SU-ITP-03/31
SLAC-PUB-10255
hep-th/0312155

Giant Hedge-Hogs: Spikes on Giant Gravitons

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Abstract

We consider giant gravitons on the maximally supersymmetric plane-wave background of type IIB string theory. Fixing the light-cone gauge, we deduce the low energy effective light-cone Hamiltonian of the three-sphere giant graviton. At first order, this is a $U(1)$ gauge theory on $\mathbb{R} \times S^3$. We place sources in this effective gauge theory. Although non-vanishing net electric charge configurations are disallowed by Gauss' law, electric dipoles can be formed. From the string theory point of view these dipoles can be understood as open strings piercing the three-sphere, generalizing the usual BIons to the giant gravitons (BIGGons). Our results can be used to give a two dimensional (worldsheet) description of giant gravitons, similar to Polchinski's description for the usual D-branes, in agreement with the discussions of hep-th/0204196.

Work supported in part by the Department of Energy contract DE-AC03-76SF00515.

1 Introduction

Giant gravitons [1] were first discussed in the context of $m-2$-branes moving on the sphere in an $AdS_n \times S^m$ background, where it was observed that such particles blow up inside S^m, losing their point-like structure, and where their size was related to their angular momentum. In fact they are branes which couple to background form fields as dipoles, in contrast to the manner in which flat branes couple to form fields; they carry zero net form field charge, but a non-vanishing dipole moment. It is this dipole coupling that is responsible for their blowing up. In the $AdS_5 \times S^5$ background, they are three dimensional branes. Initial interest in these objects arose from their connection to non-commutative physics, and the scaling of their size with angular momentum was recognized as a hallmark of non-commutativity. The giant gravitons preserve the same supercharges as the graviton multiplet [2], and are 1/2 BPS, forming short representations of the superalgebra.

Giant gravitons which expand into the AdS part of the space-time have also been constructed, and they carry the same quantum numbers as sphere giant gravitons [2, 3]. The vibration spectrum of small fluctuations for giant gravitons which have expanded in either the AdS or the sphere directions have been studied [4], where it was found that the masses were independent of the radius and angular momentum of the giant graviton, depending only on the curvature scale of the background.

BPS solutions of type IIB supergravity describing giant gravitons carrying angular momentum along the sphere are available [5], and collections of giant gravitons act as external sources which give rise to extremal limits of charged black holes (superstars), where the horizon coincides with the singularity (which is hence naked) in the AdS component of the space-time. Solutions of eleven dimensional supergravity (again BPS) characterizing giant gravitons on $AdS_7 \times S^4$ and $AdS_4 \times S^7$ appeared in [6], where they also found to contain naked singularities, sourced by giant gravitons interpreted as spherical $M2$ and $M5$ branes.

The AdS/CFT duality suggests that giant gravitons should correspond to some operators in a dual conformal field theory, where these operators are chiral primary. The dual operators (for both sphere and AdS giants) have been constructed [7, 8, 9, 10, 11], and some correlation functions have also been computed. The sphere giant gravitons correspond to operators constructed from determinants and sub-determinants of the scalar fields in the $\mathcal{N} = 4$ super Yang-Mills theory (e.g. see footnote 4). The determinants are associated with maximum size giant gravitons, carrying the maximum angular momentum on the S^5. Similarly there have been proposals for the operators dual to giant gravitons inside AdS [8].

A new maximally supersymmetric type IIB supergravity solution ("the" plane-wave), arising as the Penrose limit of $AdS_5 \times S^5$ [12], has attracted much interest in the literature, largely because of its connection to the AdS/CFT duality [13], and the fact that the Green-Schwarz superstring action, in light-cone gauge, is exactly solvable [14, 15]. This Plane-wave/super-Yang-Mills duality is a specification of the usual AdS/CFT correspondence in the Penrose limit; it states that strings on a plane-wave background are dual to a particular large R-charge sector of $\mathcal{N} = 4$, $D = 4$ superconformal $U(N)$ gauge theory.[1] The study of giant gravitons was then extended from $AdS_5 \times S^5$ to the plane-wave, and interesting issues stemming from the nature of the dual operators were addressed [9, 17], in particular the question of open strings in the dual gauge theory.

In a different line of pursuit, Callan and Maldacena & Gibbons [18, 19] considered the low energy effective theory for a single brane, which gives rise via the Dirac-Born-Infeld action, to a $U(1)$ gauge theory, and showed that electric point charges could be interpreted as end-points of fundamental strings ending on the brane, and the dual magnetic charges as D-strings similarly ending on the brane. They demonstrated the profile these strings took and showed that they are in fact BPS solutions. These BPS solutions of the linearized (Maxwell) equations match the solutions of the full non-linear Born-Infeld theory equations, hence called BIons. A similar setup was argued to hold for other BPS brane junctions, giving a characterization in terms of local configurations of fields in the effective description of the brane in terms of a gauge theory.

We study giant gravitons on the plane-wave appearing as the Penrose limit of $AdS_5 \times S^5$, with a particular focus on the behaviour of charges in the worldvolume gauge theory and their interpretation in terms of open strings. We find solutions which generalize the usual BIons to giant gravitons, allowing an open string world-sheet description of such giant gravitons. The outline of our paper follows: In section 2 we present the low energy effective theory describing giant gravitons on the plane-wave, working in light-cone gauge. We find two zero-energy configurations (vacua), corresponding to a zero-size giant graviton and one of finite size, with radius given in terms of the string coupling g_s, the light-cone momentum p^+, and a scale μ for measuring energies. We then analyze the spectrum of fluctuations, writing their eigenfrequencies and eigenmodes. We find agreement between the physical modes and those of $\mathcal{N} = 4$ super-Yang-Mills on $\mathbb{R} \times S^3$. Higher order corrections are studied,

[1] A review can be found in [16].

extracting the effective coupling of the theory, and the relation of this coupling to the dual BMN gauge theory parameters is discussed. In section 3, we analyze the behaviour of the worldvolume theory when gauge fields are turned on, presenting the spectrum of the gauge field. In section 4, we turn our attention to the BIon solutions on giant gravitons (BIGGons), explicitly solving for the scalar and gauge field configurations, and interpret them via energy considerations as fundamental strings piercing the giant gravitons. The supersymmetry and stability of the configurations is also addressed. In a final section, we summarize our conclusions and outline possible future directions for pursuit. An appendix is included, summarizing the harmonics which appear in the main body of the paper.

2 Giant Gravitons in the Plane-Wave Background

In this section we focus on the $3+1$ dimensional Dirac-Born-Infeld action in the plane-wave background. First we note that due to symmetries of the plane-wave background, in particular translational symmetry along the light-like directions x^+ and x^- [16] (similar to the case of strings on the same background [14]), fixing the light-cone gauge will simplify considerably the action. The zero energy solutions to the light-cone Hamiltonian in the sector with light-cone momentum μp^+ is a sphere of radius $R^2 = \mu p^+ g_s$. This sphere is a giant graviton [1]. It is worth noting that fixing the light-cone gauge, in the language of rotating (orbiting) branes of [1, 3], corresponds to going to the rest frame of the giant graviton.

We also study fluctuation modes of the giant gravitons by expanding the light-cone Hamiltonian about the zero energy solutions. The frequencies of these modes, as in the case of giant gravitons on the $AdS_5 \times S^5$ background [4], are independent of their radius. We next turn on the fermions and work out the full fermionic terms of the light-cone Hamiltonian and the frequencies of their small fluctuations. We also briefly discuss higher order interaction terms in the Hamiltonian and the fact that they may be analyzed in a systematic perturbation expansion with the effective coupling g_{eff} (2.40).

2.1 Low energy effective dynamics in light-cone gauge

The low energy effective action for a D-brane is

$$S = S_{DBI} + S_{CS}\,, \tag{2.1}$$

with the Dirac-Born-Infeld action

$$S_{DBI} = -T_p \int d^{p+1}\zeta \, e^{-\phi} \sqrt{-det\left(G_{\hat{\mu}\hat{\nu}} + B_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}}\right)}\,, \tag{2.2}$$

where hatted Greek indices are used for the worldvolume coordinates ranging from zero to p. We have set $2\pi\alpha' = 1$; factors of α' can be reintroduced on dimensional grounds when necessary. We will consider D3-branes, for which $p = 3$ and the dilaton background is constant, in which case $g_s = e^{\phi}$. We first consider the case where, in addition to the constant dilaton, only the metric is turned on, and drop (consistently) the other forms. The gauge field $F_{\hat{\mu}\hat{\nu}}$, however, would be considered in section 3. Our metric conventions are those of Polchinski [20]; we work with a mostly plus metric for the worldvolume and target space. Note that the physical tension for this D-brane is T_p/g_s. The Chern-Simons term describing the coupling to the background RR four form is

$$S_{CS} = q \int C_4\,, \tag{2.3}$$

with q the charge of the brane. For BPS configurations the charge and tension are equal. $G_{\hat{\mu}\hat{\nu}}$ is the pullback of the space-time metric onto the worldvolume of the brane, and C_4 is the pullback of the RR four-form. They are given by

$$G_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} X^{\mu} \partial_{\hat{\nu}} X^{\nu} g_{\mu\nu} \tag{2.4}$$

and

$$C_4 = \frac{1}{4!}\left(\partial_{\hat{\mu}_0} X^{\mu_0} \cdots \partial_{\hat{\mu}_3} X^{\mu_3}\, C_{\mu_0 \dots \mu_3}\right) d\zeta^{\hat{\mu}_0} \wedge \cdots \wedge d\zeta^{\hat{\mu}_3}\,. \tag{2.5}$$

The X^{μ} give the embedding coordinates of the brane in the target space-time, i.e. $\mu = 0, \cdots, 9$, and $\zeta^{\hat{\mu}}$ are local coordinates on the brane worldvolume. $C_{\mu_0 \cdots \mu_3}$ is the space-time RR four-form coupling to the worldvolume.

We are working in a background specified by the maximally supersymmetric type IIB plane-wave (here we will follow the notation and conventions of [16])

$$ds^2 = -2dx^+dx^- - \mu^2(x^i x^i + x^a x^a)(dx^+)^2 + dx^i dx^i + dx^a dx^a\,, \tag{2.6a}$$

$$F_{+ijkl} = \frac{4}{g_s}\mu\, \epsilon_{ijkl}\,, \qquad F_{+abcd} = \frac{4}{g_s}\mu\, \epsilon_{abcd}\,. \tag{2.6b}$$

From (2.6b) it is easy to read off the RR four-form potential C, as $F = dC$, and we have

$$C_{+ijk} = \frac{\mu}{6g_s}\epsilon_{ijkl} x^l\,, \qquad C_{+abc} = \frac{\mu}{6g_s}\epsilon_{abcd} x^d\,, \tag{2.7}$$

which has the virtue of maintaining the translational symmetry along x^+. We have chosen our coordinates to make manifest the $SO(4) \times SO(4)$ symmetry of the transverse directions, labeling the two $SO(4)$'s with $i,j = 1,2,3,4\,;\quad a,b,c,d = 5,6,7,8$. For a more detailed discussion on the isometries of the background we refer the reader to [16].

We separate the space and time indices on the brane worldvolume as $\zeta = (\tau = \sigma^0, \sigma^r)$, with $p,q,r = 1,2,3$, the space indices. We will fix the light-cone gauge, setting

$$X^+ = \tau\,. \tag{2.8}$$

In order to ensure that the above solution for X^+ is maintained by the dynamics, we should use a part of the gauge symmetries of the DBI action, which are the area preserving diffeomorphisms on the brane worldvolume, to set

$$G_{0r} = -\partial_r X^- + \partial_\tau X^I \partial_r X^I = 0\,. \tag{2.9}$$

We have used upper-case indices to denote all eight transverse coordinates, where $I = (i,a) = 1,2,\cdots,8$.

Next we note that the background (2.6) is X^- independent, (it is a cyclic coordinate), and hence the momentum conjugate to X^-, the light-cone momentum p^+, is a constant of motion:

$$\begin{aligned} p^+ = -\frac{\partial \mathcal{L}}{\partial \partial_\tau X^-} &= -\frac{1}{g_s} G^{00}\sqrt{-det\ G} \\ &= -\frac{1}{g_s}\sqrt{\frac{-det\ G_{rs}}{G_{00}}}\,. \end{aligned} \tag{2.10}$$

To obtain (2.10) we have used the fact that $G_{0r} = 0$ implies $G^{00} = 1/G_{00}$. The light-cone Hamiltonian P^- (i.e. momentum conjugate to X^+) is then found to be

$$\begin{aligned} P^- \equiv -\frac{\partial \mathcal{L}}{\partial \partial_\tau X^+} &= -\frac{1}{g_s} G^{00}\sqrt{-det\ G}(\partial_\tau X^- + \mu^2 X^I X^I) - \frac{1}{6}\epsilon^{rps} C_{+IJK}\partial_r X^I \partial_p X^J \partial_s X^K \\ &= p^+(\partial_\tau X^- + \mu^2 X^I X^I) - \frac{1}{6}\epsilon^{rps} C_{+IJK}\partial_r X^I \partial_p X^J \partial_s X^K. \end{aligned} \tag{2.11}$$

Using (2.10) we can solve G_{00} and hence $\partial_\tau X^-$ for p^+ and $det\, G_{rs}$:

$$G_{00} = -2\partial_\tau X^- - \mu^2 X^I X^I + \partial_\tau X^I \partial_\tau X^I = -\frac{detG_{rs}}{(p^+ g_s)^2}\,. \tag{2.12}$$

Inserting $\partial_\tau X^-$ from (2.12) into the light-cone Hamiltonian and noting that the momenta conjugate to X^I are

$$P_I = \frac{\partial \mathcal{L}}{\partial \partial_\tau X^I} = p^+ \partial_\tau X^I\,, \tag{2.13}$$

we obtain the light-cone Hamiltonian density

$$H_{l.c.} = \frac{1}{2p^+} P^I P^I + V(X^i, X^a) \,, \tag{2.14}$$

where

$$\begin{aligned} V(X^i, X^a) &= \frac{\mu^2 p^+}{2}(X_i^2 + X_a^2) + \frac{1}{2p^+ g_s^2} det\, G_{rs} \\ &- \frac{\mu}{6g_s}\Big(\epsilon^{ijkl} X^i \{X^j, X^k, X^l\} + \epsilon^{abcd} X^a \{X^b, X^c, X^d\}\Big) \,. \end{aligned} \tag{2.15}$$

In the above,

$$G_{rs} = \partial_r X^i \partial_s X^i + \partial_r X^a \partial_s X^a \,,$$

and the brackets are "Nambu brackets" defined as

$$\{F, G, K\} = \epsilon^{pqr} \partial_p F \partial_q G \partial_r K \,, \tag{2.16}$$

where the antisymmetrization is with respect to worldvolume coordinates. It is worth noting that as a result of light-cone gauge fixing the square-root in the DBI action has disappeared (see (2.15)). This will help us perform a more detailed analysis of the light-cone Hamiltonian. We should also keep in mind that in the light-cone gauge, $\partial_r X^-$ are totally determined in terms of X^I through (2.9), i.e.

$$-P_I \partial_r X^I + p^+ \partial_r X^- \approx 0 \,, \tag{2.17}$$

where $\approx$ is the "weak" equality, meaning that (2.17) should hold on the solutions of the equations of motion of the light-cone Hamiltonian.

2.2 Zero energy configurations

We now search for classical minima of the light-cone Hamiltonian, and expand the potential $V(X^i, X^a)$ around these vacua to find the spectrum of small fluctuations about the vacua. First we note that if we set $X^a = 0$, then

$$det\ G_{rs} = det(\partial_r X^i \partial_s X^i) = \frac{1}{3!}\{X^i, X^j, X^k\}\{X^i, X^j, X^k\} \,, \tag{2.18}$$

and hence the potential becomes a perfect square

$$V(X^i, X^a = 0) = \frac{1}{2p^+}\left(\mu p^+ X^i - \frac{1}{6g_s}\epsilon_{ijkl}\{X^j, X^k, X^l\}\right)^2 . \tag{2.19}$$

The above potential has a minimum at

$$\mu p^+ g_s \epsilon_{ijkl} X^l = \{X^i, X^j, X^k\} \, . \tag{2.20}$$

Eq.(2.20) has two solutions, one is the "trivial" vacuum, $X^i = 0$, and the other one is a three-sphere of radius R, where[2] [13]

$$R^2 = \mu p^+ g_s \tag{2.21}$$

In other words if we set

$$X^i = R x^i, \qquad \sum_{i=1}^{4} x_i^2 = 1 \ , \tag{2.22}$$

it is easy to check that $\epsilon_{ijkl} x^i = \{x^j, x^k, x^l\}$. This three sphere is a giant graviton.

Both of these vacua are zero energy configurations. We could have easily found another minimum (zero energy configuration) corresponding to a three sphere grown in the X^a directions sitting at $X^i = 0$. Note also that both of these vacua are 1/2 BPS; they annihilate all the dynamical supercharges of the background. In other words, all the fermionic generators of the $PSU(2|2) \times PSU(2|2) \times U(1)$ superalgebra would kill these states.[3]

2.3 Spectrum about the vacua

We now study the spectrum of small fluctuations about these vacua. To do so, we expand the theory about the vacua to second order in fluctuations.

2.3.1 Spectrum about $X = 0$ vacuum

In this case the $detG_{rs}$ and the bracket terms would not contribute to the quadratic Hamiltonian; they appear in the interactions and the quadratic parts of the Hamiltonian are

$$H^{(2)}_{X=0} = \frac{1}{2p^+} P_i P_i + \frac{1}{2p^+} P_a P_a + \frac{\mu^2 p^+}{2} X_i^2 + \frac{\mu^2 p^+}{2} X_a^2 \ . \tag{2.23}$$

[2] Note that all lengths are measured in units of the string scale α'. Recovering the α'-factors, we have $R^2/\alpha' = (\mu p^+ \alpha') g_s$.

[3] Although both of these vacua are BPS, it has been argued that the $X = 0$ vacuum might be unstable under certain quantum corrections [1, 3]. These arguments were finally confirmed for the case of spherical membranes and fivebranes in M-theory in a detailed analysis of the Matrix theory describing M-theory on the maximally supersymmetric eleven dimensional plane-wave background, the BMN matrix theory. It has been shown that the $X = 0$ vacuum for the membrane/fivebrane case is in fact a finite size fivebrane/membrane [21]. For the case of spherical three-branes, to the authors' knowledge the issue is not yet fully answered.

Therefore, there are eight modes, all with frequency μ, that is the modes are particles of mass μ. Of course for a generic low energy state one may excite many of these modes.

2.3.2 Spectrum about the three-sphere vacuum

If we parameterize the small fluctuations in the X^i directions by Y^i, i.e. $X^i = Rx^i + Y^i$, the quadratic Hamiltonian becomes

$$\begin{aligned} H^{(2)}_{X=R} = \frac{1}{2p^+}P_iP_i + \frac{1}{2p^+}P_aP_a &+ \frac{1}{2p^+}\left(\mu p^+ Y_i - \frac{R^2}{2g_s}\epsilon_{ijkl}\{x^j, x^k, Y^l\}\right)^2 \\ &+ \frac{1}{2p^+}\left((\mu p^+)^2 X_a^2 + \frac{R^4}{g_s^2}\partial_r X^a \partial_s X^a g_0^{rs}\right) , \end{aligned} \quad (2.24)$$

where g_0^{rs} is the inverse of the metric on a unit three-sphere. The bracket can be used to obtain generators of $SO(4)$ rotations along the three-sphere, explicitly:

$$\mathcal{L}_{ij}\Phi \equiv (x_j\partial_i - x_i\partial_j)\Phi = -\frac{1}{2}\epsilon_{ijkl}\{x^k, x^l, \Phi\} . \quad (2.25)$$

In terms of $\mathcal{L}_{ij}$ the Hamiltonian takes a simple form

$$H^{(2)}_{X=R} = \frac{1}{2p^+}P_iP_i + \frac{1}{2p^+}P_aP_a + \frac{1}{2}\mu^2 p^+(Y_i + \mathcal{L}_{ij}Y^j)^2 + \frac{1}{2}\mu^2 p^+(X_a^2 + \frac{1}{2}X^a\mathcal{L}_{ij}\mathcal{L}^{ij}X^a). \quad (2.26)$$

The normal modes for the Y^i directions about this vacuum satisfy the eigenvalue equation

$$\mathcal{L}_{ij}Y^j = \lambda Y_i ,$$

with masses given by

$$M^2 = \mu^2 (1+\lambda)^2 . \quad (2.27)$$

The eigenvectors are vector spherical harmonics of the form

$$Y_l^i = S_{ii_1\cdots i_l}\, x^{i_1}\cdots x^{i_l} , \quad (2.28a)$$

$$\tilde{Y}_l^i = x^i \tilde{S}_{i_1\cdots i_{l-1}}\, x^{i_1}\cdots x^{i_{l-1}} - \frac{l-1}{2l}\tilde{S}_{ii_1\cdots i_{l-2}} x^{i_1}\cdots x^{i_{l-2}} , \quad (2.28b)$$

where S and $\tilde{S}$ are symmetric traceless $SO(4)$ tensors. The eigenvectors (2.28a,b) correspond to the eigenvalues $\lambda = l, -(l+2)$, respectively. Both of these modes, although in different $SO(4)$ representations, would have the same mass:

$$M_i = \mu(l+1). \quad (2.29)$$

Physically these two modes correspond to geometric fluctuations of the brane in the radial directions.

Of the modes describing the five directions X^-, Y^i, there remain three zero modes

$$\hat{Y}_l^i = A^i_{i_1\cdots i_l}\, x^{i_1}\cdots x^{i_{l-1}}\ , \tag{2.30}$$

where A is symmetric in all lower indices and antisymmetric in the first upper and first lower index (and hence $x_i\hat{Y}_l^i = 0$ for any l). These eigenvectors are associated with the eigenvalue $\lambda = -1$. These zero modes are not physical and correspond to gauge degrees of freedom associated with the area preserving diffeomorphisms on the three-sphere.

The masses for X^a fluctuations can be easily obtained, noting that the eigenvalues for the $SO(4)$ Casimir $\mathcal{L}^2$, are $l(l+2)$, with the corresponding X^a

$$X_l^a = S^a_{i_1\cdots i_l} x^{i_1}\cdots x^{i_l}\ . \tag{2.31}$$

The masses are

$$M_a^2 = \mu^2[l(l+2)+1] = \mu^2(l+1)^2\ . \tag{2.32}$$

As we see, all the modes, Y^i's and X^a's, have the same mass. This is a direct result of the supersymmetry algebra of this background, which as discussed in [16] is $PSU(2|2)\times PSU(2|2)\times U(1)$, and the fact that the light-cone Hamiltonian commutes with the supercharges; as a result all the states in the same supermultiplet should have the same mass. This is in contrast with the eleven dimensional plane-wave superalgebra [22].[4]

[4]The $X^i = Rx^i$ vacuum, being a 1/2 BPS state, should, in the dual $\mathcal{N}=4,\ D=4$ gauge theory, be represented by a chiral primary operator. And in our case, since we are working in the plane-wave background, it should be a BMN [13] type operator. The corresponding operators have been introduced and studied in [7, 8, 9, 17, 11]. Let $Z^i_j,\ i,j=1,2,\cdots,N$, be one of the three complex scalar fields of an $\mathcal{N}=4,\ D=4,\ U(N)$ gauge theory (for more on conventions and notations see [16]). Then [7]

$$\mathcal{O}_J^{S^5} = \mathcal{N}_J \frac{1}{J!(N-J)!}\epsilon_{i_1 i_2\cdots i_J k_{J+1}\cdots k_N}\ \epsilon^{j_1 j_2\cdots j_J k_{J+1}\cdots k_N} Z^{i_1}_{j_1} Z^{i_2}_{j_2}\cdots Z^{i_J}_{j_J}\ , \tag{2.33}$$

is the operator dual to a giant graviton grown in the S^5 direction (the normalization factor $\mathcal{N}_J^2 = \frac{(N-J)!}{N!}$ is chosen so that $\langle \mathcal{O}_J^{S^5}\bar{\mathcal{O}}_J^{S^5}\rangle = 1$).

$$\mathcal{O}_J^{AdS} = \frac{1}{J!}\sum_{\sigma\in\mathcal{S}_J} Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}}\cdots Z^{i_J}_{i_{\sigma(J)}}\ ,$$

with $\mathcal{S}_J$ being the permutation group of length J, is proposed to describe giant gravitons grown in the AdS directions [8]. Note that in the plane-wave case (after the Penrose limit) the two giant gravitons grown in S^5 and in AdS essentially become indistinguishable.

2.3.3 Fermionic modes

For completeness we also work out the spectrum of fluctuations for the fermionic modes about both vacua. The fermionic contributions to the DBI and CS parts of the action can found using superspace coset techniques which make the superalgebra manifest (see for example [23], and [15], and also [21, 24] for a similar treatment for the case of the membrane and fivebrane on the eleven dimensional maximally supersymmetric plane-wave). After fixing κ symmetry in light-cone gauge [25], the new contributions to the potential in the Hamiltonian are quadratic in the fermions, and are given by

$$\begin{aligned} V^{\psi} =& \mu \psi^{\dagger\alpha\beta}\psi_{\alpha\beta} + \frac{2}{p^+ g_s}\left(\psi^{\dagger\alpha\beta}(\sigma^{ij})_{\alpha}{}^{\delta}\left\{X^i, X^j, \psi_{\delta\beta}\right\} + \psi^{\dagger\alpha\beta}(\sigma^{ab})_{\alpha}{}^{\delta}\left\{X^a, X^b, \psi_{\delta\beta}\right\}\right) + \\ & \mu \psi^{\dagger\dot\alpha\dot\beta}\psi_{\dot\alpha\dot\beta} + \frac{2}{p^+ g_s}\left(\psi^{\dagger\dot\alpha\dot\beta}(\sigma^{ij})_{\dot\alpha}{}^{\dot\delta}\left\{X^i, X^j, \psi_{\dot\delta\dot\beta}\right\} + \psi^{\dagger\dot\alpha\dot\beta}(\sigma^{ab})_{\dot\alpha}{}^{\dot\delta}\left\{X^a, X^b, \psi_{\dot\delta\dot\beta}\right\}\right). \end{aligned} \tag{2.34}$$

We have chosen to decompose the $SO(4) \times SO(4)$ fermions in terms of representations of the two $SU(2)$'s appearing in each $SO(4)$, following the notation of [16], making manifest the fermion representations under the $PSU(2|2) \times PSU(2|2)$ part of the superalgebra of the maximally supersymmetric plane-wave we are considering. The explicit mass terms in V^{ψ} for the fermions come from a shift in $G_{\tau\tau}$ arising from fermionic contributions to the super-vielbein and the terms involving Nambu brackets from the Chern-Simons terms. Following the notation of [16], the fermions ψ are spinors of two different $SU(2)'s$, one coming from the decomposition of each of the two $SO(4)$'s into $SU(2) \times SU(2)$; in other words, ψ above carries two spinor indices (sitting in the same chirality representations). More details on our spinor conventions can be found in [16]. Note that in the potential (2.34), the two sets of fermions with dotted and undotted indices do not couple to each other.

Expanding the potential around the three-sphere solution, setting $X^i = Rx^i$ and $X^a = 0$,

Using the above operators one may construct the dual gauge theory operators corresponding to the fluctuation modes of the giant graviton we have studied here. This can be done by insertion of "impurities" in the sequence of Z's, much like what has been done in [13] for strings. For example, if $\phi_a,\ a = 1,2,3,4$ denote the other four scalars of the $\mathcal{N} = 4,\ D = 4$ gauge multiplet, then the dual gauge theory operators for $l = 0, 1$ states of (2.31) are [9, 11]

$$\mathcal{O}_J^{X^a_{l=0}} = \mathcal{N}_{J+1}\frac{1}{(J+1)!(N-J-1)!}\epsilon_{i_1 i_2 \cdots i_{J+1} k_{J+2} \cdots k_N}\ \epsilon^{j_1 j_2 \cdots j_{J+1} k_{J+2} \cdots k_N}(\phi^a)^{i_1}_{j_1} Z^{i_2}_{j_2} Z^{i_3}_{j_3} \cdots Z^{i_J}_{j_J}\ ,$$

$$\mathcal{O}_J^{X^a_{l=1}} = \mathcal{N}_{J+2}\frac{1}{(J+2)!(N-J-2)!}\epsilon_{i_1 i_2 \cdots i_{J+2} k_{J+3} \cdots k_N}\epsilon^{j_1 j_2 \cdots j_{J+2} k_{J+3} \cdots k_N}(\phi^a)^{i_1}_{j_1} Z^{i_2}_{j_2} Z^{i_3}_{j_3} \cdots (\phi^b)^{i_l}_{j_l} \cdots Z^{i_J}_{j_J}\ .$$

Clearly these operators have $\Delta - J = 1, 2$, respectively. Similarly, one may construct higher l excitations by more insertions of ϕ's.

the quadratic part of the potential (2.34) becomes, after using (2.21) and (2.25)

$$\begin{aligned} V^{\psi}_{(2)} = \mu \left(\psi^{\dagger\alpha\beta}\psi_{\alpha\beta} - \psi^{\dagger\alpha\beta}(\sigma^{ij})_{\alpha}{}^{\delta}\, \epsilon^{ijkl}\mathcal{L}_{kl}\psi_{\delta\beta}\right) + \\ \mu \left(\psi^{\dagger\dot\alpha\dot\beta}\psi_{\dot\alpha\dot\beta} - \psi^{\dagger\dot\alpha\dot\beta}(\sigma^{ij})_{\dot\alpha}{}^{\dot\delta}\, \epsilon^{ijkl}\mathcal{L}_{kl}\psi_{\dot\delta\dot\beta}\right) . \end{aligned} \tag{2.35}$$

The spectrum of small fluctuations around this vacuum are given by solutions of the eigenvalue equation

$$\epsilon^{ijkl}(\sigma^{ij})_{\alpha}{}^{\beta}\, \mathcal{L}_{kl}\psi_{\beta} = \lambda\, \psi_{\alpha} \,, \tag{2.36}$$

with similar equations for the other modes. We have for clarity suppressed one of the indices since it is a bystander in the eigenvalue equation.

The frequencies (masses) are then given by

$$\omega = \mu\,(1-\lambda) \ , \tag{2.37}$$

where λ is the eigenvalue corresponding to the excitation mode. The eigenfunctions and corresponding eigenvalues (suppressing the inactive spinor index) are

$$\begin{aligned} \psi^l_\alpha &= \left(\theta_{\alpha i_1\ldots i_l} + \epsilon^{j i_1 kl}(\sigma^{kl})_{\alpha}{}^{\beta}\theta_{\beta j i_2\ldots i_l}\right) x^{i_1}\cdots x^{i_l} & \lambda &= -l \,, \\ \tilde\psi^l_\alpha &= \left(l\,\theta_{\alpha i_1\ldots i_l} + (l+2)\epsilon^{i_1 jkl}(\sigma^{kl})_{\alpha}{}^{\beta}\theta_{\beta j i_2\ldots i_l}\right) x^{i_1}\cdots x^{i_l} & \lambda &= l+2 \,, \end{aligned} \tag{2.38}$$

where θ, carrying the spinorial index, forms a totally symmetric traceless representation of $SO(4)$ in the indices $j, i_1, \ldots, i_l$.[5] Therefore both $\tilde\psi^l_\alpha$ and ψ^l_α excitations have the same mass, $|\omega|$, equal to $\mu(l+1)$. As it is clear from (2.35), fermions $\psi_{\dot\alpha\dot\beta}$ would also have the same mass. Hence all the bosonic and fermionic excitations about the $X^i = Rx^i$ vacuum $(Y^i_l, \tilde Y^i_l, X^a_l; \psi^l_{\alpha\beta}, \tilde\psi^l_{\alpha\beta}, \psi^l_{\dot\alpha\dot\beta}, \tilde\psi^l_{\dot\alpha\dot\beta})$ have the same mass, as expected from the $PSU(2|2)\times PSU(2|2)\times U(1)$ superalgebra, and fall into the same supermultiplet. However, these modes do not complete the multiplet (as there are two more fermions than bosons). These two extra bosonic modes correspond to a $U(1)$ gauge field living on the giant graviton; we will come back to this point in section 3.1.

The frequencies of small perturbations around the zero size giant graviton are simply given by μ, arising from the explicit mass term in the potential. The masses are then the

[5] It is straightforward but tedious to check that these are indeed eigenfunctions of (2.36), making use of the identity

$$\begin{aligned} (\sigma^{ij})_{\alpha}{}^{\beta}(\sigma^{kl})_{\beta}{}^{\rho} = -\frac{1}{4}\Big[& \delta^{\rho}_{\alpha}\left(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk} + i\epsilon^{ijkl}\right) \\ & + 2\left(\delta^{ik}(\sigma^{jl})_{\alpha}{}^{\rho} + \delta^{jl}(\sigma^{ik})_{\alpha}{}^{\rho} - \delta^{il}(\sigma^{jk})_{\alpha}{}^{\rho} - \delta^{jk}(\sigma^{il})_{\alpha}{}^{\rho}\right)\Big] . \end{aligned}$$

same as the bosonic spectrum. In the $X = 0$ vacuum, as opposed to the spherical vacuum, the number of scalar and fermionic excitations are both eight, i.e. there are no gauge field modes.

Finally we would like to point out that, although we do not explicitly show it here, all the modes, about both vacua, fall into a BPS (short) multiplet of the $PSU(2|2) \times PSU(2|2) \times U(1)$ superalgebra. A study of representation theory of this superalgebra is an interesting open problem in need of a thorough analysis.

2.4 Interaction terms

So far we have only considered the quadratic terms around each of the two vacua. One may study the theory perturbatively about the spherical or $X = 0$ solution. The purpose of this section is to find the effective coupling about these vacua and discuss under what conditions the expansion around these vacua can be trusted.

Let us first consider the spherical vacuum. Expanding (2.14) about the $X^i = Rx^i$ solution, we obtain the interaction terms which are from cubic up to sixth order in fluctuations Y^i or X^a. In order to read the coupling constant, however, we should redefine (rescale) the fluctuations so that the quadratic part of the Hamiltonian takes the standard canonically normalized form of $\sum_i \hbar\omega_l a_l^\dagger a_l$, where $a_l^\dagger$ is the corresponding creation operator and ω_l is the mass of the mode, which in our case is $\mu(l+1)$. For this we need to rescale Y^i and X^a as

$$Y^i, X^a \to \frac{1}{\sqrt{\mu p^+}} Y^i, X^a \ . \tag{2.39}$$

As can be seen from (2.34), for the fermions no rescaling is needed. It is straightforward to see that the cubic term is suppressed by a factor of g_{eff}, and likewise terms of order n in fields are accompanied by a factor of g_{eff}^{n-2}, where

$$g_{eff} = \frac{1}{\mu p^+ \sqrt{g_s}} = \frac{1}{R\sqrt{\mu p+}} \ . \tag{2.40}$$

(Note that energy is measured in units of μ and hence one should take out a factor of μ from the potential. This can be done systematically if we scale time with $1/\mu$.)

One might rewrite g_{eff} in terms of the BMN gauge theory parameters, J, N and g_{YM}^2, where [16]

$$\frac{1}{(\mu p^+)^2} = \frac{g_{YM}^2 N}{J^2} \equiv \lambda' \ , \qquad g_2 \equiv \frac{J^2}{N} \ .$$

Then $R^2 = \sqrt{\lambda'} g_2$ and $g_{eff} = 1/g_2$. Noting that g_2 is the genus counting parameter for strings on plane-waves, (2.40) suggests that our giant graviton theory is somehow S-dual to string theory on the plane-wave.[6]

One may repeat the same analysis for the $X = 0$ vacuum, for which we should use the same scaling as above and hence we again end up with the same coupling as (2.40). We caution that the above coupling should be thought of as a "bare" coupling and in a properly quantized system this coupling may be dressed with some other factors of μp^+ and also this dressing factor can be different for different vacua. In this respect the situation is quite similar to the membrane case which was analyzed in detail in [24]. However, in our case we do not know how to quantize the Nambu brackets.

3 Gauge Theory on Giant Gravitons

We are now ready to include the contribution from the gauge fields on the brane. The analysis of section 2 is modified slightly in this case when $F_{\hat{\mu}\hat{\nu}} = (dA)_{\hat{\mu}\hat{\nu}}$ is turned on. The equations giving the metric on the brane as the pullback of the plane-wave space-time metric are of course unaffected, but the determinant appearing in the DBI action receives contributions from the gauge field strength. We write

$$M_{\hat{\mu}\hat{\nu}} = G_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}} ,$$

where as before, $G_{\hat{\mu}\hat{\nu}}$ is the pullback of the space-time metric given by equation (2.4). As a part of the light-cone gauge fixing, as we did in section 2.1, we set $G_{0r} = 0$ and hence

$$M_{00} = G_{00} \ , \qquad M_{0r} = F_{0r} = -F_{r0} \equiv E_r \ , \qquad M_{rs} = G_{rs} + F_{rs} \ , \tag{3.1}$$

where E_r is the electric field and G_{00} is still given by (2.12). The Chern-Simons terms and also the fermionic contributions (2.34) are unaffected by the appearance of the gauge fields. The contribution of the gauge field to the momentum conjugate to X^- results in a modification of (2.10), as

$$p^+ = -\frac{1}{g_s} M^{00} \sqrt{-\det M} \, , \tag{3.2}$$

[6]It is interesting to note that the three point function of $\mathcal{O}_J^{S^5}$ (2.33) is given by [17]

$$\langle \mathcal{O}_J^{S^5} \bar{\mathcal{O}}_{rJ}^{S^5} \bar{\mathcal{O}}_{(1-r)J}^{S^5} \rangle \simeq e^{-g_2 r(1-r)/2} \sim e^{\frac{-r(1-r)}{g_{eff}}} \ , \tag{2.41}$$

where $0 \le r < 1$ and by $\simeq$ we mean that the result is presented after the BMN limit, i.e. $J, N \to \infty$ and $J^2/N =$ fixed. (2.41) shows that the above three point function corresponds to tunneling between two different giant graviton states which is a non-perturbative effect in the giant graviton theory.

but now $M^{00} \neq 1/M^{00}$ because of the off-diagonal electric field appearing in $M_{\hat{\mu}\hat{\nu}}$; it becomes

$$M^{00} = \frac{det\ (G_{rs} + F_{rs})}{det\ M}\ . \tag{3.3}$$

The determinant appearing in the action is also modified

$$det\ M = \Big(det\ (G_{rs} + F_{rs})\Big)\left(G_{00} + E_r G^{rs} E_s\right). \tag{3.4}$$

Using (3.2), we can write

$$-\frac{det\ (G_{rs} + F_{rs})}{(p^+ g_s)^2} = G_{00} + E_r G^{rs} E_s\,. \tag{3.5}$$

The expression for the momentum conjugate to X^+ (the light-cone Hamiltonian) remains as in (2.11), but with p^+ now given by (3.2), i.e.

$$P^- = p^+\left(\partial_\tau X^- + \mu^2 X^I X^I\right) - \frac{1}{6}\epsilon^{rps} C_{+IJK}\partial_r X^I \partial_p X^J \partial_s X^K.$$

The momenta conjugate to X^I (2.13) are unaffected. We can solve for ∂X^- in terms of X^I and their conjugate momenta P_I, and also E_r and F_{rs}, using (3.2), (3.3) and (3.5). Since the computations are very similar to those of section 2.1 we do not repeat them here. Gathering all the terms, the total light-cone Hamiltonian density becomes

$$\begin{aligned}\mathcal{H}_{l.c} = & \frac{P_I^2}{2p^+} + \frac{1}{2}\mu^2 p^+ X_I^2 + \frac{1}{2p^+ g_s^2} det\ (G_{rs} + F_{rs}) + \frac{1}{2}p^+ E_r G^{rs} E_s \\ & + \frac{\mu}{6g_s}\Big(\epsilon^{ijkl} X^i \{X^j, X^k, X^l\} + \epsilon^{abcd} X^a \{X^b, X^c, X^d\}\Big).\end{aligned} \tag{3.6}$$

3.1 Spectrum of small fluctuations of the gauge field

From (3.6) it is readily seen that $X = 0$ and $X^i = Rx^i$ (and of course together with $E_r = F_{rs} = 0$) are still the only zero energy configurations. Then one may expand the theory about each of these vacua. The spectrum of X^i, X^a modes is the same as those we studied in section 2.3. (This statement is also true for fermionic modes. In fact one can show that the full supersymmetric version of the Hamiltonian (3.6) is obtained by adding (2.34) to (3.6). This in particular means that fermions do not directly couple to gauge fields. The latter could be understood by noting that we are only dealing with a $U(1)$ gauge theory where fermions sit in the adjoint representation, i.e. they are neutral.)

For the $X = 0$ vacuum, there are no gauge field contributions, because the gauge field terms only appear in quartic or higher powers. For example the induced metric G_{rs} is second order in X fluctuations and hence the $E_r G^{rs} E_s$ term is at least quartic. This is compatible with our earlier discussions in section 2.3.3, that the fluctuations of X^i's, X^a's and the corresponding fermionic modes, complete a $PSU(2|2) \times PSU(2|2) \times U(1)$ (short) multiplet. In this case, gauge fields only couple to "scalar" bosonic modes with the "bare" coupling given by (2.40).

As for the $X^i = Rx^i$ vacuum, expanding (3.6) up to second order in all fluctuations we obtain

$$\mathcal{H}^{(2)}_{l.c.} = H^{(2)}_{l.c.} + \frac{1}{2\mu g_s}\left(E_r g_0^{rs} E_s + \mu^2 B_r g_0^{rs} B_s\right) , \tag{3.7}$$

where $H^{(2)}_{l.c.}$ is given in (2.14), g_0^{rs} is the (inverse) metric on the unit three-sphere and $g_0^{rp} B_p = \epsilon^{rqs} F_{qs}$ is the magnetic field. It is worth noting that $\mathcal{H}^{(2)}_{l.c.}$ (plus (2.34)) is exactly the Hamiltonian for an $\mathcal{N} = 4$ $U(1)$ gauge theory on $\mathbb{R} \times S^3$ (the latter action may be found in [26]). Among the fields in the four dimensional $\mathcal{N} = 4$ gauge multiplet the gauge field, four scalars, the X^a modes, and 16 fermions (which are in the correct representation, e.g. see [16]) are explicit. The other two scalars modes, however, are a combination of Y^i's. Although the explicit expressions defining these two scalars in terms of Y^i's is not so simple, as we discussed in section 2.3.2, Y^i's only contain two physical modes, with the masses equal to the other four scalar, X^a modes, and also the fermions. Moreover, the above argument would imply that the $SO(4)$ symmetry rotating X^a's among each other, can be generalized to $SO(6)$ including these other two scalar modes, giving rise to the full R-symmetry group of the $\mathcal{N} = 4$ gauge theory.

In the same manner that we argued that the fluctuation modes of section 2.3.2 complete a $PSU(2|2) \times PSU(2|2) \times U(1)$ multiplet, we should also work out the spectrum of the gauge fields, i.e. photons, on the three-sphere, and show that the two polarizations of the photon have the same "mass" as the fermions and scalars. Let us start with the equation of motion for the gauge field, $A_{\hat{\mu}}$:

$$\left(g^0_{\hat{\mu}\hat{\nu}} \nabla^2 - \frac{1}{2}(\nabla_{\hat{\mu}}\nabla_{\hat{\nu}} + \nabla_{\hat{\nu}}\nabla_{\hat{\mu}})\right) A^{\hat{\nu}} = 0 \ .$$

We choose to work in Coulomb gauge, i.e. $A_0 = 0$, $\nabla_{\hat{\mu}} A^{\hat{\mu}} = 0$.[7] Next we note that ∇_r and the gradient on the unit three-sphere do not commute. In fact they commute to the Riemann tensor. For a unit sphere $R_{rs} = 2\, g_{rs}$, where R_{rs} is the Ricci tensor, and we have

$$\left(\omega^2 - \mu^2(\nabla_r^2 + 1)\right) A_s = 0 \, . \tag{3.8}$$

[7] We would like to point out that working with the light-cone gauge in the bulk, as we have done here, *does not* necessarily imply that for the worldvolume gauge theory we have also fixed the same gauge. In fact these are two independent gauge symmetries and the $U(1)$ gauge theory should be fixed separately.

If we take A_r to be in the spin l representation of $SO(4)$, i.e. $\nabla^2 A_r^l = l(l+2)A_r^l$, the spectrum of the two photon polarizations is obtained to be

$$\omega = \mu(l+1). \tag{3.9}$$

As we see the "mass" for photons has a purely geometric origin. (The same is also true for fermions). This is in contrast with that of scalars, where we have an explicit mass term. Equation (3.9), as we expected from the superalgebra arguments, leads exactly to the same spectrum as scalars and fermions. These two photon modes together with the other excitations studied in section 2.3.2 complete a multiplet of the $PSU(2|2)\times PSU(2|2)\times U(1)$ algebra.

4 BIGGons: BIons on Giant Gravitons

In this section we focus on a $U(1)$ gauge theory on $\mathbb{R}\times S^3$ and study static, BPS charge configurations. We cannot have non-zero electric net charge (S^3 is compact), however, higher-pole configurations are allowed. We study the dipole configurations in some detail. Looking for BPS configurations, we are forced to turn on the scalar fields as well. This is a direct generalization of Callan-Maldacena [18] and Gibbons' BIons [19] argument to the "compact" branes. These configurations, from the point of view of an observer far away in the bulk, have the interpretation of (fundamental) strings piercing the S^3. The points of attachment (the north and south pole of the three-sphere), carry positive and negative electric charge. The open strings, locally, can be thought of as Polchinski's open strings ending on the brane (giant graviton in our case) with Dirichlet boundary conditions [27]. As evidence that this configuration is really a string we show that the energy is proportional to the distance to the brane (at least for far distances).

4.1 Solutions

We would now like to consider placing charges on the giant graviton. We take for the embedding coordinates of a unit three-sphere

$$\begin{aligned} x_1 &= \sin\psi\ \sin\theta\ \cos\phi \\ x_2 &= \sin\psi\ \sin\theta\ \sin\phi \\ x_3 &= \sin\psi\ \cos\theta \\ x_4 &= \cos\psi \end{aligned} \tag{4.1}$$

with $0 \leq \psi, \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. The metric on the three-sphere in the coordinate system (4.1) is

$$ds^2 \; = \; d\psi^2 + \sin^2\psi d\Omega_2^2 \,, \tag{4.2}$$

with $d\Omega_2^2 = \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$ the metric on the unit two-sphere, and $\sqrt{\det g} = \sin^2\psi \, \sin\theta$. The Laplacian acting on a scalar field Φ is $\nabla^2\Phi = \frac{1}{\sqrt{\det g}}\partial_\mu \left(\sqrt{\det g} g^{\mu\nu}\partial_\nu\Phi\right)$, which on the three-sphere for Φ's with only ψ dependence becomes

$$\nabla^2\Phi(\psi) = \frac{1}{\sin^2\psi}\partial_\psi \left(\sin^2\psi \, \partial_\psi\Phi(\psi)\right) \,, \tag{4.3}$$

The three-sphere is compact, and hence the giant graviton cannot support single charges. It does, however, support dipoles (as well as higher poles) with vanishing total charge. Consider a dipole with two opposite charges placed at the two poles ($\psi = 0, \pi$). The charge density of such a configuration, as seen by the gauge theory, is

$$\rho \; = \; \frac{Q}{\sin^2\psi}\Big[\delta(\psi) - \delta(\psi - \pi)\Big]\delta(\cos\theta)\delta(\phi) \,. \tag{4.4}$$

A few words about the normalizations of the fields are in order: The scalar and gauge fields appearing in the Hamiltonian (2.24) and (3.7) are not canonically normalized. The normalization of the gauge field in (3.7) is such that gauge theory action carries an overall factor of $1/g_{YM}^2$, where $g_{YM} = \mu\sqrt{g_s}$, a convenient choice for studying gauge theories on curved backgrounds. Canonical normalization of the gauge field can be achieved by taking $A_\mu \to \mu\sqrt{g_s}A_\mu$. The coupling of the gauge field to the charges in the gauge theory is of the form $J \cdot A$, with J_0 the charge density ρ, which carries the same units as the charge Q since the angular coordinates are dimensionless. The scalar fields in (2.24), as stated in (2.39), can be normalized canonically by taking $X \to R \, g_{eff} \, \Phi$. This is the normalization in which the scalar field couples to the sources in the same way as the gauge field, via (4.4). Noting that the energy is measured in units of μ (a choice of scale for the time coordinate), we find that the charges, as seen from the DBI action (2.2), are measured in units of $\mu\sqrt{g_s}$, so $q = Q\mu\sqrt{g_s}$, with Q dimensionless. It is the canonical fields (scalar and gauge) that are sourced by Q. The fact that the gauge and scalar fields enter the Hamiltonian with different scales is an artifact of the choice of relative normalizations of the fields in the DBI action (2.2). In fact, the normalizations in the DBI action are chosen to reproduce the correct field normalizations in the gauge theory for the case of a flat background, but in a curved background do not reproduce the standard normalizations for a gauge theory coupled to a fixed curved metric, as is the case for the SYM theory on $\mathbb{R} \times S^3$. We also remind the reader that $\alpha' \mu p^+ g_s$ is a dimensionless quantity, so in our units, where we have set $\alpha' = 1/2\pi$, μp^+ and g_{eff} are both dimensionless. From this point on we shall deal only with canonically normalized fields.

We study the electrostatic problem for the gauge field in a gauge where $A_0 = \Lambda$. The equation of motion for the gauge field, arising from the quadratic Hamiltonian (3.7), is simply Poisson's equation, which in appropriate units requires $\nabla^2 \Lambda = -\rho$. We take as our ansatz a field $\Lambda(\psi)$ which is a constant along the two angular directions θ, ϕ. The solution is

$$\Lambda(\psi) = Q \cot \psi. \tag{4.5}$$

As in the discussion of [18], exciting the gauge field alone would not result in a BPS configuration (supersymmetry implies a relation between the profile of the gauge field and the other fields in the $\mathcal{N} = 4$ supermultiplet). The solution can be made BPS by turning on a non-trivial profile for the scalars, keeping a constant vanishing background fermion field.

To find the scalar profile, it will prove useful to consider the quadratic part of the Hamiltonian, expanded around the spherical vacuum, restricted to field configurations which depend only on the radial direction. The potential for the scalar field describing the radial profile can then be written

$$V_\Phi = \frac{1}{2} \left(\Phi^2 + (\nabla_{S^3} \Phi)^2 \right) , \tag{4.6}$$

where the square is with respect to the metric on the three-sphere of unit radius. The spikes can't go off to infinity in just any direction because of the potential from the plane-wave, but they can extend off to infinity along X^-, which can be taken as the radial direction.

The equation of motion for the field Φ in the gauge theory, in the presence of a dipole charge configuration is simply

$$\nabla^2_{S^3} \Phi - \Phi = \frac{Q}{\sin^2 \psi} \Big[\delta(\psi) + \delta(\psi - \pi) \Big] \delta(\cos\theta) \delta(\phi). \tag{4.7}$$

The right hand side represents the sourcing of the scalar field Φ by the charges, as required by supersymmetry. The fact that the two sources contribute with the same sign can be seen (as we will show below) from requiring that the solutions of these equations, together with the electric field, form a BPS configuration.

The equation of motion for the scalar field (4.7) is solved by taking (see the Appendix for more details)

$$\Phi(\psi) = \frac{Q}{\sin \psi} \, . \tag{4.8}$$

Measured with respect to the origin in spherical coordinates, the profile of the spike is given by $R \pm \Phi$, with R the radius of the giant graviton (see Figure 1). We would like to note that

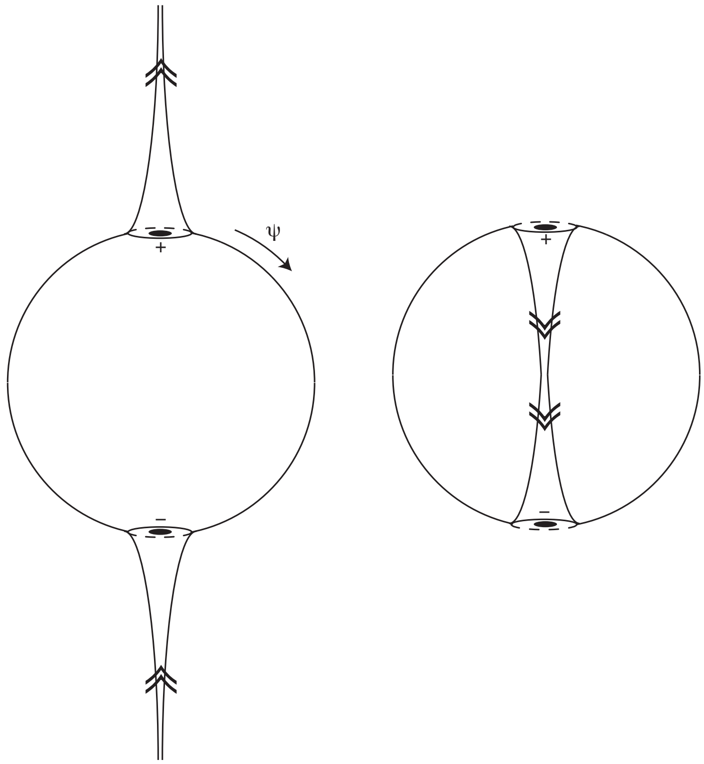

Figure 1: Two dipole configurations. The left one has an interpretation of two fundamental strings extending off the giant graviton to infinity. The configuration on the right is unstable and has no such string interpretation. The sign of the charges are indicated and the arrows denote the direction of flux of the electric field.

in our case the profile of Φ and the Coulomb potential Λ, (4.8) and (4.5) are different; this should be contrasted with the usual BIon case [18]. Near the north pole (with $\psi \approx 0$), the solution for the scalar field and gauge field are similar, while at the south pole ($\psi \approx \pi$), they differ by a sign, and for ψ away from the poles, each solution interpolates smoothly between the solutions in the two regions. The scalar field is blind to the sign of the charges which source it, while the gauge field is not.

The size of the throat, as seen by the DBI action, is Rg_{eff}, the coupling for the canonically normalized fields. Explicitly, the solution for the radial direction X is

$$X = R\left(1 \pm Q\frac{g_{eff}}{\sin\psi}\right). \tag{4.9}$$

Interestingly, the corrections to the shape of the strings arising from non-linearities are suppressed relative to the tree-level shape by the same scale that sets the throat size, i.e. g_{eff}. This basically means that the perturbative expansion of the light-cone Hamiltonian (up to quadratic order) is a good one as long as the size of the throat is much smaller than the giant graviton itself. The non-linearities and interaction terms can in fact modify the form of the potential around a given vacuum such that one solution, explicitly the solution corresponding to the choice of minus sign in (4.9), is destabilized, as happens to the dipole solution (diagram on right of Figure 1), where the strings enter the interior and meet. The energy of this arrangement can be lowered by moving the end-points of the strings closer to each other, and the charges at the ends would eventually annihilate, leaving behind a giant graviton with no strings attached. For the dipole where the strings extend off to infinity, i.e. the solution with plus sign in (4.9), the layout of the strings at opposite ends (the diagram on the left of Figure 1) is in fact a minimum, and remains so even when the interactions are included, with the interactions only modifying the profile of the string at the junction. Higher

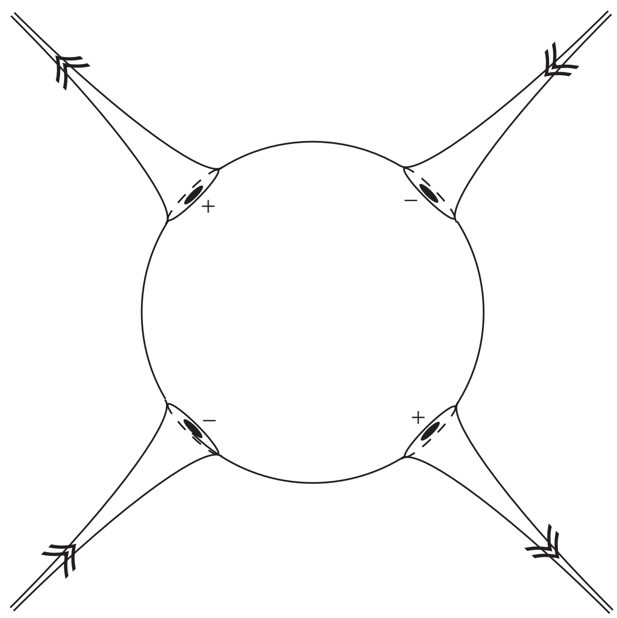

Figure 2: A quadrapole configuration of charges and the associated fundamental strings on the giant graviton.

pole solutions can be analogously constructed (see Figure 2 for a quadrapole, and Figure 3 for a more general "hedge-hog" configuration). When the coupling is of order the separation of the strings attached to the giant graviton, the end-points can meet and the string can separate from the giant graviton. From the gauge theory point of view, this corresponds to the charges annihilating each other. For finite coupling (and hence finite throat size), there will be a maximum number of strings which can attach to the giant graviton, and hence a maximum pole configuration in the gauge theory. The smallest size which can be effectively probed by the open strings is set by g_{eff}, and this leads to fuzziness of the giant gravitons in view of an open string probe.

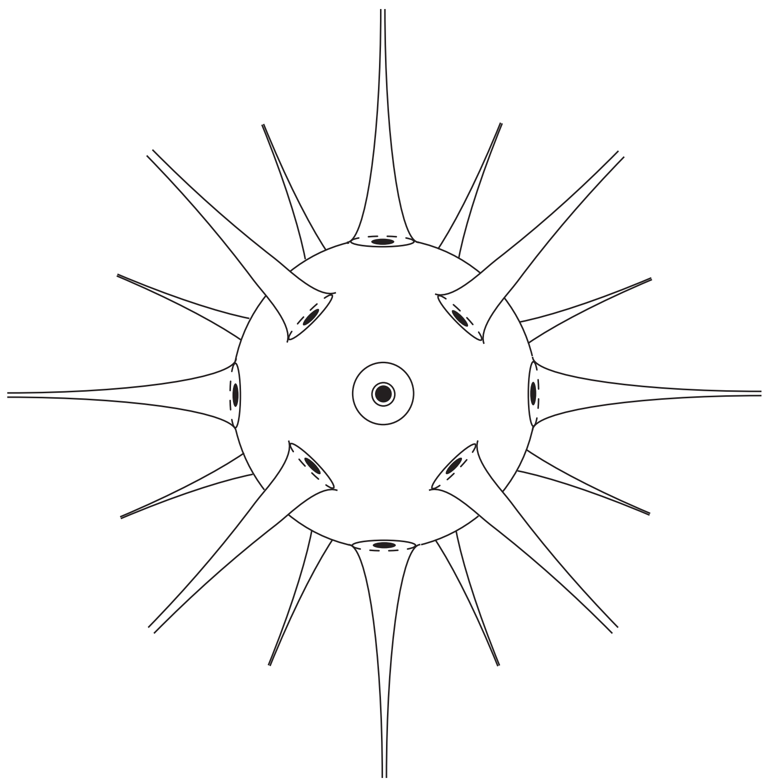

Figure 3: A generic giant graviton with multiple spikes, suggesting the hedge-hog title.

One may also consider configurations which are sourced by magnetic dipoles (and higher poles). Such configurations correspond to S-dual solutions to the ones considered above, where the strings ending on the giant graviton are D-strings. Dyonic configurations with both electric and magnetic sources can also be envisioned. More general configurations, where several giant gravitons are coincident, can also be constructed, and by analogy to the general case of coincident D-branes, would give rise a non-Abelian gauge theory on their worldvolume.

4.2 Energy

We would now like to give an interpretation to the spikes we found as solutions of the quadratic Hamiltonian in section 4.1. To do so we consider the energy of such a configuration. To find the energy density, we use the solutions (4.5) and (4.8) for the gauge field configuration and radial profile of the giant graviton in the Hamiltonian density, then integrate the density to find the total energy. For the dipole, there are two solutions, one for which the spike extends off to infinity away from the giant graviton, and one where the spikes enter the interior and join (see Figure 1). We consider both configurations. The resultant energy $\mathcal{E}$, in units of μ, for the first configuration is

$$\mathcal{E} = 4\pi Q^2 \cot\epsilon\,, \tag{4.10}$$

and the scalar and gauge fields contribute equally to the energy. We have integrated along the ψ direction from ϵ to $\pi/2$. The ϵ serves as a cut-off, since the total energy would diverge; we are interested in the scaling of this energy with length as the cut-off is removed. The $\pi/2$ captures one string (the other string would give an equal contribution). For small ϵ, the result scales as

$$\mathcal{E} \sim \Phi(\epsilon) \tag{4.11}$$

up to some fixed numerical coefficients. In other words, the energy per unit length is the same as the tension of the fundamental string.[8] We expect also that the spectrum of small fluctuations for the string should reproduce the spectrum of massive modes of the open fundamental string in the plane-wave background [9].

The profile where the "strings" enter into the interior of the giant graviton is given by $R - \Phi$, with Φ the same as for the outgoing strings, but the range of ψ is now restricted such that $R - \Phi$ is limited to only one "hemisphere" inside the giant graviton. At the lower cutoff for ψ, the string joins onto the other string originating at the other charge. In other words, $\sin^{-1}(g_{eff}) \leq \psi \leq \pi - \sin^{-1}(g_{eff})$ and only when the size of the throat is very small compared to the size of the giant graviton (i.e. when $g_{eff} \ll 1$) this spike can be interpreted as a fundamental string. For any finite value of g_{eff}, the profile never reaches that of a fundamental string.

In any case, as we have already discussed, this configuration is unstable (metastable). The dipole where the strings run off to infinity is stabilized by the fact that the endpoints

[8] In our units, the string tension $T \sim 1$.

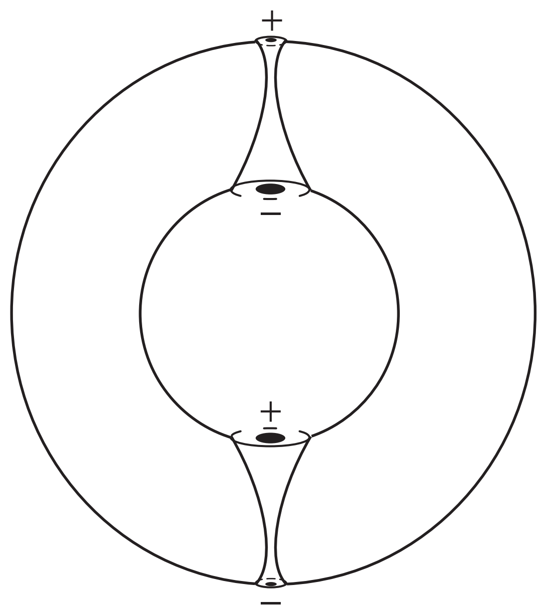

Figure 4: Two giant gravitons of different radii, connected by "strings". In the limit that the radii are equal and the giant gravitons become coincident, the gauge symmetry of the worldvolume theory is enhanced.

of the strings have their boundary conditions fixed at infinity, and any small perturbation of their junction increases the length of the string and hence the energy.

Given a total light-cone momentum p^+, one may distribute it among some number of giant gravitons, that is a configuration of concentric giant gravitons. One may wonder whether in the limit when two of these giant gravitons become coincident, analogously to the case of D-branes, one should expect enhancement of the $U(1)$ gauge symmetry to $U(2)$. That is possible if the metastable spikes (strings) depicted in Figure 4 become massless in the coincident limit, a fact which is confirmed by our energy analysis. The life-time of these spikes depends on g_{eff} as well as the difference in the radii of the two giant gravitons. Based on energy arguments we expect it to be proportional to the inverse of $\sqrt{g_s}$ as well as the difference of inverse radii squared of the two giant gravitons.

4.3 Supersymmetry

The spin connection one-forms on the three-sphere can be deduced from the metric (4.2)

$$\Omega^{12} = -\cos\psi d\theta\,, \qquad \Omega^{23} = -\cos\theta d\phi\,, \qquad \Omega^{31} = \cos\psi\sin\theta d\phi\,, \tag{4.12}$$

and the supersymmetry variation of the gaugino for the abelian theory is [28, 29]

$$\delta_\epsilon\lambda = \Big(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} - (\partial_\mu\Phi^m)\Gamma^m\Gamma^\mu - \frac{1}{2}\Phi_m\Gamma^m\Gamma^\mu\nabla_\mu\Big)\epsilon\,. \tag{4.13}$$

The derivatives on the fields are not gauge covariant since these fields transform in the adjoint of $U(1)$, and hence are neutral. Also, $\nabla_\mu = \partial_\mu + \frac{1}{4}\Omega^{ab}_\mu\Gamma_{ab}$, is the covariant derivative with the spin connection Ω^{ab}_μ.[9] Solutions ϵ in a given background, for which this variation vanishes, are the Killing spinors, and the number of such solutions gives the amount of supersymmetry preserved by the background. For the solutions we are considering, the deformation of the sphere is independent of the angular directions along an S^2 of the S^3, and preserves an $SU(2)$ symmetry of the $SO(4)$ invariant vacuum.

We make use of the solution for the electric field in terms of the scalar potential, where

$$F_{0\psi} = -\partial_\psi\Lambda(\psi) = \frac{Q}{\sin^2\psi}\,, \tag{4.14}$$

and $\Phi^m\Gamma^m = \Phi(\psi)\Gamma^r$ with r designating the radial direction in the transverse directions, and with $\Phi = \frac{Q}{\sin\psi}$. The Killing spinor equation for this background is [26]

$$\nabla_\mu\epsilon = \frac{1}{2}\Gamma^r\Gamma_\mu\epsilon\,, \tag{4.15}$$

and the square of the Killing spinors are the Killing vectors. The Killing spinor equation (4.15) has the maximal number of solutions [28]. For these Killing spinors, the condition for supersymmetry (4.13) reduces to[10]

$$\frac{Q}{\sin^2\psi}\Big(\Gamma^0 + \tilde{\Gamma}^r\Big)\epsilon' = 0\,, \tag{4.16}$$

with $\epsilon' = \Gamma^\psi\epsilon$, and

$$\tilde{\Gamma}^r = \cos\psi\Gamma^r + \sin\psi\Gamma^\psi$$

is a rotated Dirac matrix of unit norm, i.e. $(\tilde{\Gamma}^r)^2 = 1$. This implies that half the supersymmetries of the background plane-wave remain unbroken by the presence of the D-brane,

[9] Note that the indices a, b are with respect to the orthonormal tangent frame, while μ, ν are curved indices on the worldvolume and m ranges over the six $SO(6)$ components.

[10] With our metric conventions, $(\Gamma^0)^2 = -1$, with the other Dirac matrices squaring to one.

and the dipole configuration of the giant graviton state with the two spikes piercing it is 1/2 BPS, i.e. it preserves eight supercharges.

The $\tilde{\Gamma}^r$ matrix at $\psi = 0$ is Γ^r and hence the supersymmetry condition (4.16) is essentially that of the usual BIon [18] with, say a positive charge. At $\psi = \pi$, however, $\tilde{\Gamma}^r = -\Gamma^r$ reducing (4.16) to a usual BIon with negative charge and the term proportional to Φ in (4.13) makes it possible to have a smooth supersymmetric transition form a positive charge to a negative charge.

There also exists a solution of the quadratic Hamiltonian for which $\Phi = -\frac{Q}{\sin\psi}$, but with the gauge field configuration unchanged. The condition for the existence of supersymmetry for this configuration is

$$\frac{Q}{\sin^2\psi}\left(\Gamma^0 - \tilde{\Gamma}^r\right)\epsilon' = 0\,, \tag{4.17}$$

which differs from (4.16) by the relative sign between the time-like and radial Dirac matrices, but preserves precisely the same amount of supersymmetry as the original configuration, and is also 1/2 BPS. One should note that being BPS does not necessarily guarantee the stability of the solution, particularly when the interaction terms in the light-cone Hamiltonian are taken into account.[11]

5 Outlook and future directions

In this paper we have analyzed some aspects of the worldvolume theory of giant gravitons on the plane-wave background. Working out the spectrum of small fluctuations of the giant three-sphere, we argued that they fall into (short) multiplets of the $PSU(2|2) \times PSU(2|2) \times U(1)$ algebra. One of the interesting features of the three-brane light-cone Hamiltonian (2.14) is the natural appearance of Nambu brackets (2.16). In this point of view "quantization" of Nambu bracket (2.16) would provide us with a natural quantization of the theory living on the giant graviton. In the membrane case the corresponding Nambu bracket is essentially a Poisson bracket and its quantization is possible by replacing the bracket with Matrix commutators [30]. This quantization of Possion brackets from the membrane point of view can be regarded as discretization of the worldvolume, which also leads to the "noncommutative"

[11] In principle the statement that BPS configurations are protected should be taken with a grain of salt. It is possible that some multiplets which are BPS at a given value of coupling combine into a long (ordinary non-BPS) multiplet and receive corrections. For explicit examples and more detailed discussion on this point see [22].

(non-Abelian) structure of the BFSS matrix model. In the same trend quantization of the three-sphere giant graviton theory may provide us with an answer to the puzzle of finding a holographic description of type IIB string theory on the plane-wave background, which supposedly is a Matrix theory [31] (for a summary of discussions on the matter see section IX of [16]).

As another aspect of the gauge theory living on giant gravitons, we studied static configurations which source the gauge fields and also the scalar fields. The basic building blocks of such objects are dipole configurations with the largest possible dipole moment being proportional to the size of the giant graviton. We argued that the BPS dipole configurations, from the bulk viewpoint, can be understood as open strings ending on the giant graviton. These are open strings with their two ends on the north and south pole of the three-sphere. It is evident from our construction that these open strings satisfy Dirichlet boundary conditions in the directions transverse to the brane, a natural expectation generalizing Polchinski's D-brane picture [27]. We also argued that it is possible to have dipole configurations with the spike going inside the three-sphere. These states, although being metastable, are responsible for enhancing the gauge symmetry when two concentric giant gravitons become coincident.

As we discussed, since at finite g_{eff} the size of the throat of the spikes is finite, one would physically expect to have an upper limit on the highest multiple moment. In other words there is a minimum area which can be probed using these open strings and also there is a minimum size dipole moment. This suggests that the fuzzy three sphere [32] is the right description of the quantized giant graviton [31]. A description of multiple coincident giant gravitons in terms of a non-commutative three-sphere defined as a Hopf fibration over a fuzzy two-sphere is given in [33].

As a direct generalization of our giant hedge-hog configurations one can consider circular D-strings in the $AdS_3 \times S^3$ background (or the corresponding Penrose limit [34]). In that case, however, we expect that similar to the flat D-string case [35], the spike touches the giant circle to form a "string junction". This leads to a pair of three string junctions, two of the legs of each are connected and make a deformed half circle. This construction can then be generalized to junctions of (p, q) strings and string networks [36] in plane-wave backgrounds [37].

Acknowledgements

We would like to thank Keshav Dasgupta, Michal Fabinger, Sergey Prokushkin, and especially Leonard Susskind for fruitful discussions. The work of M. M. Sh-J. is supported in part by NSF grant PHY-9870115 and in part by funds from the Stanford Institute for Theoretical Physics. The work of D. S. is supported by the Department of Energy, Contract DE-AC03-76SF00515.

Appendix: $SO(4)$ Harmonics in terms of usual Y_{lm}'s

The Laplacian on the three-sphere in the coordinate system we have adopted is

$$\nabla^2_{S^3} = \frac{1}{\sin^2\psi}\partial_\psi\left(\sin^2\psi\partial_\psi\right) + \frac{1}{\sin^2\psi}\nabla^2_{S^2}\,, \tag{A.1}$$

where $\nabla^2_{S^2} = \frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) + \frac{1}{\sin\theta}\partial^2_\phi$. One may use (A.1) to write $SO(4)$ harmonics in terms of the $SO(3)$ Y_{lm}'s. Explicitly, let us consider the (source free) equation of motion for the Coulomb potential Λ:

$$\nabla^2_{S^3}\,\Lambda(\psi,\theta,\phi) = 0\,. \tag{A.2}$$

Separating variables as $\Lambda(\psi,\theta,\phi) = \Lambda_l(\psi)Y_{lm}(\theta,\phi)$, (A.2) can be cast in the form

$$\frac{1}{\sin^2\psi}\partial_\psi\left(\sin^2\psi\Lambda_l\right) - \frac{1}{\sin^2\psi}l(l+1)\Lambda_l = 0\ . \tag{A.3}$$

After the change of variable $u = \cot\psi$, (A.3) takes the form

$$(1+u^2)\Lambda_l'' - l(l+1)\Lambda_l = 0\ , \tag{A.4}$$

where $\Lambda' = \frac{d}{du}\Lambda$. For $l=0$, (A.4) is simply solved by $\Lambda_0 = u = \cot\psi$ (the solution we have already discussed as the dipole (4.5)), and for $l=1$, $\Lambda_1 = 1+u^2 = 1/\sin^2\psi$. For general l, (A.4) can be solved using a series expansion for $\Lambda_l(u)$

$$\Lambda_l(u) = \sum_{k=0}^{l+1} a_k u^k\,,$$

where $a_{l+1} = 1$, $a_l = 0$ and

$$a_k = \frac{(k+1)(k+2)}{l(l+1)-k(k-1)}\ a_{k+2}\ , \qquad 0\le k\le l-1\ .$$

Similarly, solutions to the equation for the scalar field, namely $\nabla^2_{S^3}\Phi - \Phi = 0$ can be decomposed as

$$\Phi = \Phi_l(\psi)Y_{lm}(\theta,\phi)\,.$$

Taking $v = 1/\sin\psi$ and $\frac{d}{dv}\Phi = \Phi'$, then

$$v^2(v^2-1)\Phi_l'' + v\Phi_l' - [l(l+1)v^2+1]\Phi_l = 0 \ . \tag{A.5}$$

For the $l = 0$ case, as we discussed in (4.8), the solution is $\Phi_{l=0} = v$, and for general l, as in the previous case, (A.5) may be solved using Taylor expansion techniques, inserting

$$\Phi_l(v) = \sum_{k=0}^{l+1} b_k v^k$$

into the equation. It turns out that (A.5) has only solutions for *even* l with $b_0 = 0$, $b_{l+1} = 1$, and

$$b_k = -\frac{(k+1)^2}{l(l+1)-k(k-1)}\ b_{k+2}\ , \qquad 1 \le k \le l-1\ .$$

The fact that (A.5) has (polynomial) solutions only for even l can physically be understood by noting that the source term for the scalars is a *sum* of delta-functions (whereas that of the Coulomb potential is an alternating sum, so that the total net charge is zero).

Finally, we would like to mention that in our expansions the 2^{l+1}-poles of $SO(4)$ are related to Y_{lm} (i.e. 2^l-pole of $SO(3)$). For example, our "dipole" configurations correspond to the $l = 0$ case. Also note that the dipole configuration can be thought of as a Dirac string on the sphere where $\psi = 0$ corresponds to the monopole and $\psi = \pi$ corresponds to the end of the Dirac string tail, which in the flat space language is at infinity.

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