predicted form of $\delta\rho/\rho$. Failing that, one might be able to bound the earliest time at which one is free to set initial conditions on the state of the inflaton and other fields in the system[3]. In other words, either there may well be measureable consequences following from the quantum nature of the problem at early times, or one will have to face up to the problem of how and when to set initial conditions.

While, as it stands, the formalism we have presented is by no means a good candidate for a theory of everything, we feel that the interesting results obtained by proceeding along these lines suggests it is a very good candidate for a theory of something. Namely, a fully quantum theory of the measured fluctuations in the CMB radiation.

IX. ACKNOWLEDGEMENTS

We would like to thank J. D. Bjorken for helpful communications.

[1] M. Weinstein and R. Akhoury, arXiv:hep-th/0311189.
[2] For references to the general theory of computing cosmological perturbations see V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rept. **215**, 203-333 (1992), and references cited therein.
[3] For the original model of inflation which did not really work see A. H. Guth, Phys. Rev. **D 23**, 347356(1981). The current paradigm of slow-roll inflation which improved the situation was developed separately by Linde and Albrecht and Steinhardt, A. D. Linde, Phys. Lett. **B108** 389393 (1982) A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett.**48**, 12201223 (1982).